Problem of the Week

Problems and Solutions 2024-2025

Problem A

Grade 3/4

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

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Algebra (A)

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Problem A
Tapping to Success

For their dance unit, Yelena, Jackson, and Todd had to come up with a 30 second dance routine. All three of them are tap dancers, so they decided that each of them would do a 5 second solo tap, and the remaining 15 seconds would be a group tap.

During Yelena's solo tap section, she tapped at a rate of 5 taps per second.

During Jackson's solo tap section, he tapped at a rate of 4 taps per second.

During Todd's solo tap section, he tapped at a rate of 3 taps per second.

For their group tap section, they all tapped at the slowest rate so that they could all keep up.

How many taps did each dancer do in total during the full routine?

Problem of the Week
Problem A and Solution
Tapping to Success

Problem

For their dance unit, Yelena, Jackson, and Todd had to come up with a 30 second dance routine. All three of them are tap dancers, so they decided that each of them would do a 5 second solo tap, and the remaining 15 seconds would be a group tap.

During Yelena's solo tap section, she tapped at a rate of 5 taps per second.

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During Todd's solo tap section, he tapped at a rate of 3 taps per second.

For their group tap section, they all tapped at the slowest rate so that they could all keep up.

How many taps did each dancer do in total during the full routine?

Solution

We use a table to keep track of how many taps each dancer did during their solo.

Dancer	Total Taps after 1 s	Total Taps after 2 s	Total Taps after 3 s	Total Taps after 4 s	Total Taps after 5 s
Yelena	5	10	15	20	25
Jackson	4	8	12	16	20
Todd	3	6	9	12	15

Since they all tapped at the slowest rate for the group tap, then they all tapped at 3 taps per second, which is the same as Todd's rate in the solo. From the table above, we know that in 5 seconds, Todd tapped 15 times. Then at this rate, in 10 seconds they would tap 30 times, and in 15 seconds they would tap 45 times.

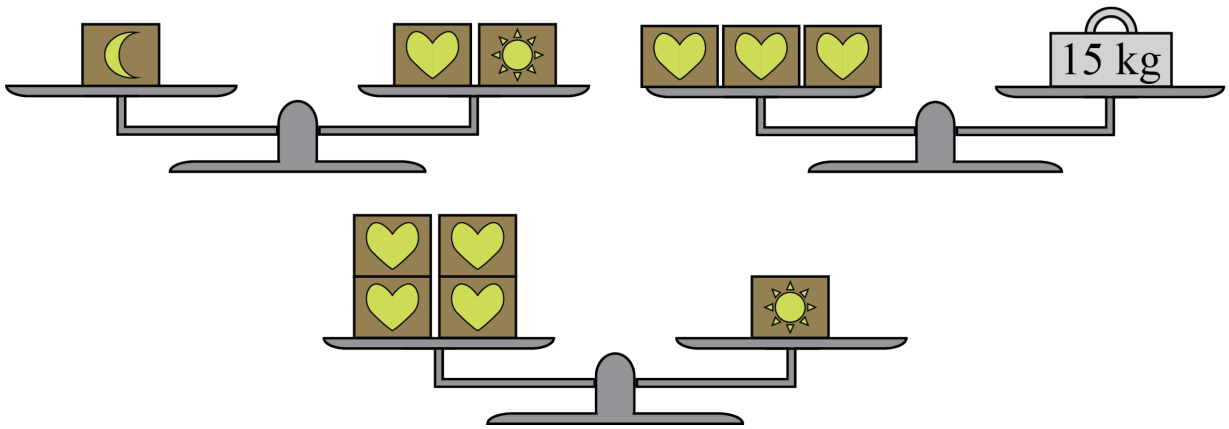
Then we can conclude the following:

- Yelena tapped a total of $25 + 45 = 70$ times during the full routine.
- Jackson tapped a total of $20 + 45 = 65$ times during the full routine.
- Todd tapped a total of $15 + 45 = 60$ times during the full routine.

Problem of the Week
Problem A
Balancing Act

James is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass.

The following diagrams show what James observed when arranging some of the boxes and standard weights on a scale.



Given that each scale is balanced, determine the mass of each box.

Theme: Algebra

Problem of the Week

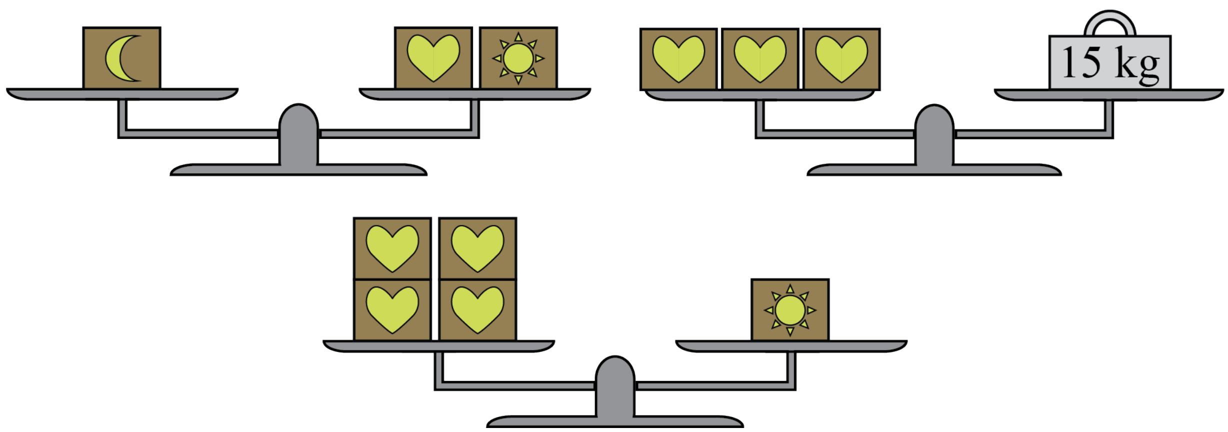
Problem A and Solution

Balancing Act

Problem

James is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass.

The following diagrams show what James observed when arranging some of the boxes and standard weights on a scale.



Given that each scale is balanced, determine the mass of each box.

Solution

From the diagrams we notice the following.

- One moon box has the same mass as the sum of the mass of a heart box and the mass of a sun box.
- Three heart boxes have a total mass of 15 kg.
- Four heart boxes have the same total mass as one sun box.

Since 3 heart boxes have a total mass of 15 kg, then the mass of 1 heart box must be $\frac{1}{3}$ of 15 kg. Therefore, 1 heart box has a mass of 5 kg.

Since 4 heart boxes have the same mass as 1 sun box, then 1 sun box must have a mass of $5 \times 4 = 20$ kg.

Since 1 moon box has the same mass as the sum of the mass of 1 heart box and the mass of 1 sun box, then 1 moon box must have a mass of $20 + 5 = 25$ kg.

Therefore, 1 moon box has a mass of 25 kg, 1 heart box has a mass of 5 kg, and 1 sun box has a mass of 20 kg.

Problem of the Week

Problem A

Delivery Dilemma

Soraya and Aydin both work as delivery drivers for a local business. Today they have 64 packages to deliver in total. Soraya was given three times as many packages as Aydin to deliver.

They decide it would be better for each person to deliver the same number of packages. How many packages should Soraya give Aydin so that they have the same number of packages?

Themes: Algebra, Number Sense

Problem of the Week

Problem A and Solution

Delivery Dilemma

Problem

Soraya and Aydin both work as delivery drivers for a local business. Today they have 64 packages to deliver in total. Soraya was given three times as many packages as Aydin to deliver.

They decide it would be better for each person to deliver the same number of packages. How many packages should Soraya give Aydin so that they have the same number of packages?

Solution

Solution 1

One way to solve the problem is to guess and check to figure out how many packages were given to each driver. We can organize our guesses in a table where we keep track of how many packages Aydin has, how many packages Soraya has, and how many packages they have in total. Let's start by guessing that Aydin has 10 packages until we find a combination that results in a total of 64 packages.

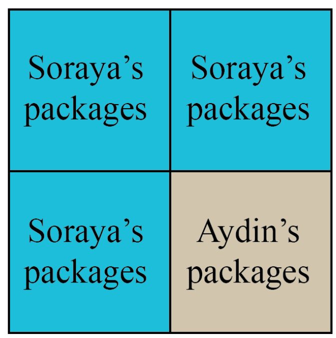
Aydin's Packages	Soraya's Packages	Total Packages
10	$3 \times 10 = 30$	$10 + 30 = 40$
11	$3 \times 11 = 33$	$11 + 33 = 44$
12	$3 \times 12 = 36$	$12 + 36 = 48$
13	$3 \times 12 = 39$	$13 + 39 = 52$
14	$3 \times 14 = 42$	$14 + 42 = 56$
15	$3 \times 15 = 45$	$15 + 45 = 60$
16	$3 \times 16 = 48$	$16 + 48 = 64$

Thus, we see that Aydin started with 16 packages and Soraya started with 48 packages. If they want to deliver the same number of packages, each should take half of the total number of packages. Half of 64 is 32 packages.

So if Soraya gives Aydin $48 - 32 = 16$ packages, then Aydin will have $16 + 16 = 32$ packages and each of them will have the same number to deliver.

Solution 2

Another way to solve this problem is to use fractions. If Soraya was given three times as many packages as Aydin to deliver, then adding Soraya and Aydin's packages together should give us four times as many packages as Aydin has. This means that Aydin has $\frac{1}{4}$ of the total number of packages and Soraya has $\frac{3}{4}$ of the total number of packages. This is shown in the following diagram. The large square represents the total number of packages. The large square is divided into quarters, with three of the quarters representing Soraya's packages and one of the quarters representing Aydin's packages.



In order for Aydin and Soraya to each have the same number of packages, Soraya must give Aydin $\frac{1}{4}$ of the total number of packages, so that they each have $\frac{1}{2}$. Since there are 64 packages in total, $\frac{1}{4}$ of 64 is equal to $64 \div 4 = 16$. Thus, Soraya should give Aydin 16 packages.

Problem of the Week
Problem A
Circular Calculations

Lorna uses the following instructions to write sequences of numbers.

Step 1: Start with a whole number greater than 0.

Step 2: If the number is even, divide it by 2 to get the next number. If the number is odd, multiply it by 3 and add 1 to get the next number.

Step 3: Repeat Step 2 to continue the sequence.

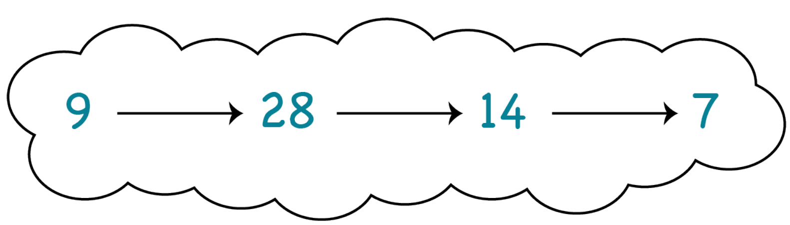
For example, suppose Lorna starts with 9.
Since this number is odd, the next number is $9 \times 3 + 1 = 27 + 1 = 28$.
Since this number is even, the next number is $28 \div 2 = 14$.
Since this number is even, the next number is $14 \div 2 = 7$.
Thus, the first four numbers in the sequence are 9, 28, 14, and 7.

(a) Follow the instructions using the given starting number and write the first 12 numbers in each sequence.

(i) 3 (ii) 13

(b) What do you notice about each sequence in part (a)? What would happen if you continued each sequence?

(c) Use your answer to part (b) to predict the 20^{th} number in the sequence starting with 13.



Themes: Algebra, Computational Thinking

Problem of the Week
Problem A and Solution
Circular Calculations

Problem

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(a) Follow the instructions using the given starting number and write the first 12 numbers in each sequence.
(i) 3 (ii) 13

(b) What do you notice about each sequence in part (a)? What would happen if you continued each sequence?

(c) Use your answer to part (b) to predict the 20^{th} number in the sequence starting with 13.

Solution

(a) The first 12 numbers in each sequence are as follows.

(i) 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4

(ii) 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2

(b) After each sequence reaches 1, the numbers 4, 2, and 1 repeat over and over.

(c) We can write the first 20 numbers in the sequence starting with 13 by first using part (b) to write out the first 12 numbers in the sequence, and then repeating 4, 2, and 1 until we have 20 numbers.

$$13,\ 40,\ 20,\ 10,\ 5,\ 16,\ 8,\ 4,\ 2,\ 1,\ 4,\ 2,\ 1,\ 4,\ 2,\ 1,\ 4,\ 2,\ 1,\ 4$$

Thus, the 20^{th} number is 4.

Teacher's Notes

As an extra challenge, students could attempt this starting with the number 27. Although it takes a long time when you start with the number 27, the sequence will eventually reach 1. It takes 111 numbers to reach 1, as shown.

$27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1$

Mathematicians believe that starting with any positive integer, the instructions will always lead to a sequence that reaches 1 (or *converges* to 1). This is known as the *Collatz Conjecture*. However, proving this is true for all positive integers is an open problem. There is experimental evidence that shows this is true for very large numbers; however, there is no formal proof that the conjecture holds for all positive integers.

Computational Thinking (C)

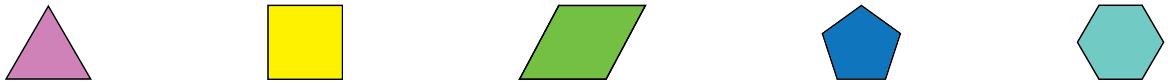
Take me to the cover

Problem of the Week

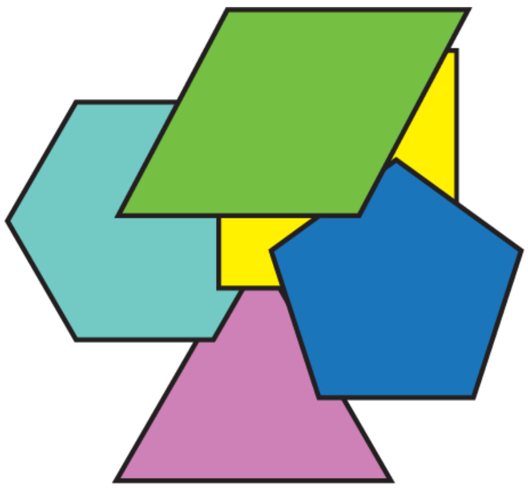
Problem A

Stacking Shapes

Anna cut out the following five polygons.



She then placed them on a table. The top view after doing so is shown.

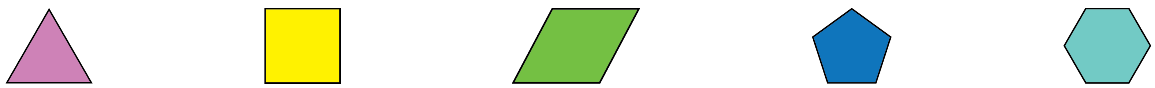


In what order did she place the polygons on the table?

