

Problem of the Week

Problem A and Solution

Fencepost Problem

Problem

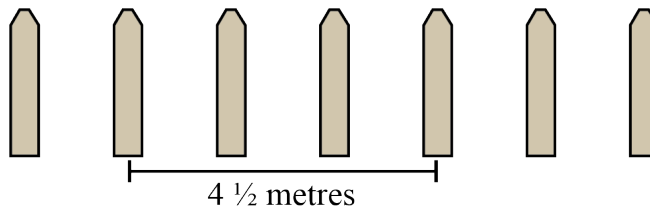
Elyas helps his parents install fence posts in their backyard. They install a row of 7 equally-spaced posts. The posts are all the same size. The distance between the middle of the 2nd post and the middle of the 5th post is four and a half meters.

What is the distance between the middle of the first post and the middle of the last post?

HINT: You might start by drawing a diagram of the fence posts and labelling it with the distance you know.

Solution

There are many ways to calculate the distance. We start with a diagram of the posts, labelled with the distance of $4\frac{1}{2}$ metres between the middle of the 2nd and 5th posts.



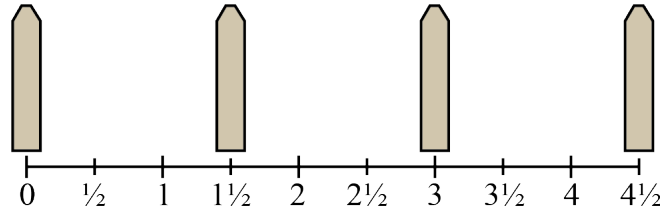
Solution 1

Notice that there are a total of three gaps between the 2nd and 5th posts. There are also a total of three gaps between the 1st and 2nd posts and the 5th and 7th posts. Since the distances between the middle of adjacent posts are all the same, then the total distance between the 1st and 2nd posts and the 5th and 7th posts must also be $4\frac{1}{2}$ m. Since there are six gaps in total, the distance between the middle of the first post and the middle of the last post is $4 + \frac{1}{2} + 4 + \frac{1}{2} = 9$ m.



Solution 2

Another way to solve this problem is to draw a number line from 0 to $4\frac{1}{2}$ m and space the 2nd, 3rd, 4th, and 5th fenceposts evenly along this line.

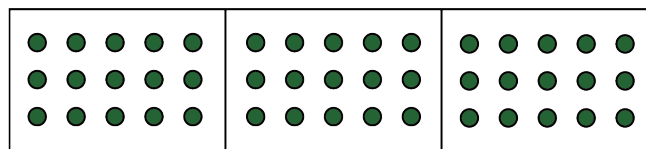


From this, we observe that the distance from the middle of one post to the middle of an adjacent post is $1\frac{1}{2}$ m. Since there are six gaps in total between adjacent posts, the distance between the middle of the first post and the middle of the last post is:

$$1 + \frac{1}{2} + 1 + \frac{1}{2} + 1 + \frac{1}{2} + 1 + \frac{1}{2} + 1 + \frac{1}{2} + 1 + \frac{1}{2} = 9 \text{ m}$$

Solution 3

First we convert the distance to centimetres: $4\frac{1}{2}$ m = 450 cm. Then we use a tape diagram. We observe that there are 3 gaps between the 2nd and 5th posts. That means we need to divide 450 cm into three equal distances. To make it easier, since $450 = 45 \times 10$, we let one dot represent 10 cm, and then distribute 45 dots into the three equal pieces of our tape diagram.



Each piece has a total of 15 dots. This means the distance between adjacent posts is $15 \times 10 = 150$ cm. Since there are 6 gaps in total between the first post and the last post, the total distance is:

$$150 + 150 + 150 + 150 + 150 + 150 = 900 \text{ cm or } 9 \text{ m}$$