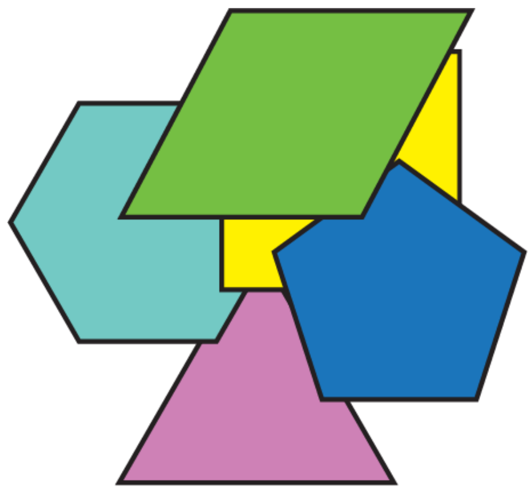
Problem of the Week
Problem A and Solution
Stacking Shapes

Problem

Anna cut out the following five polygons.



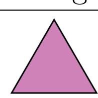
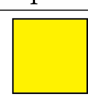
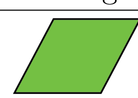
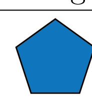
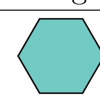
She then placed them on a table. The top view after doing so is shown.



In what order did she place the polygons on the table?

Solution

First, we recall the names of the five polygons.

Name	Triangle	Square	Parallelogram	Pentagon	Hexagon
Image					

Since the parallelogram has no shapes covering part of it, it must have been the last polygon placed on the table.

The pentagon has the parallelogram covering part of it, so the pentagon must have been placed before the parallelogram.

The square has the pentagon and parallelogram covering part of it, so the square must have been placed before the pentagon and parallelogram.

The hexagon has the square and parallelogram covering part of it, so the hexagon must have been placed before the square and parallelogram.

The triangle has the hexagon, square, and pentagon covering part of it, so the triangle must have been placed before the hexagon, square, and pentagon.

Thus, Anna must have first placed the triangle, then the hexagon, then the square, then the pentagon, then the parallelogram.

Problem of the Week
Problem A
What Number Am I?

I am a 3-digit number.

The sum of my digits is 11.

The product of my digits is 16.

My digits are in decreasing order from the hundreds digit to the ones digit.

I have no repeated digits.

What number am I?

Themes: Computational Thinking, Number Sense

Problem of the Week
Problem A and Solution
What Number Am I?

Problem

I am a 3-digit number.

The sum of my digits is 11.

The product of my digits is 16.

My digits are in decreasing order from the hundreds digit to the ones digit.

I have no repeated digits.

What number am I?

Solution

We start by determining the ways to multiply three single-digits to get a product of 16. Here are the possibilities:

$1 \times 4 \times 4$ (in any order)
$1 \times 2 \times 8$ (in any order)
$2 \times 2 \times 4$ (in any order)

Since the number we are looking for does not have any repeated digits, then the digits in the number must be 1, 2, and 8. We can confirm this conclusion by noticing that the sum of these digits is $1 + 2 + 8 = 11$.

Since the digits appear in decreasing order, the number must be 821.

Problem of the Week

Problem A

Entry Code

Janet has a safe that can be opened with a 4-digit code. Janet has set up her safe to open for any 4-digit code that satisfies the following rules:

1. The first digit and the last digit cannot be equal to each other.
2. The second digit must be greater than the third digit.
3. The last digit must be greater than either the third digit or greater than the first digit.
4. At least one digit must be an even number.

Which of the following codes would unlock the safe? Justify your answers.

(a) 1234

(b) 4321

(c) 5313

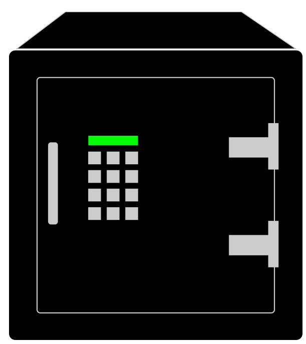
(d) 2644

(e) 3333

(f) 5312

(g) 7437

(h) 5857



Problem of the Week
Problem A and Solution
Entry Code

Problem

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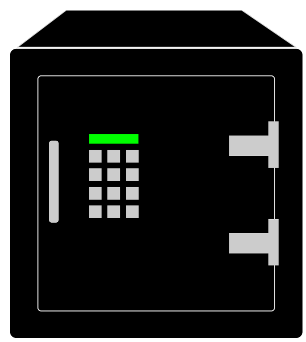


Solution

We can check each code to see if satisfies the given rules.

(a) 1234 would *not* unlock the safe because the second digit is less than the third digit, so it does not follow rule 2.

(b) 4321 would *not* unlock the safe because the last digit is less than both the first digit and the third digit, so it does not follow rule 3.

(c) 5313 would *not* unlock the safe because there are no even digits, so it does not follow rule 4.

(d) 2644 would unlock the safe.

(e) 3333 would *not* unlock the safe because it breaks all four rules.

(f) 5312 would unlock the safe.

(g) 7437 would *not* unlock the safe because the first and last digit are equal to each other, so it does not follow rule 1.

(h) 5857 would unlock the safe.

Problem of the Week

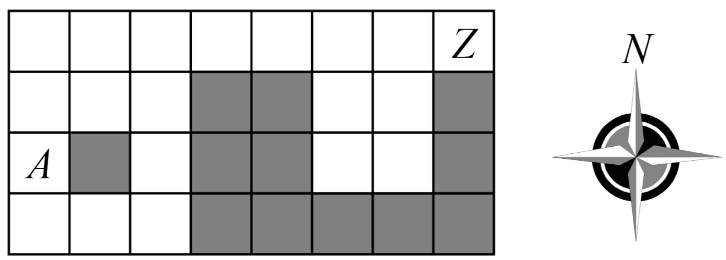
Problem A

Amazing Navigation

Juanita and AJ create mazes on grid paper. Each maze is a rectangular grid containing white squares and grey squares. One white square is marked A and another is marked Z.

To complete a maze, they start at A and need to reach Z by moving one square at a time in one of the following directions: north (N), east (E), south (S), or west (W), where the top of the page is considered north. They *cannot* go through any of the grey squares and must go through each of the white squares *exactly once*. That is, they must go through all of the white squares but cannot go through any of them more than once.

(a) Determine the directions they need to follow to successfully complete the given maze.



(b) AJ creates another maze by changing where the grey squares are in the maze from part (a). (The locations of A and Z remain unchanged.) Juanita successfully completes this new maze by following these directions:

$$E,\quad S,\quad E,\quad E,\quad E,\quad N,\quad E,\quad N,\quad W,\quad W,$$
$$W,\quad W,\quad N,\quad E,\quad E,\quad E,\quad E,\quad E,\quad E$$

What does AJ's maze look like?

Themes: Computational Thinking, Geometry & Measurement

Problem of the Week

Problem A and Solution

Amazing Navigation

Problem

Juanita and AJ create mazes on grid paper. Each maze is a rectangular grid containing white squares and grey squares. One white square is marked A and another is marked Z.

To complete a maze, they start at A and need to reach Z by moving one square at a time in one of the following directions: north (N), east (E), south (S), or west (W), where the top of the page is considered north. They *cannot* go through any of the grey squares and must go through each of the white squares *exactly once*. That is, they must go through all of the white squares but cannot go through any of them more than once.

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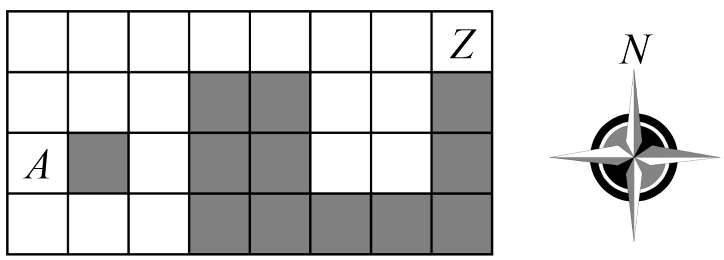


(b) AJ creates another maze by changing where the grey squares are in the maze from part (a). (The locations of A and Z remain unchanged.) Juanita successfully completes this new maze by following these directions:

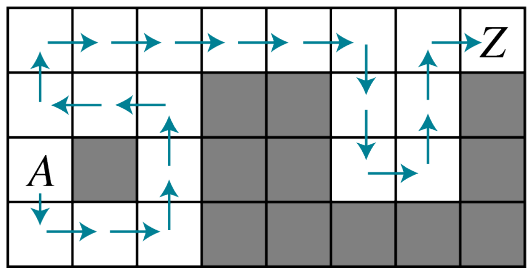
$$E, \quad S, \quad E, \quad E, \quad E, \quad N, \quad E, \quad N, \quad W, \quad W,$$
$$W, \quad W, \quad N, \quad E, \quad E, \quad E, \quad E, \quad E, \quad E$$

What does AJ's maze look like?

Solution

(a) The first direction must be either N or S, because there is a grey square to the right of A. Suppose they start with N. Then at some point, they must go through the square that is directly below A. However, once they reach that square they will be stuck because they can't go through any white square more than once. So the first direction must be S.

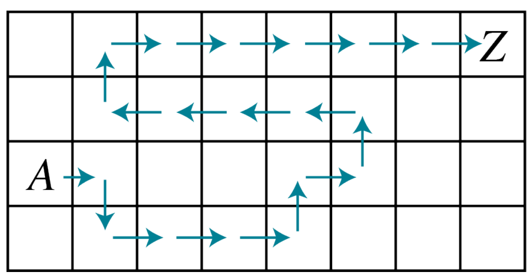
Using similar reasoning, we can complete the maze as shown.



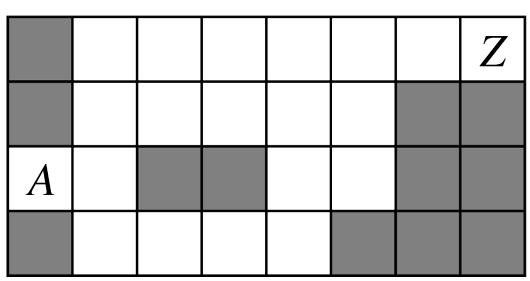
The directions followed are then

$$S, \quad E, \quad E, \quad N, \quad N, \quad W, \quad W, \quad N, \quad E, \quad E,$$
$$E, \quad E, \quad E, \quad S, \quad S, \quad E, \quad N, \quad N, \quad E$$

(b) We first mark the given directions on the maze, as shown.



Since Juanita cannot go through any of the grey squares, and must go through each of the white squares exactly once, the squares that are blank after marking the path must be the grey squares. Thus, AJ's maze is as shown.



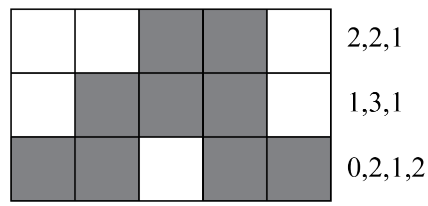
Problem of the Week

Problem A

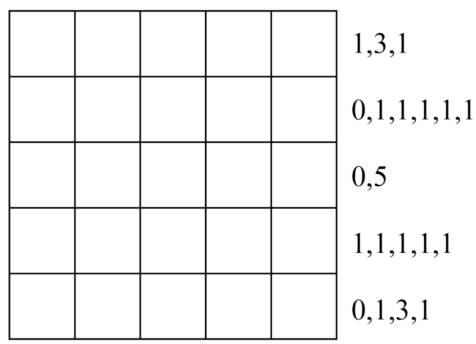
Picture Pixels

Akhil sends codes to his friends that tell them how to shade in squares of a grid to reveal a secret picture. Each code contains a blank grid with a list of numbers next to each row, which are used to shade in the row from left to right. The first number in a list represents the number of blank squares at the beginning of the row. The second number represents the number of squares that need to be shaded next. After that, the numbers alternate between the number of blank squares and the number of shaded squares until you reach the end of the row.

For example, if the list of numbers was 2, 2, 1, then the leftmost 2 squares would be blank, the next 2 squares would be shaded, and the next square would be blank. A completed picture is shown.

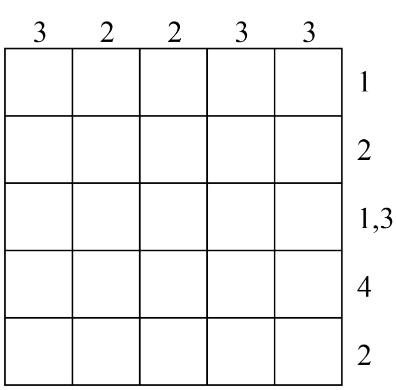


(a) Draw the picture for the given code.

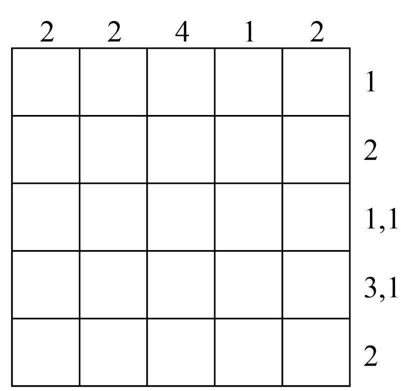


(b) Akhil turns some of his codes into puzzles. For each column, he adds a list of numbers representing the number of blank and shaded squares, as he has for the rows. Then, for each row and column, he removes the numbers representing blank squares. Two of his puzzles are shown. Try to solve them.

(i)



(ii)



Theme: Computational Thinking

Problem of the Week

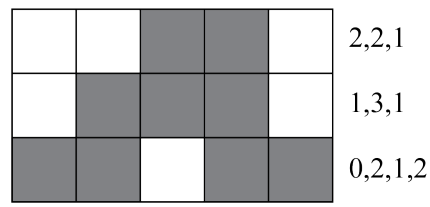
Problem A and Solution

Picture Pixels

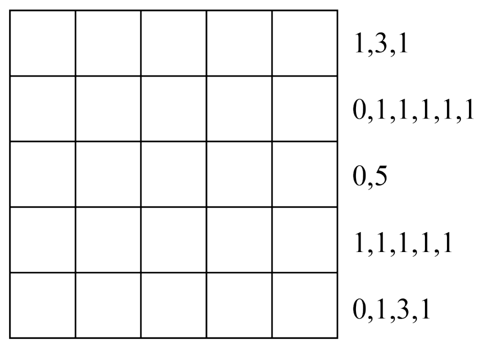
Problem

Akhil sends codes to his friends that tell them how to shade in squares of a grid to reveal a secret picture. Each code contains a blank grid with a list of numbers next to each row, which are used to shade in the row from left to right. The first number in a list represents the number of blank squares at the beginning of the row. The second number represents the number of squares that need to be shaded next. After that, the numbers alternate between the number of blank squares and the number of shaded squares until you reach the end of the row.

For example, if the list of numbers was 2, 2, 1, then the leftmost 2 squares would be blank, the next 2 squares would be shaded, and the next square would be blank. A completed picture is shown.

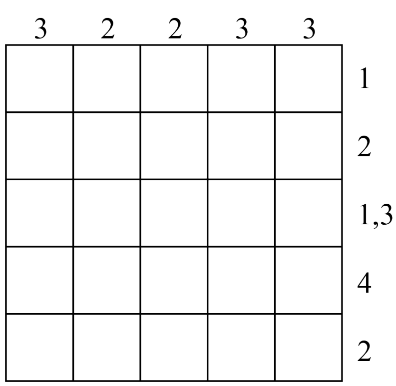


(a) Draw the picture for the given code.

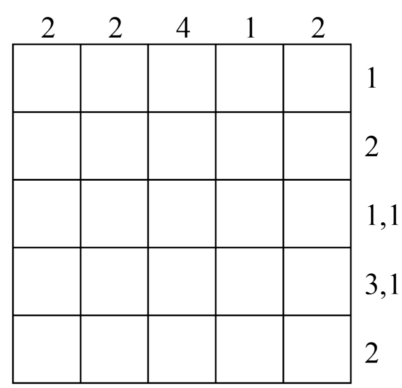


(b) Akhil turns some of his codes into puzzles. For each column, he adds a list of numbers representing the number of blank and shaded squares, as he has for the rows. Then, for each row and column, he removes the numbers representing blank squares. Two of his puzzles are shown. Try to solve them.

(i)

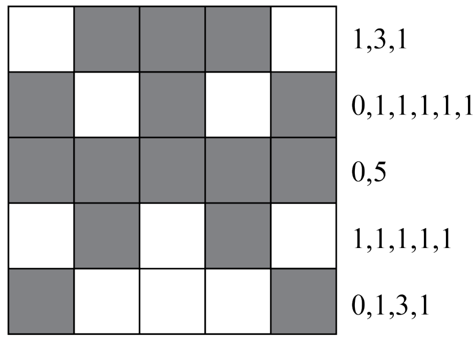


(ii)



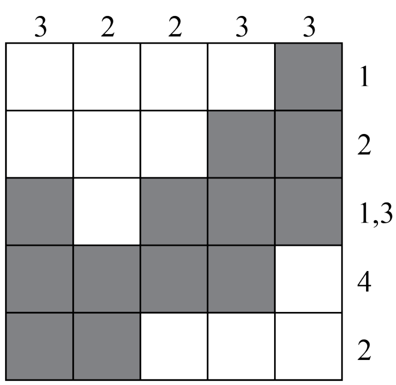
Solution

(a) The completed picture is shown.

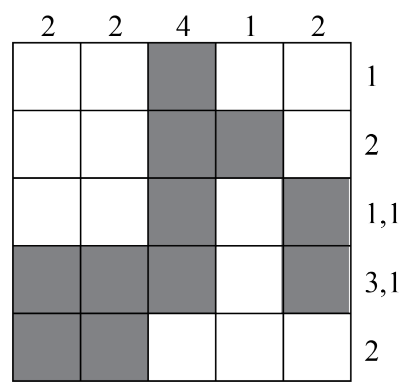


(b) For each puzzle, start by identifying any squares that you know must be either blank or filled. For example, for puzzle (i), the row with list 1, 3 must have at least one blank between the 1 shaded square and the 3 shaded squares. Since there are 5 squares in the row, this means that, from left to right, the row must contain one shaded square, then one blank square, then three shaded squares. It also helps to mark the blank squares with a dot. The completed puzzles are shown. Note that these kind of puzzles are often called *Nonograms*.

(i)



(ii)



Teacher's Notes

The original code in this problem uses *run length encoding* to describe images. Run length encoding is a way to describe large amounts of data in a more compact way; it is an example of a *compression* technique.

This particular compression technique was used by fax machines to send information more efficiently. Since fax images were black and white, rather than sending the image data one pixel at a time, it took less data to identify the image by grouping white pixels and black pixels in each row as is shown in this problem.

Although fax machines are not used much these days, run length encoding is still a technique used by computer scientists to help reduce the size of large data to be more manageable. One place where it is used is to describe information in gene sequences. Genes can be described as long sequences made up of four different proteins: adenine (A), thymine (T), cytosine (C), and guanine (G). The long sequences that describe the building blocks of DNA can be compressed by grouping common protein letters and using the count of how many of the same letter appears consecutively.

Problem of the Week

Problem A

Circular Calculations

Lorna uses the following instructions to write sequences of numbers.

Step 1: Start with a whole number greater than 0.

Step 2: If the number is even, divide it by 2 to get the next number. If the number is odd, multiply it by 3 and add 1 to get the next number.

Step 3: Repeat Step 2 to continue the sequence.

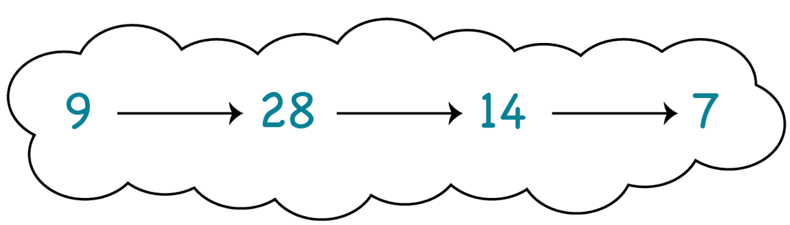
For example, suppose Lorna starts with 9.
Since this number is odd, the next number is $9 \times 3 + 1 = 27 + 1 = 28$.
Since this number is even, the next number is $28 \div 2 = 14$.
Since this number is even, the next number is $14 \div 2 = 7$.
Thus, the first four numbers in the sequence are 9, 28, 14, and 7.

(a) Follow the instructions using the given starting number and write the first 12 numbers in each sequence.

(i) 3 (ii) 13

(b) What do you notice about each sequence in part (a)? What would happen if you continued each sequence?

(c) Use your answer to part (b) to predict the 20^{th} number in the sequence starting with 13.



Themes: Algebra, Computational Thinking

Problem of the Week
Problem A and Solution
Circular Calculations

Problem

Lorna uses the following instructions to write sequences of numbers.

Step 1: Start with a whole number greater than 0.

Step 2: If the number is even, divide it by 2 to get the next number.
If the number is odd, multiply it by 3 and add 1 to get the next number.

Step 3: Repeat Step 2 to continue the sequence.

For example, suppose Lorna starts with 9.
Since this number is odd, the next number is $9 \times 3 + 1 = 27 + 1 = 28$.
Since this number is even, the next number is $28 \div 2 = 14$.
Since this number is even, the next number is $14 \div 2 = 7$.
Thus, the first four numbers in the sequence are 9, 28, 14, and 7.

(a) Follow the instructions using the given starting number and write the first 12 numbers in each sequence.
(i) 3 (ii) 13

(b) What do you notice about each sequence in part (a)? What would happen if you continued each sequence?

(c) Use your answer to part (b) to predict the 20^{th} number in the sequence starting with 13.

Solution

(a) The first 12 numbers in each sequence are as follows.

(i) 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4

(ii) 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2

(b) After each sequence reaches 1, the numbers 4, 2, and 1 repeat over and over.

(c) We can write the first 20 numbers in the sequence starting with 13 by first using part (b) to write out the first 12 numbers in the sequence, and then repeating 4, 2, and 1 until we have 20 numbers.

$$13,\ 40,\ 20,\ 10,\ 5,\ 16,\ 8,\ 4,\ 2,\ 1,\ 4,\ 2,\ 1,\ 4,\ 2,\ 1,\ 4,\ 2,\ 1,\ 4$$

Thus, the 20^{th} number is 4.

Teacher's Notes

As an extra challenge, students could attempt this starting with the number 27. Although it takes a long time when you start with the number 27, the sequence will eventually reach 1. It takes 111 numbers to reach 1, as shown.

27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1

Mathematicians believe that starting with any positive integer, the instructions will always lead to a sequence that reaches 1 (or *converges* to 1). This is known as the *Collatz Conjecture*. However, proving this is true for all positive integers is an open problem. There is experimental evidence that shows this is true for very large numbers; however, there is no formal proof that the conjecture holds for all positive integers.

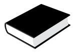
Data Management (D)

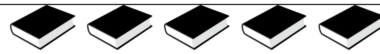
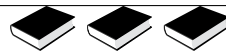
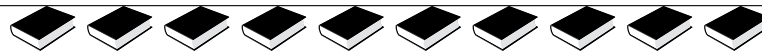
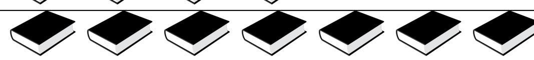
Take me to the cover

Problem of the Week

Problem A

Books, Books, Books

The pictograph below shows how many books five students have each read this month. Each  represents a fixed number of books.

Student	**Books Read**
Xuan	
Javya	
Natasha	
Sanan	
Brandon	

(a) Brandon read 28 books this month. How many books does each  represent in the pictograph?

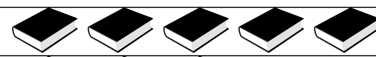
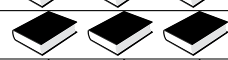
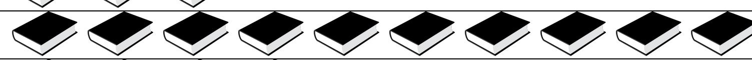
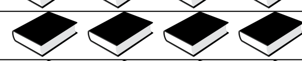
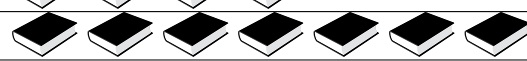
(b) How many books were read in total by these students this month?

Theme: Data Management

Problem of the Week
Problem A and Solution
Books, Books, Books

Problem

The pictograph below shows how many books five students have each read this month. Each  represents a fixed number of books.

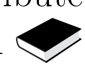
Student	Books Read
Xuan	
Javya	
Natasha	
Sanan	
Brandon	

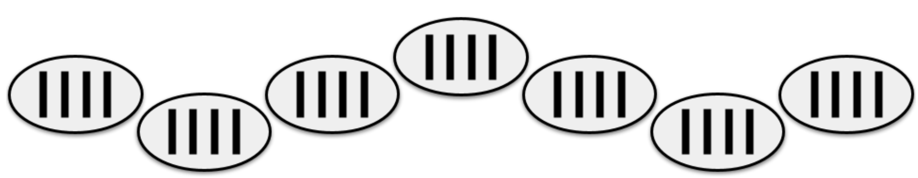
(a) Brandon read 28 books this month. How many books does each  represent in the pictograph?

(b) How many books were read in total by these students this month?

Solution

(a) Since Brandon has 7 , we can skip count by 7s until we get to 28. Doing this gives 7, 14, 21, 28. This means that each  represents 4 books read by a student.

Alternatively, we could use a fair share strategy to determine how many books each  represents. We draw seven ovals, and add a tally to each oval one at a time until 28 tallies have been distributed. Then we end up with 4 tallies in each oval, which means that each  represents 4 books.



(b) Since each  represents 4 books read, we know that Xuan read $5 \times 4 = 20$ books, Javya read $3 \times 4 = 12$ books, Natasha read $10 \times 4 = 40$ books, Sanan read $4 \times 4 = 16$ books, and Brandon read $7 \times 4 = 28$ books.

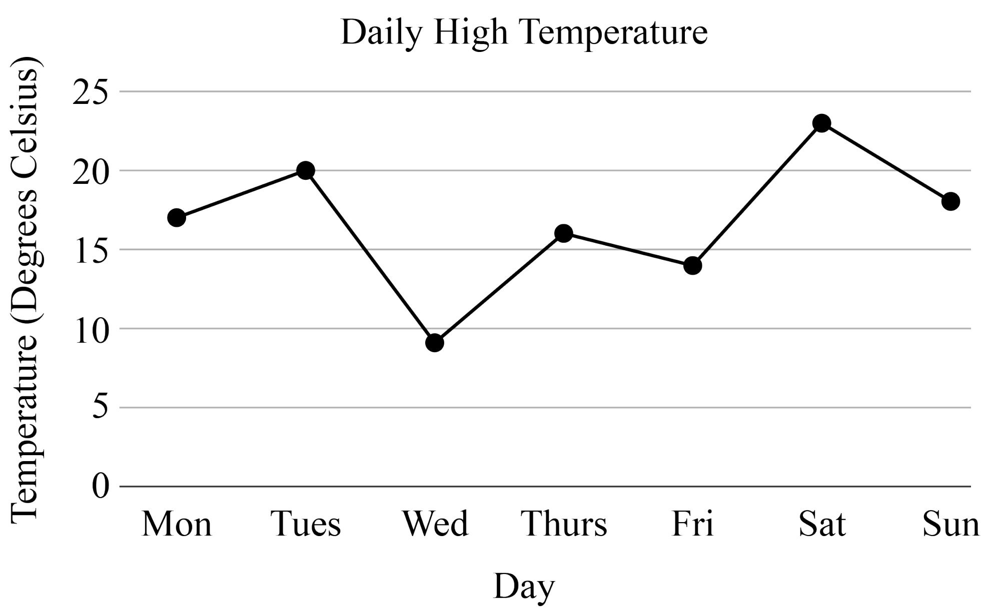
Thus, in total these students read $20 + 12 + 40 + 16 + 28 = 116$ books.

Problem of the Week

Problem A

Tracking Temperatures

The weather station at the University of Waterloo records information every day. The graph below shows the high temperature each day for one week.



(a) Estimate the highest temperature recorded during this week.

(b) Estimate lowest temperature recorded during this week.

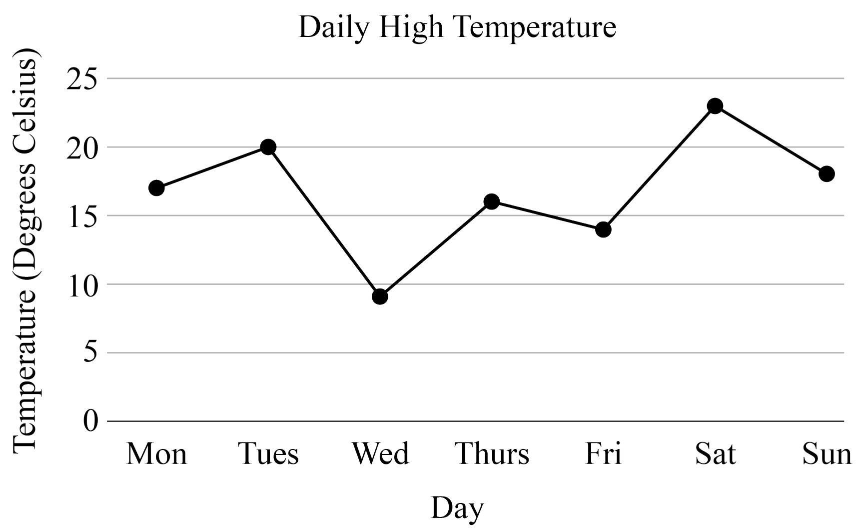
(c) Estimate the biggest change of temperature from one day to the next during this week.

Themes: Data Management, Number Sense

Problem of the Week
Problem A and Solution
Tracking Temperatures

Problem

The weather station at the University of Waterloo records information every day. The graph below shows the high temperature each day for one week.



(a) Estimate the highest temperature recorded during this week.

(b) Estimate lowest temperature recorded during this week.

(c) Estimate the biggest change of temperature from one day to the next during this week.

Solution

(a) The highest temperature occurs on Saturday. It is between 20 degrees and 25 degrees. It appears to be slightly closer to 25 degrees than 20 degrees, so we will estimate the temperature is 23 degrees.

(b) The lowest temperature occurs on Wednesday. It is between 5 degrees and 10 degrees, but is very close to 10 degrees. We will estimate the temperature is 9 degrees.

(c) We estimate the temperature on Monday to be 17 degrees, the temperature on Tuesday to be 20 degrees, the temperature on Thursday to be 16 degrees, the temperature on Friday to be 14 degrees, and the temperature on Sunday to be 18 degrees, along with the estimations in parts (a) and (b).
Thus, from Monday to Tuesday we estimate that it became $20 - 17 = 3$ degrees warmer.
From Tuesday to Wednesday we estimate that it became $20 - 9 = 11$ degrees colder.
From Wednesday to Thursday we estimate that it became $16 - 9 = 7$ degrees warmer.
From Thursday to Friday we estimate that it became $16 - 14 = 2$ degrees colder.
From Friday to Saturday we estimate that it became $23 - 14 = 9$ degrees warmer.
From Saturday to Sunday we estimate that it became $23 - 18 = 5$ degrees colder.
Therefore, we can estimate that the biggest change of temperature from one day to the next is 11 degrees between Tuesday and Wednesday.

Problem of the Week

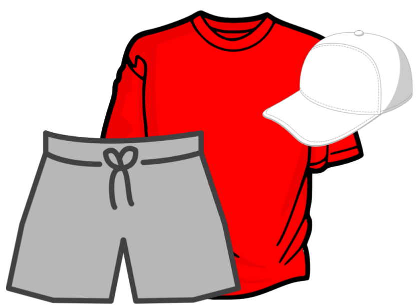
Problem A

Summer Selections

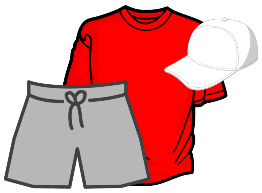
Robbie is trying to simplify his summer wardrobe. He always wears a pair of shorts, a shirt, and a hat. He has:

- 3 pairs of shorts in the colours of red, grey, and yellow
- 3 shirts in the colours of red, white, and green
- 3 hats in the colours of yellow, purple, and white

How many different combinations of a pair of shorts, a shirt, and a hat can he put together so that each item is of a different colour?



Theme: Data Management



Problem of the Week
Problem A and Solution
Summer Selections

Problem

Robbie is trying to simplify his summer wardrobe. He always wears a pair of shorts, a shirt, and a hat. He has:

- 3 pairs of shorts in the colours of red, grey, and yellow
- 3 shirts in the colours of red, white, and green
- 3 hats in the colours of yellow, purple, and white

How many different combinations of a pair of shorts, a shirt, and a hat can he put together so that each item is of a different colour?

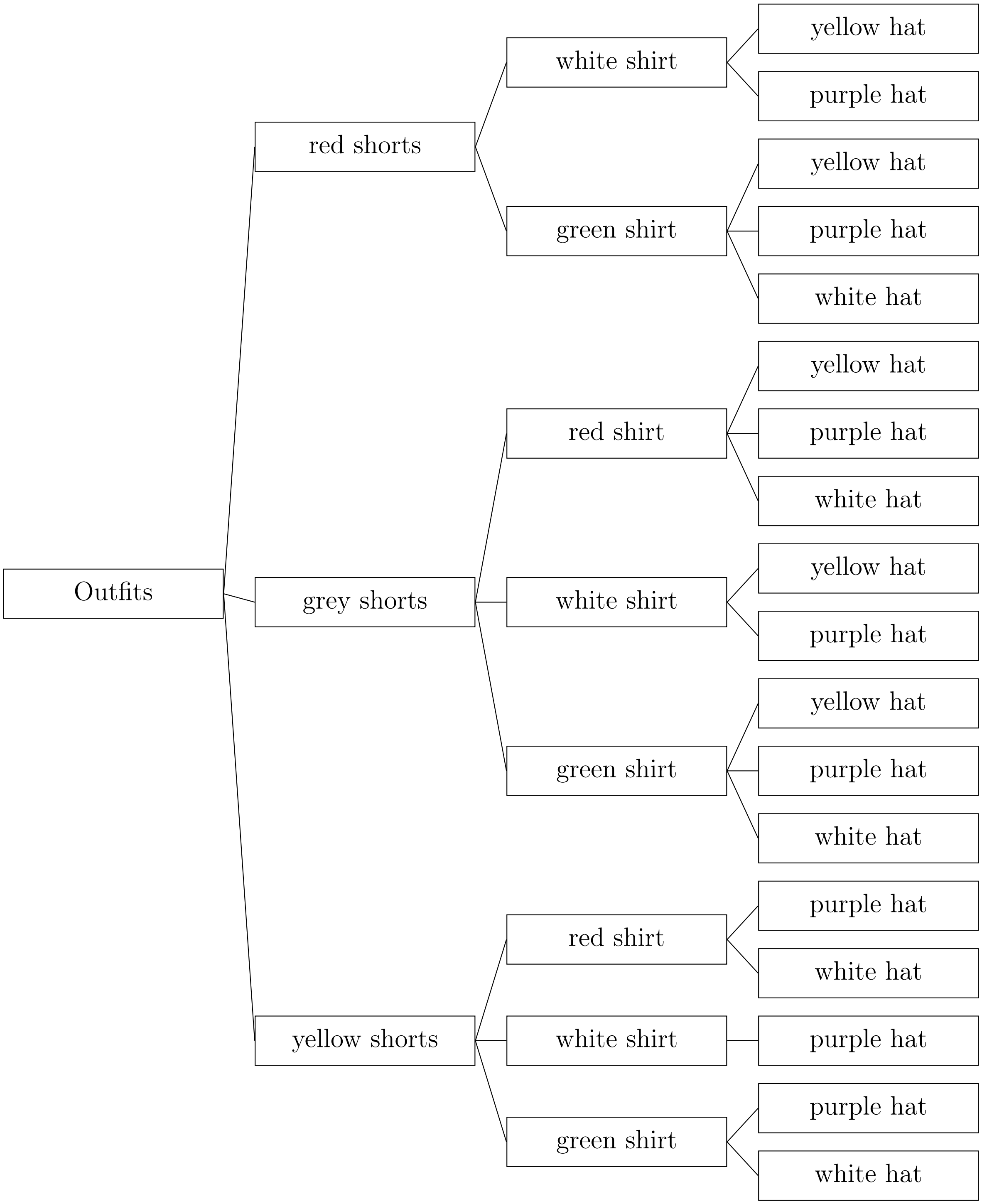
Solution

One way to solve this problem is to create a tree that shows all possible combinations of shorts, shirts, and hats that Robbie can wear. Then we can count the number of leaves of the tree (items in the tree that do not have any branches leading away from them) to determine the number of possible colour combinations.

When creating the tree, it is important that we avoid duplicating colours in each outfit. We can check this by following a path from the root of the tree ("Outfits") to each leaf and ensuring there are no duplicate colours. Each path from the root to a leaf describes a colour combination of a single outfit.

For example **red shorts** → **green shirt** → **purple hat** is one outfit and **grey shorts** → **red shirt** → **white hat** is another outfit. There are no duplicate colours in those outfits.

Here is tree showing those combinations:



Since there are 18 leaves in this diagram, there are 18 possible outfits Robbie could wear without duplicating colours.

Geometry & Measurement (G)

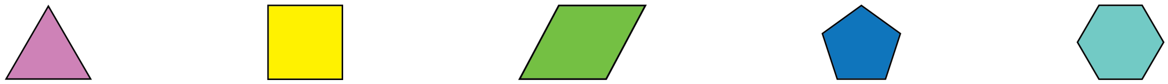
Take me to the cover

Problem of the Week

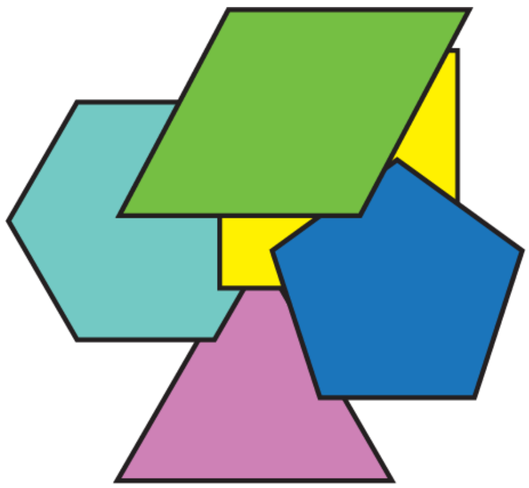
Problem A

Stacking Shapes

Anna cut out the following five polygons.



She then placed them on a table. The top view after doing so is shown.

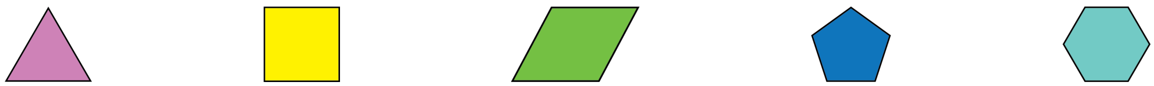


In what order did she place the polygons on the table?

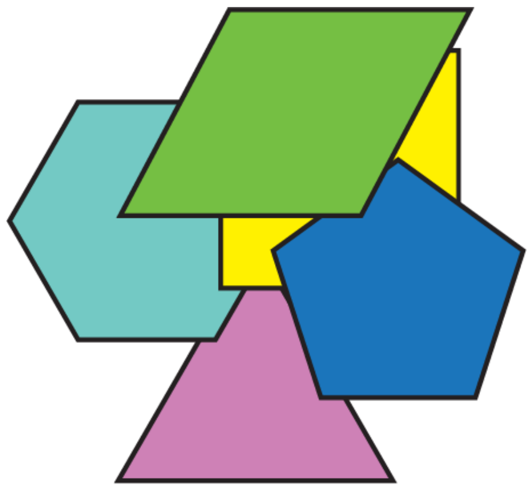
Problem of the Week
Problem A and Solution
Stacking Shapes

Problem

Anna cut out the following five polygons.



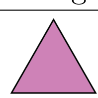
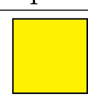
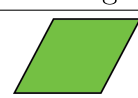
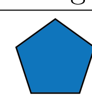
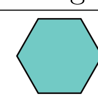
She then placed them on a table. The top view after doing so is shown.



In what order did she place the polygons on the table?

Solution

First, we recall the names of the five polygons.

Name	Triangle	Square	Parallelogram	Pentagon	Hexagon
Image					

Since the parallelogram has no shapes covering part of it, it must have been the last polygon placed on the table.

The pentagon has the parallelogram covering part of it, so the pentagon must have been placed before the parallelogram.

The square has the pentagon and parallelogram covering part of it, so the square must have been placed before the pentagon and parallelogram.

The hexagon has the square and parallelogram covering part of it, so the hexagon must have been placed before the square and parallelogram.

The triangle has the hexagon, square, and pentagon covering part of it, so the triangle must have been placed before the hexagon, square, and pentagon.

Thus, Anna must have first placed the triangle, then the hexagon, then the square, then the pentagon, then the parallelogram.

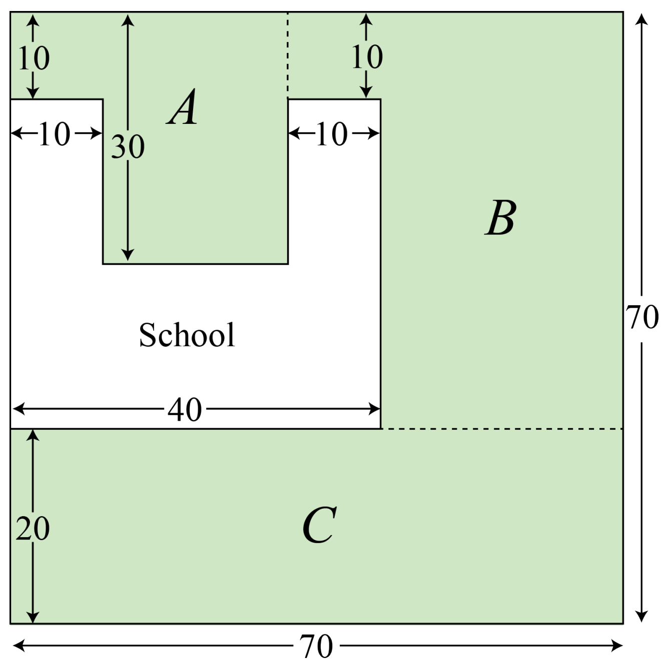
Problem of the Week

Problem A

School Clean Up

Every year the students at Spotless Elementary do an outdoor cleanup. First, the school yard is divided into three sections: A, B, and C.

A map of the school yard is shown, with all given lengths in metres. Note that the school yard is a square, the school is a rectangular U-shape, and the diagram is not drawn to scale.



The Grade 1 & 2 students are assigned the section with the smallest area, the Grade 5 & 6 students are assigned the section with the largest area, and the Grade 3 & 4 students are assigned the remaining section. Determine which section is assigned to each grade. Justify your answer.

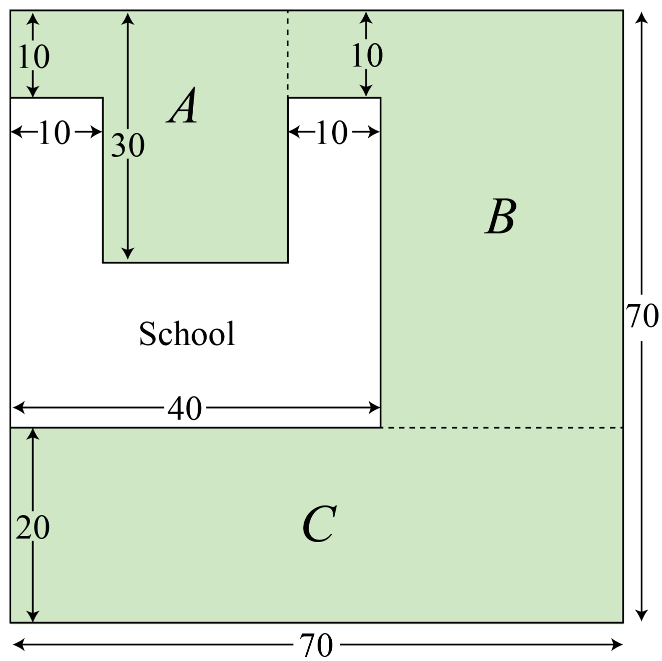
Theme: Geometry & Measurement

Problem of the Week
Problem A and Solution
School Clean Up

Problem

Every year the students at Spotless Elementary do an outdoor cleanup. First, the school yard is divided into three sections: A, B, and C.

A map of the school yard is shown, with all given lengths in metres. Note that the school yard is a square, the school is a rectangular U-shape, and the diagram is not drawn to scale.



The Grade 1 & 2 students are assigned the section with the smallest area, the Grade 5 & 6 students are assigned the section with the largest area, and the Grade 3 & 4 students are assigned the remaining section. Determine which section is assigned to each grade. Justify your answer.

Solution
Solution 1

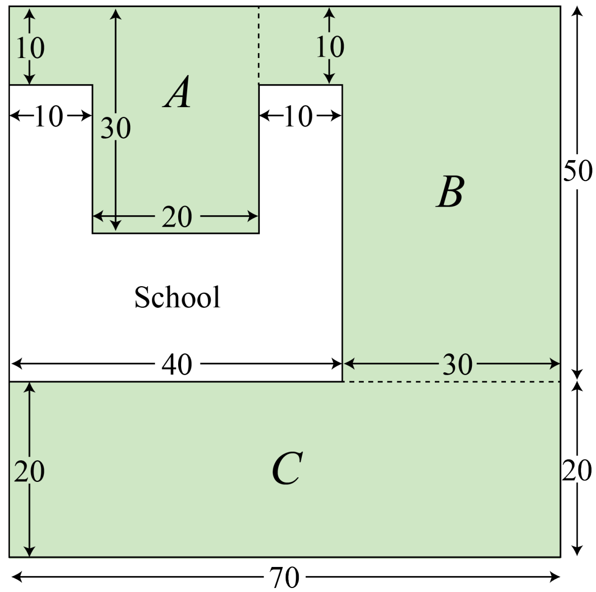
One way to solve this problem is to calculate the area of each section. Since Section C is a rectangle with length 70 m and width 20 m, its area is $70 \times 20 = 1400$ m^2.

We are missing some dimensions needed to calculate the areas of the other two sections. However, we can determine these missing dimensions from the information in the diagram.

Since the bottom side of the school is 40 m and the widths of the ends of the U-shape are each 10 m, then the length in between the ends of the U-shape is $40 - 10 - 10 = 20$ m.

Since the bottom side of the yard is 70 m and the bottom side of the school is 40 m, then the distance from the right side of the school to the right side of the yard must be $70 - 40 = 30$ m.

Since the right side of the yard is 70 m and the right side of Section C is 20 m, then the right side of Section B is $70 - 20 = 50$ m. The following diagram shows all the dimensions.



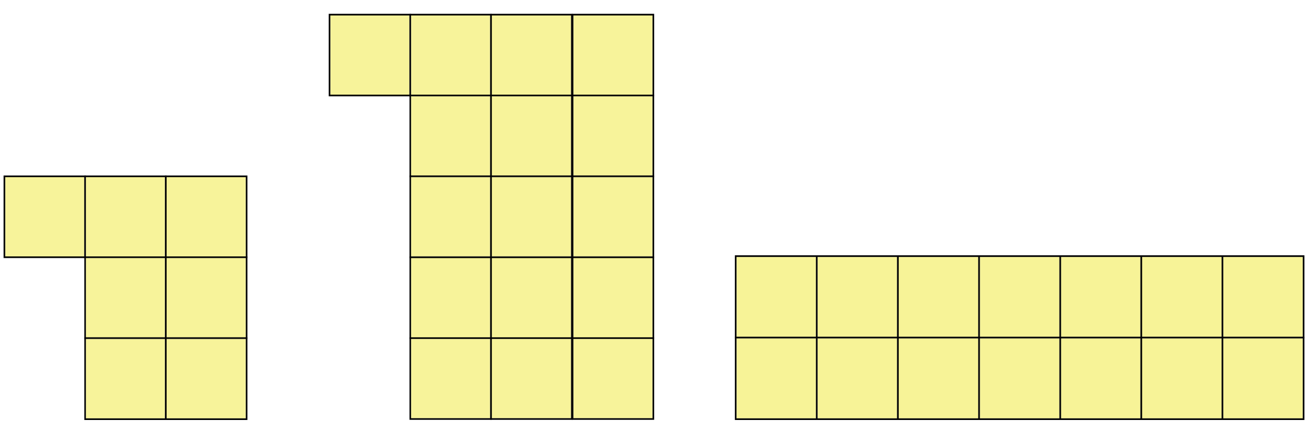
Section A is formed by a square with side length 10 m, and a rectangle with length 30 m and width 20 m. The area of the square is $10 \times 10 = 100$ m^2, and the area of the rectangle is $30 \times 20 = 600$ m^2. The total area of Section A is then $100 + 600 = 700$ m^2.

Similarly, Section B is formed by a square with side length 10 m, and a rectangle with length 50 m and width 30 m. The area of the square is $10 \times 10 = 100$ m^2, and the area of the rectangle is $50 \times 30 = 1500$ m^2. The total area of Section B is then $100 + 1500 = 1600$ m^2.

Thus, the area of Section A is 700 m^2, the area of Section B is 1600 m^2, and the area of Section C is 1400 m^2. Therefore Grades 1 & 2 are assigned to Section A, Grades 5 & 6 are assigned to Section B, and Grades 3 & 4 are assigned to Section C.

Solution 2

Another way to solve this problem is to build each section using 10 m $\times$ 10 m squares, as shown.



Section A Section B Section C

Using this approach, we can count the number of 10×10 squares that form each section. Section A is formed with 7 squares, Section B is formed with 16 squares, and Section C is formed with 14 squares. Therefore, Grades 1 & 2 are assigned to Section A, Grades 5 & 6 are assigned to Section B, and Grades 3 & 4 are assigned to Section C.

Problem of the Week

Problem A

Time After Time

Assume that the current time matches the time shown on the clock, and the current time is in the morning.

The school day ends in 5 hours and 6 minutes from the time shown on the clock.

(a) How many more minutes are left until the end of the school day?

(b) What time does school end?

(c) School started at 8:55 a.m. How much time has passed, in hours and minutes, between the start of the school day and the time shown on the clock?

Themes: Geometry & Measurement, Number Sense

Problem of the Week

Problem A and Solution

Time After Time

Problem

Assume that the current time matches the time shown on the clock, and the current time is in the morning.

The school day ends in 5 hours and 6 minutes from the time shown on the clock.

(a) How many more minutes are left until the end of the school day?

(b) What time does school end?

(c) School started at 8:55 a.m. How much time has passed, in hours and minutes, between the start of the school day and the time shown on the clock?

Solution

(a) Since each hour has 60 minutes, then 5 hours have a total of $5 \times 60 = 300$ minutes. So the total number of minutes left until the end of the school day is $300 + 6 = 306$ minutes.

(b) The time showing on the clock is 10:09 a.m. We will add 5 hours and 6 minutes to this time. Two hours after 10:09 a.m. it will be 12:09 p.m. Three hours after that it will be 3:09 p.m. Six minutes later it will be 3:15 p.m. Thus, school ends at 3:15 p.m.

(c) We start at 8:55 a.m. One hour after that it will be 9:55 a.m. Five minutes after that it will be 10:00 a.m. Nine minutes after that it will be 10:09 a.m. Thus, a total of 1 hour and $5 + 9 = 14$ minutes have passed between the start of the school day and the time shown on the clock.

Problem of the Week

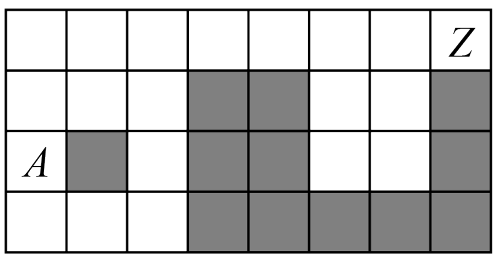
Problem A

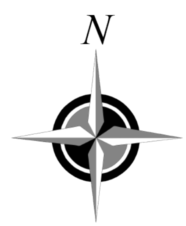
Amazing Navigation

Juanita and AJ create mazes on grid paper. Each maze is a rectangular grid containing white squares and grey squares. One white square is marked A and another is marked Z.

To complete a maze, they start at A and need to reach Z by moving one square at a time in one of the following directions: north (N), east (E), south (S), or west (W), where the top of the page is considered north. They *cannot* go through any of the grey squares and must go through each of the white squares *exactly once*. That is, they must go through all of the white squares but cannot go through any of them more than once.

(a) Determine the directions they need to follow to successfully complete the given maze.





(b) AJ creates another maze by changing where the grey squares are in the maze from part (a). (The locations of A and Z remain unchanged.) Juanita successfully completes this new maze by following these directions:

$$E, \quad S, \quad E, \quad E, \quad E, \quad N, \quad E, \quad N, \quad W, \quad W,$$
$$W, \quad W, \quad N, \quad E, \quad E, \quad E, \quad E, \quad E, \quad E$$

What does AJ's maze look like?

Problem of the Week

Problem A and Solution

Amazing Navigation

Problem

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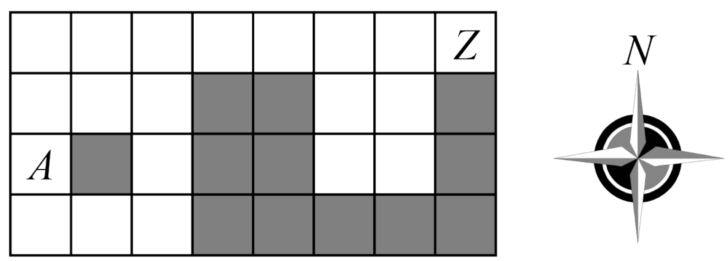


(b) AJ creates another maze by changing where the grey squares are in the maze from part (a). (The locations of A and Z remain unchanged.) Juanita successfully completes this new maze by following these directions:

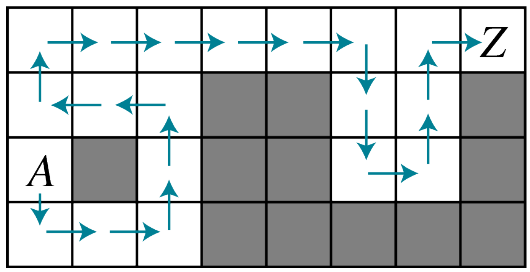
$$E,\quad S,\quad E,\quad E,\quad E,\quad N,\quad E,\quad N,\quad W,\quad W,$$
$$W,\quad W,\quad N,\quad E,\quad E,\quad E,\quad E,\quad E,\quad E$$

What does AJ's maze look like?

Solution

(a) The first direction must be either N or S, because there is a grey square to the right of A. Suppose they start with N. Then at some point, they must go through the square that is directly below A. However, once they reach that square they will be stuck because they can't go through any white square more than once. So the first direction must be S.

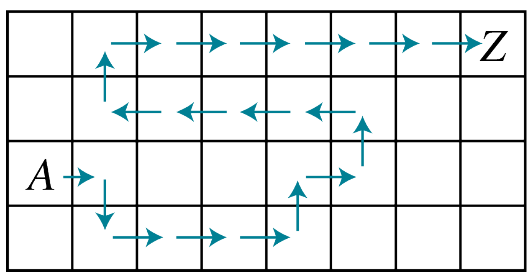
Using similar reasoning, we can complete the maze as shown.



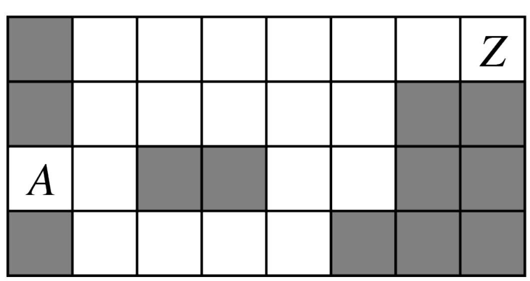
The directions followed are then

$$S,\quad E,\quad E,\quad N,\quad N,\quad W,\quad W,\quad N,\quad E,\quad E,$$
$$E,\quad E,\quad E,\quad S,\quad S,\quad E,\quad N,\quad N,\quad E$$

(b) We first mark the given directions on the maze, as shown.



Since Juanita cannot go through any of the grey squares, and must go through each of the white squares exactly once, the squares that are blank after marking the path must be the grey squares. Thus, AJ's maze is as shown.



Problem of the Week

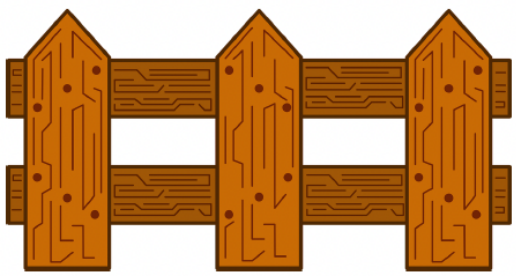
Problem A

Fencepost Problem

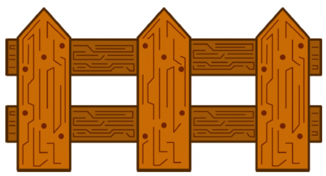
Elyas helps his parents install fence posts in their backyard. They install a row of 7 equally-spaced posts. The posts are all the same size. The distance between the middle of the 2^{nd} post and the middle of the 5^{th} post is four and a half meters.

What is the distance between the middle of the first post and the middle of the last post?

Hint: You might start by drawing a diagram of the fence posts and labelling it with the distance you know.



Themes: Geometry & Measurement, Number Sense



Problem of the Week
Problem A and Solution
Fencepost Problem

Problem

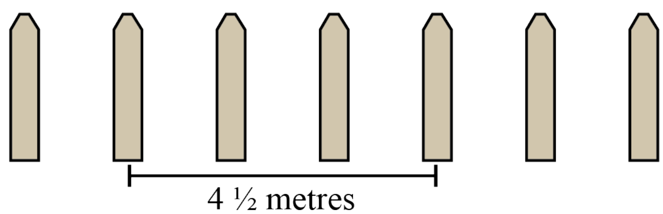
Elyas helps his parents install fence posts in their backyard. They install a row of 7 equally-spaced posts. The posts are all the same size. The distance between the middle of the 2^{nd} post and the middle of the 5^{th} post is four and a half meters.

What is the distance between the middle of the first post and the middle of the last post?

HINT: You might start by drawing a diagram of the fence posts and labelling it with the distance you know.

Solution

There are many ways to calculate the distance. We start with a diagram of the posts, labelled with the distance of $4\frac{1}{2}$ metres between the middle of the 2^{nd} and 5^{th} posts.

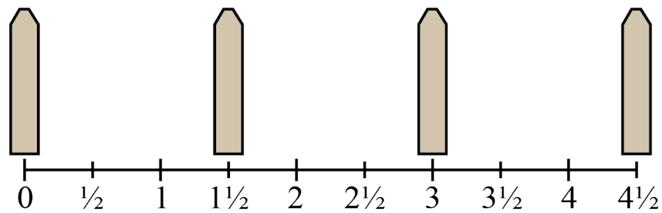


Solution 1

Notice that there are a total of three gaps between the 2^{nd} and 5^{th} posts. There are also a total of three gaps between the 1^{st} and 2^{nd} posts and the 5^{th} and 7^{th} posts. Since the distances between the middle of adjacent posts are all the same, then the total distance between the 1^{st} and 2^{nd} posts and the 5^{th} and 7^{th} posts must also be $4\frac{1}{2}$ m. Since there are six gaps in total, the distance between the middle of the first post and the middle of the last post is $4 + \frac{1}{2} + 4 + \frac{1}{2} = 9$ m.

Solution 2

Another way to solve this problem is to draw a number line from 0 to $4\frac{1}{2}$ m and space the 2^{nd}, 3^{rd}, 4^{th}, and 5^{th} fenceposts evenly along this line.

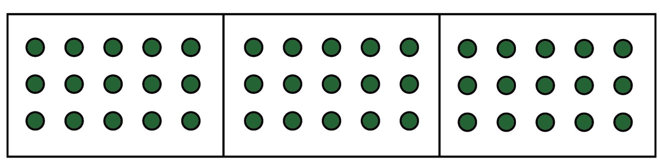


From this, we observe that the distance from the middle of one post to the middle of an adjacent post is $1\frac{1}{2}$ m. Since there are six gaps in total between adjacent posts, the distance between the middle of the first post and the middle of the last post is:

$$1 + \frac{1}{2} + 1 + \frac{1}{2} + 1 + \frac{1}{2} + 1 + \frac{1}{2} + 1 + \frac{1}{2} + 1 + \frac{1}{2} = 9 \text{ m}$$

Solution 3

First we convert the distance to centimetres: $4\frac{1}{2}$ m $= 450$ cm. Then we use a tape diagram. We observe that there are 3 gaps between the 2^{nd} and 5^{th} posts. That means we need to divide 450 cm into three equal distances. To make it easier, since $450 = 45 \times 10$, we let one dot represent 10 cm, and then distribute 45 dots into the three equal pieces of our tape diagram.



Each piece has a total of 15 dots. This means the distance between adjacent posts is $15 \times 10 = 150$ cm. Since there are 6 gaps in total between the first post and the last post, the total distance is:

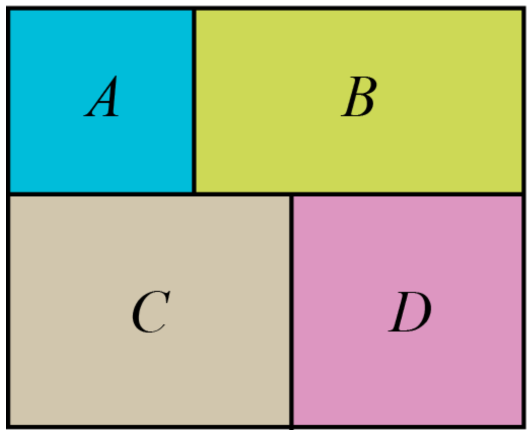
$$150 + 150 + 150 + 150 + 150 + 150 = 900 \text{ cm or } 9 \text{ m}$$

Problem of the Week

Problem A

Quilting Puzzle

The employees of Sew Inspired need to make a tiny quilt for a special project using four different colours of fabric. Their quilt pattern is a rectangle divided into four smaller rectangles, marked A, B, C, and D.



Piece A is a square with area 16 cm^2 and piece D is a square with area 25 cm^2. The horizontal distance between the right side of piece A and the left side of piece D is 2 cm. What is the area of the entire quilt?

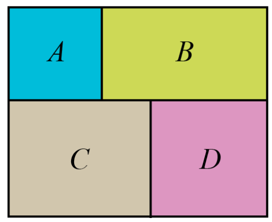
Problem of the Week

Problem A and Solution

Quilting Puzzle

Problem

The employees of Sew Inspired need to make a tiny quilt for a special project using four different colours of fabric. Their quilt pattern is a rectangle divided into four smaller rectangles, marked A, B, C, and D.



Piece A is a square with area 16 cm^2 and piece D is a square with area 25 cm^2. The horizontal distance between the right side of piece A and the left side of piece D is 2 cm. What is the area of the entire quilt?

Solution

To calculate the area of the quilt we will determine the lengths of its sides. We know that piece A is a square with area 16 cm^2. We also know that the lengths of the sides of a square must be the same. So if the length of one side of a square is n, then the area of the square must be $n \times n$. By trial and error, we determine $4 \times 4 = 16$, and so the length of each side of piece A must be 4 cm. Another way to determine the lengths of the sides of piece A is to start with 16 unit squares (using blocks or cut out of paper) and determine how to arrange them into a larger square. The only possible arrangement is a 4×4 square.

Similarly, piece D is a square with area 25 cm^2. Since $5 \times 5 = 25$, the length of each side of piece D must be 5 cm.

The opposite sides of a rectangle must be the same length, so the width of the quilt (i.e. the vertical side) is equal to the sum of the side lengths of pieces A and D. Thus, the width of the quilt is equal to $4 + 5 = 9$ cm. Similarly, we know the bottom of piece A is on the same line as the top of piece D, and the horizontal distance between the two pieces is 2 cm. Therefore the length of the quilt (i.e. the horizontal side) is equal to the sum of the side lengths of pieces A and D, plus 2. Thus, the length of the quilt is equal to $4 + 5 + 2 = 11$ cm.

Now we can calculate the area of the entire quilt. The area of this rectangle is the product of its length and width. So the area of the entire quilt is $9 \times 11 = 99$ cm^2.

Problem of the Week

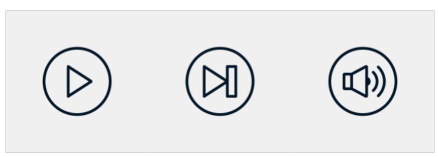
Problem A

Video Viewing

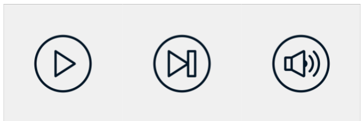
Jo likes to watch videos. Sometimes she watches them at normal speed. However, she also has the option to watch them at $\frac{1}{4}$ as fast as normal speed, $\frac{1}{2}$ as fast as normal speed, or 2 times as fast as normal speed. The table below lists the normal play times for four videos and the speed at which Jo watches each one.

	Normal Speed Time	**Speed Jo Watches Video At**
Video A	50 seconds	$\frac{1}{4}$ speed
Video B	2 minutes 15 seconds	normal speed
Video C	1 minute 40 seconds	$\frac{1}{2}$ speed
Video D	6 minutes 20 seconds	2 times speed

What is the total time Jo spent watching these four videos?



Themes: Geometry & Measurement, Number Sense



Problem of the Week
Problem A and Solution
Video Viewing

Problem

Jo likes to watch videos. Sometimes she watches them at normal speed. However, she also has the option to watch them at $\frac{1}{4}$ as fast as normal speed, $\frac{1}{2}$ as fast as normal speed, or 2 times as fast as normal speed. The table below lists the normal play times for four videos and the speed at which Jo watches each one.

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Video A	50 seconds	$\frac{1}{4}$ speed
Video B	2 minutes 15 seconds	normal speed
Video C	1 minute 40 seconds	$\frac{1}{2}$ speed
Video D	6 minutes 20 seconds	2 times speed

What is the total time Jo spent watching these four videos?

Solution

If a video is playing at $\frac{1}{4}$ speed, then it will take 4 times as long to play as normal speed. So a 50 second video will take $50 \times 4 = 200$ seconds to watch.

If a video is playing at $\frac{1}{2}$ speed, then it will take 2 times as long to play as normal speed. So a 1 minute and 40 second video will take 2 minutes and $40 \times 2 = 80$ seconds to watch.

If a video is playing at 2 times speed then it will take half as long to play as normal speed. Half of 6 minutes is 3 minutes and half of 20 seconds is 10 seconds.

Now we can add up all the minutes and seconds of playing times of all four videos:

$$2 + 2 + 3 = 7 \text{ minutes and } 200 + 15 + 80 + 10 = 305 \text{ seconds}$$

There are 60 seconds in 1 minute. When we skip count by 60, we get 60, 120, 180, 240, 300. Thus, there are 5 minutes in 305 seconds with $305 - 300 = 5$ seconds left over.

So Jo spent $7 + 5 = 12$ minutes and 5 seconds watching these four videos.

Number Sense (N)

Take me to the cover

Problem of the Week
Problem A
Movie Magic

Comfort and her family are planning to see a movie on the weekend. Comfort is 9 years old. Her twin brothers, who are 5 years older than Comfort, and both her parents will be going to the movie. The prices for tickets, including all taxes, are:

- Adults: \$14 each
- Children between 12 and 18: \$12 each
- Children under 12: \$8 each

They have a gift card worth \$25 that they can use to pay for part of the cost of the movie tickets. How much more money do they need to pay for the tickets for the entire family? Justify your answer.

Theme: Number Sense

Problem of the Week
Problem A and Solution
Movie Magic

Problem

Comfort and her family are planning to see a movie on the weekend. Comfort is 9 years old. Her twin brothers, who are 5 years older than Comfort, and both her parents will be going to the movie. The prices for tickets, including all taxes, are:

- Adults: $14 each
- Children between 12 and 18: $12 each
- Children under 12: $8 each

They have a gift card worth $25 that they can use to pay for part of the cost of the movie tickets. How much more money do they need to pay for the tickets for the entire family? Justify your answer.

Solution

Since Comfort is 9 years old and her twin brothers are 5 years older, they are both $9 + 5 = 14$ years old. (Note that depending on when their birthdays are exactly, the brothers might actually be 13 or 15 at the time. However, their age would definitely be between 12 and 18.)

So, Comfort's ticket will cost $8.
Comfort's brothers' tickets will cost $12 each.
Comfort's parents' tickets will cost $14 each.

The total cost for the family will then be $\$8 + \$12 + \$12 + \$14 + \$14 = \$60$.

Since they have a $25 gift card, they will need to pay $\$60 - \$25 = \$35$ more for the tickets for the entire family.

Problem of the Week

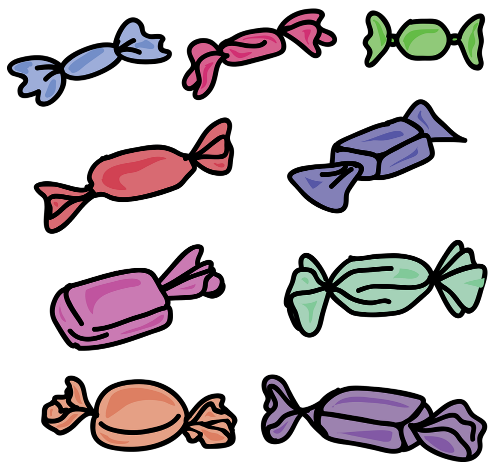
Problem A

Better Deal?

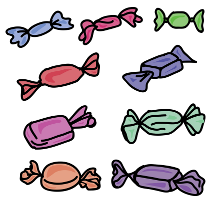
From a candy machine, I can buy 8 candies for 25 cents. Alternatively, I can buy 64 candies in a package for 2 dollars and 20 cents.

If I want to buy 128 candies for my class, how should I buy the candies in order to spend the least amount of money?

NOTE: 1 dollar is equal to 100 cents.



THEME: NUMBER SENSE



Problem of the Week
Problem A and Solution
Better Deal?

Problem

From a candy machine, I can buy 8 candies for 25 cents. Alternatively, I can buy 64 candies in a package for 2 dollars and 20 cents.

If I want to buy 128 candies for my class, how should I buy the candies in order to spend the least amount of money?

NOTE: 1 dollar is equal to 100 cents.

Solution

Since 128 is equal to 2×64, two packages of candies will be enough for the class. This would cost $2 \times \$2$ plus 2×20 cents, for a total of \$4 and 40 cents.

We can use skip counting to calculate how much the candies will cost if we buy them from the candy machine. This is summarized in the table below.

Number of Candies	Cost (cents)
8	25
16	50
24	75
32	100
40	125
48	150
56	175
64	200

Thus, 64 candies from the candy machine will cost 200 cents, which is equal to \$2. This is less than the cost of 64 candies in a package.

Therefore, to spend the least amount of money, we should buy all 128 candies from the candy machine. The total cost will then be $2 \times \$2 = \$4$.

Problem of the Week

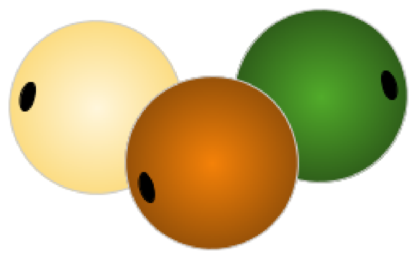
Problem A

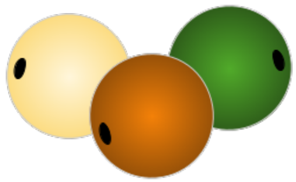
Beaded Bracelets

Aditi and Ersal have designed a bracelet that has 17 beads. The bracelet has 6 red beads, 9 green beads, and the rest of the beads are yellow.

(a) How many yellow beads are in one bracelet?

(b) They want to make a total of 21 of these bracelets so they have one for everyone in their class. Each package of 24 beads has 8 red, 8 green, and 8 yellow beads in it. How many packages of beads do they need?





Problem of the Week

Problem A and Solution

Beaded Bracelets

Problem

Aditi and Ersal have designed a bracelet that has 17 beads. The bracelet has 6 red beads, 9 green beads, and the rest of the beads are yellow.

(a) How many yellow beads are in one bracelet?

(b) They want to make a total of 21 of these bracelets so they have one for everyone in their class. Each package of 24 beads has 8 red, 8 green, and 8 yellow beads in it. How many packages of beads do they need?

Solution

(a) We can calculate the number of yellow beads by subtracting the number of red and green beads from the total, to get $17 - 6 - 9 = 2$ yellow beads.

(b) Since they need more green beads than any other colour, to make 21 bracelets they need to buy enough packages for $21 \times 9 = 189$ green beads. Since each package has 8 green beads, one way to calculate how many packages we need is to divide $189 \div 8 = 23$ with a remainder of 5. This means they need to buy 24 packages to get enough green beads.

Alternatively, we know that 21 packages will contain 8 green beads that can be used to make each bracelet. However, they need one more green bead for each of the 21 bracelets. Now we calculate how many packages are needed for 21 more green beads. We can skip count: 8, 16, 24 to see we need 3 more packages to have at least 21 more green beads. From this we know that Aditi and Ersal need a total of $21 + 3 = 24$ packages of beads to make the bracelets.

Problem of the Week

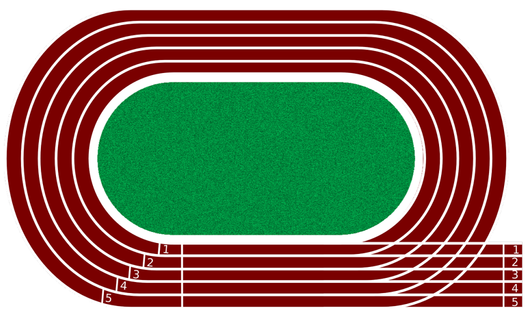
Problem A

Cross Training

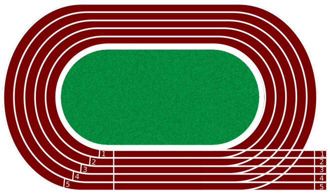
Jelena is training at the track at her school. She does interval training, which means that she runs for some distance then stops to do other exercises. Each time around the track is called a lap. This is her training plan:

- Run half of the way around the track, stop and do 10 push-ups.
- Run three quarters of the way around the track, stop and do 5 burpees.
- Run one and a quarter of the way around the track, stop and do 15 jumping jacks.

How many laps of the track has Jelena completed after doing the jumping jacks?



Theme: Number Sense



Problem of the Week
Problem A and Solution
Cross Training

Problem

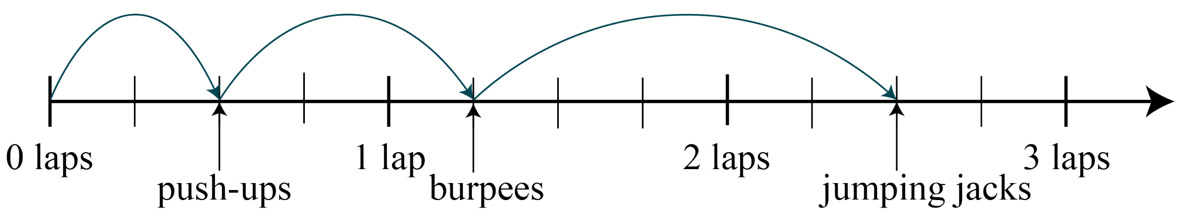
Jelena is training at the track at her school. She does interval training, which means that she runs for some distance then stops to do other exercises. Each time around the track is called a lap. This is her training plan:

- Run half of the way around the track, stop and do 10 push-ups.
- Run three quarters of the way around the track, stop and do 5 burpees.
- Run one and a quarter of the way around the track, stop and do 15 jumping jacks.

How many laps of the track has Jelena completed after doing the jumping jacks?

Solution

One way to solve this problem is to use a timeline broken up into quarter laps.



From this, we determine that after completing the jumping jacks, Jelena has completed 2 and a half laps of the track.

Another way to determine the answer is to add fractions together. To add fractions, we need the fractions written with a common denominator. We know that $\frac{1}{2} = \frac{2}{4}$ and that $1 = \frac{4}{4}$, so we can add to get

$$\frac{2}{4} + \frac{3}{4} + \frac{4}{4} + \frac{1}{4} = \frac{10}{4} = 2\frac{1}{2}$$

Teacher's Notes

Using a *number line* can help with adding fractions, especially when the sum produces a *mixed fraction*.

A number line often includes arrows at one or both ends to indicate that positive and negative numbers continue to increase in magnitude indefinitely in each direction. In particular, an arrow pointing to the right indicates that there are an *infinite* number of positive integers. There are also an infinite number of values between any two marked points on the number line. For example, there are an infinite number of values between two consecutive positive integers. Some of those values are *rational* numbers that can be represented by fractions in the form:

$$\frac{a}{b}, \text{ where } a \text{ and } b \text{ are integers, and } b \text{ is not } 0$$

There are other values, such as $\sqrt{2}$, that cannot be represented in this form. They are known as *irrational* numbers.

Problem of the Week

Problem A

Holiday Cheer

Students at Giving Elementary School are spreading holiday cheer this year by collecting donations for families in need within their community. Their goal this year was to collect 2000 items in one week, and they exceeded their goal! The table below shows the number of donations each day during the week.

Day	Number of Items Donated
Monday	392
Tuesday	46
Wednesday	877
Thursday	?
Friday	229

Someone accidentally tore the collection sheet with the data on it, so they don't have the number of items they collected on Thursday. If we know the school exceeded their goal of 2000 items in a week, what is the minimum number of items the students collected on Thursday?

Theme: Number Sense

Problem of the Week
Problem A and Solution
Holiday Cheer

Problem

Students at Giving Elementary School are spreading holiday cheer this year by collecting donations for families in need within their community. Their goal this year was to collect 2000 items in one week, and they exceeded their goal! The table below shows the number of donations each day during the week.

Day	Number of Items Donated
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Someone accidentally tore the collection sheet with the data on it, so they don't have the number of items they collected on Thursday. If we know the school exceeded their goal of 2000 items in a week, what is the minimum number of items the students collected on Thursday?

Solution

We start by adding up the number of items for the days we know, and we get $392 + 46 + 877 + 229 = 1544$.

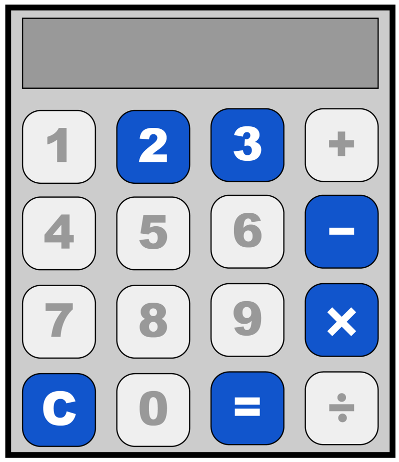
Since we know the students exceeded their goal, then they must have collected at least 2001 items in total. Thus, the minimum number of items they collected on Thursday must be $2001 - 1544 = 457$.

Problem of the Week

Problem A

Calculator Catastrophe

I dropped my calculator and my dog stepped on it. Now, most of the buttons are broken. The only working buttons are the ones with the dark background in the image below:



I want to have the calculator display each of the numbers from 1 to 12, by using only the unbroken buttons. The easiest ones to display are the numbers 2 and 3 since I can just enter them alone. However, all the others will need more buttons pressed. For example, to display a 7, I could use the fact that $7 = 3 \times 3 - 2$.

This is a very simple calculator, so I have to press the $\boxed{=}$ button after each part of the calculation. If I want to complete the calculation for $7 = 3 \times 3 - 2$, I would press the $\boxed{3}$ button, then the $\boxed{\times}$ button, then the $\boxed{3}$ button, and then the $\boxed{=}$ button to get the number 9. Then I would press the $\boxed{-}$ button, then the $\boxed{2}$ button, and then the $\boxed{=}$ button to get the number 7. This calculation takes 7 button presses to get the number 7 to appear in the calculator's display.

For each whole number from 1 to 12, show how you can display the number using at most 10 button presses.

NOTE: Your calculations can include 2-digit numbers. For example, if you wish to enter the number 32 you would press the $\boxed{3}$ button and then the $\boxed{2}$ button.

THEME: NUMBER SENSE

Problem of the Week

Problem A and Solution

Calculator Catastrophe

Problem

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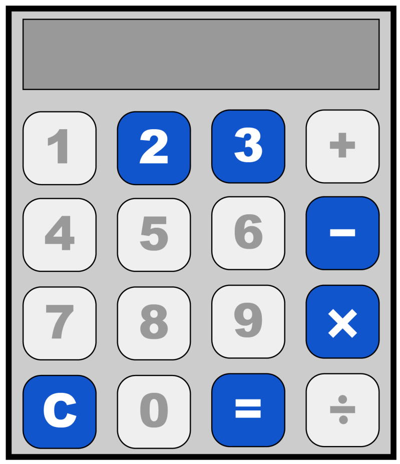

I want to have the calculator display each of the numbers from 1 to 12, by using only the unbroken buttons. The easiest ones to display are the numbers 2 and 3 since I can just enter them alone. However, all the others will need more buttons pressed. For example, to display a 7, I could use the fact that $7 = 3 \times 3 - 2$.

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For each whole number from 1 to 12, show how you can display the number using at most 10 button presses.

NOTE: Your calculations can include 2-digit numbers. For example, if you wish to enter the number 32 you would press the $\boxed{3}$ button and then the $\boxed{2}$ button.

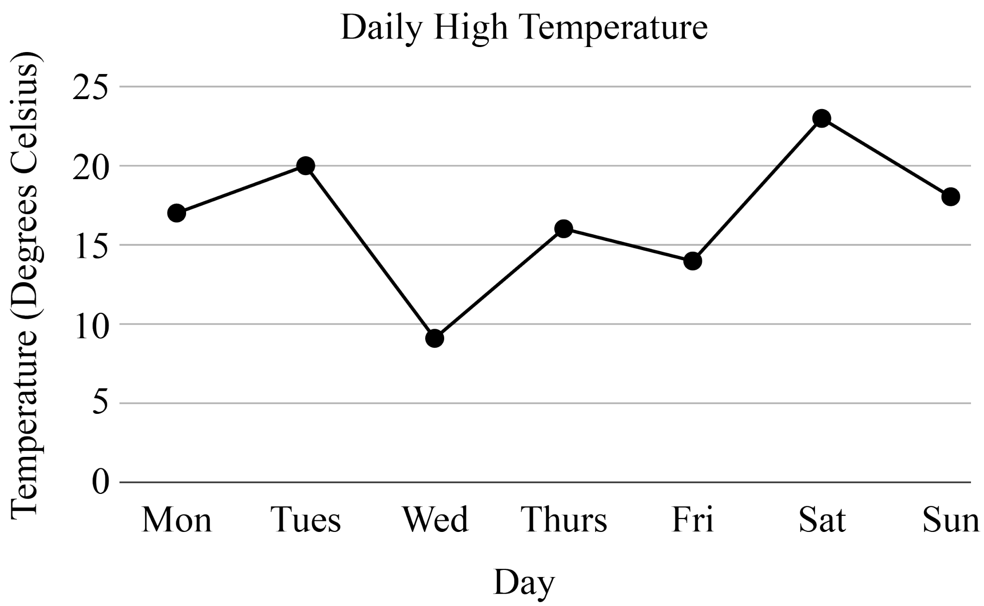
Solution

Answers will vary. Here is a table summarizing possible solutions for each whole number from 1 to 12.

Number	Calculation	Button Presses	Number of Button Presses
1	$3 - 2$	[3] [−] [2] [=]	4
2	2	[2]	1
3	3	[3]	1
4	2×2	[2] [×] [2] [=]	4
5	$3 \times 3 - 2 - 2$	[3] [×] [3] [=] [−] [2] [=] [−] [2] [=]	10
6	2×3	[2] [×] [3] [=]	4
7	$3 \times 3 - 2$	[3] [×] [3] [=] [−] [2] [=]	7
8	$2 \times 2 \times 2$	[2] [×] [2] [=] [×] [2] [=]	7
9	3×3	[3] [×] [3] [=]	4
10	$33 - 23$	[3] [3] [−] [2] [3] [=]	6
11	$33 - 22$	[3] [3] [−] [2] [2] [=]	6
12	$3 \times 2 \times 2$	[3] [×] [2] [=] [×] [2] [=]	7

Problem of the Week
Problem A
Tracking Temperatures

The weather station at the University of Waterloo records information every day. The graph below shows the high temperature each day for one week.



(a) Estimate the highest temperature recorded during this week.

(b) Estimate lowest temperature recorded during this week.

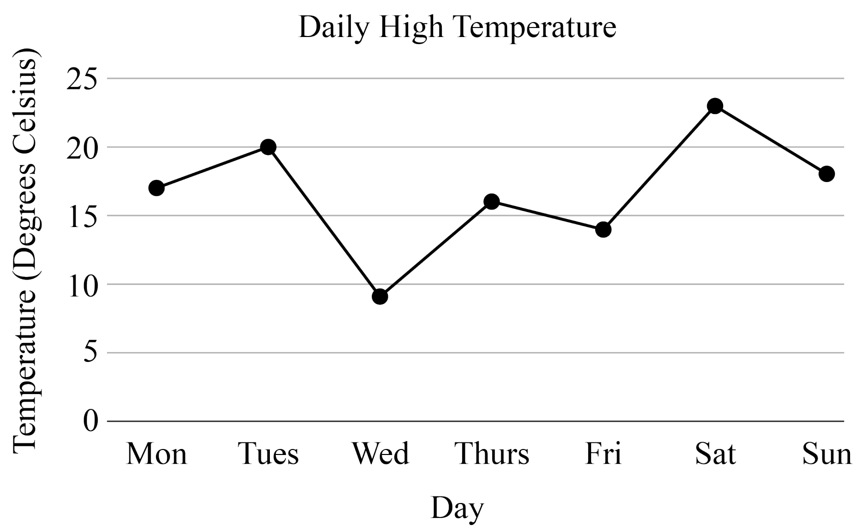
(c) Estimate the biggest change of temperature from one day to the next during this week.

Themes: Data Management, Number Sense

Problem of the Week
Problem A and Solution
Tracking Temperatures

Problem

The weather station at the University of Waterloo records information every day. The graph below shows the high temperature each day for one week.



(a) Estimate the highest temperature recorded during this week.

(b) Estimate lowest temperature recorded during this week.

(c) Estimate the biggest change of temperature from one day to the next during this week.

Solution

(a) The highest temperature occurs on Saturday. It is between 20 degrees and 25 degrees. It appears to be slightly closer to 25 degrees than 20 degrees, so we will estimate the temperature is 23 degrees.

(b) The lowest temperature occurs on Wednesday. It is between 5 degrees and 10 degrees, but is very close to 10 degrees. We will estimate the temperature is 9 degrees.

(c) We estimate the temperature on Monday to be 17 degrees, the temperature on Tuesday to be 20 degrees, the temperature on Thursday to be 16 degrees, the temperature on Friday to be 14 degrees, and the temperature on Sunday to be 18 degrees, along with the estimations in parts (a) and (b).
Thus, from Monday to Tuesday we estimate that it became $20 - 17 = 3$ degrees warmer.
From Tuesday to Wednesday we estimate that it became $20 - 9 = 11$ degrees colder.
From Wednesday to Thursday we estimate that it became $16 - 9 = 7$ degrees warmer.
From Thursday to Friday we estimate that it became $16 - 14 = 2$ degrees colder.
From Friday to Saturday we estimate that it became $23 - 14 = 9$ degrees warmer.
From Saturday to Sunday we estimate that it became $23 - 18 = 5$ degrees colder.
Therefore, we can estimate that the biggest change of temperature from one day to the next is 11 degrees between Tuesday and Wednesday.

Problem of the Week

Problem A

What Number Am I?

I am a 3-digit number.

The sum of my digits is 11.

The product of my digits is 16.

My digits are in decreasing order from the hundreds digit to the ones digit.

I have no repeated digits.

What number am I?

Themes: Computational Thinking, Number Sense

Problem of the Week

Problem A and Solution

What Number Am I?

Problem

I am a 3-digit number.

The sum of my digits is 11.

The product of my digits is 16.

My digits are in decreasing order from the hundreds digit to the ones digit.

I have no repeated digits.

What number am I?

Solution

We start by determining the ways to multiply three single-digits to get a product of 16. Here are the possibilities:

$1 \times 4 \times 4$ (in any order)
$1 \times 2 \times 8$ (in any order)
$2 \times 2 \times 4$ (in any order)

Since the number we are looking for does not have any repeated digits, then the digits in the number must be 1, 2, and 8. We can confirm this conclusion by noticing that the sum of these digits is $1 + 2 + 8 = 11$.

Since the digits appear in decreasing order, the number must be 821.

Problem of the Week
Problem A
Gym Budgets

Ms Lukezich needs to order sports equipment for the gym. There will be a maximum of 40 students using the equipment at any time. She needs the following equipment:

- one soccer ball for each pair of students
- one parachute for each group of 10 students
- three tennis balls for each group of 4 students

One soccer ball costs \$4. One parachute costs \$25. One tennis ball costs \$2.

How much will it cost to buy all of the required equipment?

Theme: Number Sense

Problem of the Week
Problem A and Solution
Gym Budgets

Problem

Ms Lukezich needs to order sports equipment for the gym. There will be a maximum of 40 students using the equipment at any time. She needs the following equipment:

- one soccer ball for each pair of students
- one parachute for each group of 10 students
- three tennis balls for each group of 4 students

One soccer ball costs \$4. One parachute costs \$25. One tennis ball costs \$2.

How much will it cost to buy all of the required equipment?

Solution

One way to solve this is to figure out the maximum number of groups that could be using each piece of equipment. We will do this by assuming we have 40 students, since that is the maximum number of students using the equipment at any time.

- Since she needs 1 soccer ball for every 2 students, Ms Lukezich needs $40 \div 2 = 20$ soccer balls.

  The soccer balls will cost a total of $20 \times \$4 = \$80$.

- Since she needs 1 parachute for every 10 students, Ms Lukezich needs $40 \div 10 = 4$ parachutes.

  The parachutes will cost a total of $4 \times \$25 = \$100$.

- Since she needs 3 tennis balls for every 4 students, Ms Lukezich needs $40 \div 4 = 10$ sets of 3 tennis balls. Thus, she needs $10 \times 3 = 30$ tennis balls altogether.

  The tennis balls will cost a total of $30 \times \$2 = \$60$.

Therefore, the total cost for the sports equipment is $\$80 + \$100 + \$60 = \$240$.

Alternatively, we could make a table for each piece of equipment to determine how much each will cost. First, we make a table for the soccer balls.

Number of Soccer Balls	**Number of Students**	**Total Cost, in $**
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20

We could continue writing out rows in the table until we determine that 20 balls meets the needs of 40 students, for a cost of \$80. Or we might notice at this point that since $40 = 4 \times 10$, then the cost of soccer balls for 40 students is equal to 4 times the cost of soccer balls for 10 students. So the cost for the soccer balls is $4 \times \$20 = \$80$.

Next, we make a table for the parachutes.

Number of Parachutes	**Number of Students**	**Total Cost, in $**
1	10	25
2	20	50
3	30	75
4	40	100

So the cost for the parachutes is \$100.

Finally, we make a table for the tennis balls. Note that one set of 3 tennis balls costs $3 \times \$2 = \$6$.

Number of Sets of 3 Tennis Balls	**Number of Students**	**Total Cost, in $**
1	4	6
2	8	12
3	12	18
4	16	24
5	20	30

We could continue writing out rows in the table until we determine that 10 sets of tennis balls meets the needs of 40 students for a cost of \$60. Or, we might notice at this point that since $40 = 2 \times 20$, then the cost of the tennis balls for 40 students is equal to 2 times the cost of tennis balls for 20 students. So the cost for the tennis balls is $2 \times \$30 = \$60$.

Once again, we get a total cost of $\$80 + \$100 + \$60 = \$240$ for the sports equipment.

Problem of the Week

Problem A

Apple Picking

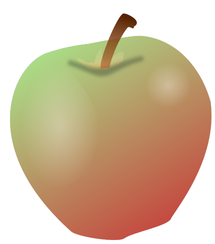
Caleb's family went apple picking. They came home with 24 apples.

(a) They dried one sixth of the apples to make snacks. How many apples did they dry?

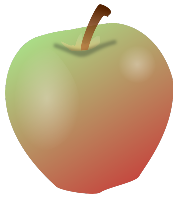
(b) They used one third of the apples to make applesauce. It takes two apples to make one jar of applesauce. How many jars did they make?

(c) They used one fourth of the apples to make two pies. How many apples were in each pie?

(d) They saved the rest of the apples to eat. How many apples did they have to eat?



Theme: Number Sense



Problem of the Week
Problem A and Solution
Apple Picking

Problem

Caleb's family went apple picking. They came home with 24 apples.

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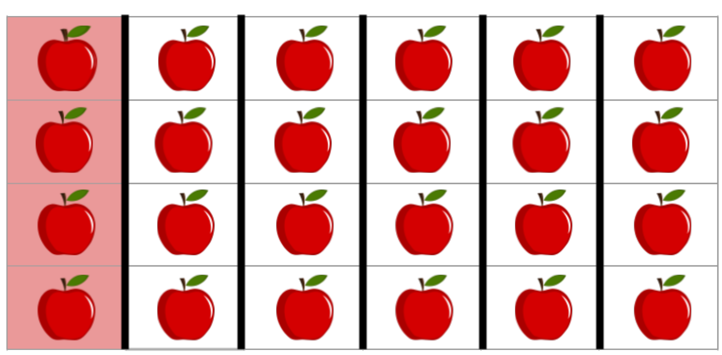
(c) They used one fourth of the apples to make two pies. How many apples were in each pie?

(d) They saved the rest of the apples to eat. How many apples did they have to eat?

Solution

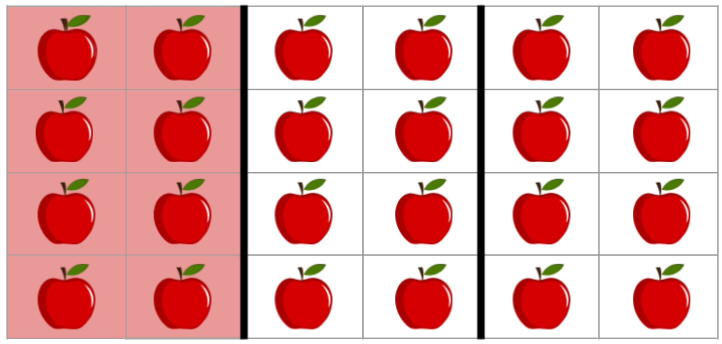
In each part, we will use a grid that has the 24 apples arranged in 4 rows and 6 columns, with 1 apple in each cell, and use this to divide the apples into smaller groups.

(a) We divide the grid of apples into six equal groups by placing a dividing line between each column.



Since each group contains 4 apples, we know that 4 apples were dried for snacks.

(b) We divide the grid of apples into three equal groups by placing a dividing line between the second and third column, and between the fourth and fifth column.

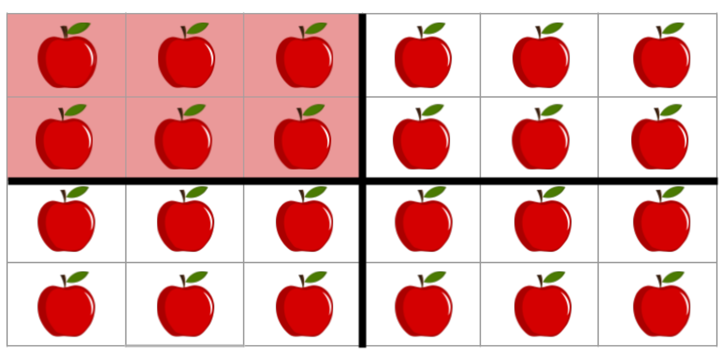


Since each group contains 8 apples, know that one third of the apples is 8 apples. Since it takes 2 apples to make one jar of applesauce, we can skip count by 2s to determine the number of jars they made. This is summarized in the table below.

Number of Apples	2	4	6	8
Number of Jars	1	2	3	4

Therefore, Caleb's family made 4 jars of applesauce.

(c) We divide the grid of apples into four equal groups by placing a dividing line between the third and fourth column, and between the second and third row.



Since each group contains 6 apples, we know that one fourth of the apples is 6 apples. Since they made 2 pies, then half of these apples were used in each pie. So there were 3 apples in each pie.

(d) From the previous three parts, we know that the family used $4 + 8 + 6 = 18$ apples. This means they have $24 - 18 = 6$ apples left for eating.

Problem of the Week
Problem A
Fishing Frenzy

In one hour, the eight person crew of the **Hook Line and Sinker** boat caught 120 fish. They then sold each fish for \$3 and divided the total amount of money equally between the crew members.

How much money did each crew member receive?

Theme: Number Sense

Problem of the Week
Problem A and Solution
Fishing Frenzy

Problem

In one hour, the eight person crew of the **Hook Line and Sinker** boat caught 120 fish. They then sold each fish for \$3 and divided the total amount of money equally between the crew members.

How much money did each crew member receive?

Solution

One way to solve this problem is to divide the number of fish by the number of people in the crew: $120 \div 8 = 15$. Since each person's share of the catch is 15 fish, and each fish is worth \$3, then each person should receive $15 \times \$3 = \$45$ after selling the fish.

Another way to determine each person's share is distribute the money earned into 8 equal piles. However, you probably don't want to do this \$1 at a time.

The total amount earned is $120 \times \$3 = \$360$. You might start by distributing \$10 at a time to each crew member, until you have less than \$80 left. After doing so, each crew member would have received \$10 four times, so \$40 each, and there would be \$40 left to share among the crew.

Now you can distribute that money \$1 or \$2 at a time, or you might try a higher number like \$5. If you distribute \$5 at a time, each crew member would receive \$5 once.

At this point there would be no money left from the original \$360.

Therefore, each person receives $\$40 + \$5 = \$45$ as their share of the catch.

Problem of the Week
Problem A
Time After Time

Assume that the current time matches the time shown on the clock, and the current time is in the morning.

The school day ends in 5 hours and 6 minutes from the time shown on the clock.

(a) How many more minutes are left until the end of the school day?

(b) What time does school end?

(c) School started at 8:55 a.m. How much time has passed, in hours and minutes, between the start of the school day and the time shown on the clock?

Themes: Geometry & Measurement, Number Sense

Problem of the Week

Problem A and Solution

Time After Time

Problem

Assume that the current time matches the time shown on the clock, and the current time is in the morning.

The school day ends in 5 hours and 6 minutes from the time shown on the clock.

(a) How many more minutes are left until the end of the school day?

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(c) School started at 8:55 a.m. How much time has passed, in hours and minutes, between the start of the school day and the time shown on the clock?

Solution

(a) Since each hour has 60 minutes, then 5 hours have a total of $5 \times 60 = 300$ minutes. So the total number of minutes left until the end of the school day is $300 + 6 = 306$ minutes.

(b) The time showing on the clock is 10:09 a.m. We will add 5 hours and 6 minutes to this time. Two hours after 10:09 a.m. it will be 12:09 p.m. Three hours after that it will be 3:09 p.m. Six minutes later it will be 3:15 p.m. Thus, school ends at 3:15 p.m.

(c) We start at 8:55 a.m. One hour after that it will be 9:55 a.m. Five minutes after that it will be 10:00 a.m. Nine minutes after that it will be 10:09 a.m. Thus, a total of 1 hour and $5 + 9 = 14$ minutes have passed between the start of the school day and the time shown on the clock.

Problem of the Week

Problem A

Bus Stops

An empty bus starts its daily route. Along the way it makes several stops. At each stop, some passengers board the bus and some passengers exit the bus.

The table shows how many passengers get on and get off the bus at each stop.

Stop Number	Number Boarding	Number Exiting
1	23	0
2	17	12
3	13	3
4	1	9
5	2	8

(a) How many passengers board the bus in total?

(b) The bus has seats for 30 passengers. Once all the seats are full, passengers must stand. Between the 3$^{\text{rd}}$ and 4$^{\text{th}}$ stops, are there enough seats for all the passengers? If not, how many passengers must stand?

(c) The bus gets a flat tire at the 6$^{\text{th}}$ stop so all the passengers need to exit. How many passengers get off the bus at the 6$^{\text{th}}$ stop?

Theme: Number Sense

Problem of the Week
Problem A and Solution
Bus Stops

Problem

An empty bus starts its daily route. Along the way it makes several stops. At each stop, some passengers board the bus and some passengers exit the bus.

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(a) How many passengers board the bus in total?

(b) The bus has seats for 30 passengers. Once all the seats are full, passengers must stand. Between the 3^{rd} and 4^{th} stops, are there enough seats for all the passengers? If not, how many passengers must stand?

(c) The bus gets a flat tire at the 6^{th} stop so all the passengers need to exit. How many passengers get off the bus at the 6^{th} stop?

Solution

(a) The total number of passengers who board the bus is: $23 + 17 + 13 + 1 + 2 = 56$.

(b) The table summarizes the number of passengers on the bus after each stop.

Stop Number	Number Boarding	Number Exiting	Total Passengers on the Bus
1	23	0	23
2	17	12	$23 + 17 - 12 = 28$
3	13	3	$28 + 13 - 3 = 38$
4	1	9	$38 + 1 - 9 = 30$
5	2	8	$30 + 2 - 8 = 24$

Thus, between the 3^{rd} and 4^{th} stops, there are 38 passengers on the bus. Since there are only 30 seats, there are not enough seats for all the passengers. The number of passengers who must stand is $38 - 30 = 8$.

(c) From the table in (b), we know that 24 passengers are on the bus when it arrives at the 6^{th} stop. They would all need to exit. Thus, 24 passengers get off the bus at the 6^{th} stop.

Problem of the Week
Problem A
Bright Beautiful Banners

A club is making a large, colourful banner to carry in a parade.

When you buy fabric, you specify the exact length of material you need. The table below shows information about the amount of fabric of each colour needed for the banner design.

Colour	**Amount of Fabric Needed**
red	two times as much fabric as purple
orange	one half as much fabric as white
purple	one third as much fabric as orange
white	four times as much fabric as green
green	half a metre

(a) How many metres of each colour fabric are needed to make the banner?

(b) The fabric costs \$6 per metre. Determine the total cost of the fabric required to make the banner.

Theme: Number Sense

Problem of the Week
Problem A and Solution
Bright Beautiful Banners

Problem

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When you buy fabric, you specify the exact length of material you need. The table below shows information about the amount of fabric of each colour needed for the banner design.

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white	four times as much fabric as green
green	half a metre

(a) How many metres of each colour fabric are needed to make the banner?

(b) The fabric costs \$6 per metre. Determine the total cost of the fabric required to make the banner.

Solution

(a) Since $\frac{1}{2}$ m of green fabric is needed, and the club needs 4 times as much white fabric as green, then they must need $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=2$ m of white fabric.

Since they need $\frac{1}{2}$ as much orange fabric as white, then they must need 1 m of orange fabric.

Since they need $\frac{1}{3}$ as much purple fabric as orange, then they must need $\frac{1}{3}$ m of purple fabric.

Since they need 2 times as much red fabric as purple, then they must need $\frac{2}{3}$ m of red fabric.

(b) The club needs $2+1+\frac{1}{3}+\frac{2}{3}+\frac{1}{2}=4\frac{1}{2}$ m of fabric in total.

Since the fabric costs \$6 per metre, the total cost of 4 m of fabric is $4\times\$6=\$24$. The cost of $\frac{1}{2}$ m of fabric is $\frac{1}{2}\times\$6=\$3$. So the total cost of the fabric required to make the banner is $\$24+\$3=\$27$.

Problem of the Week

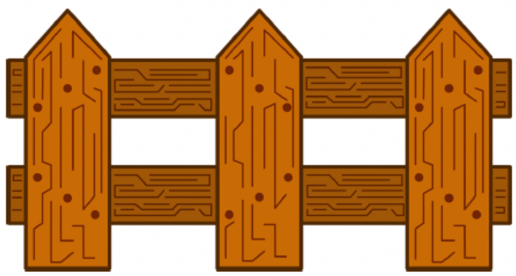
Problem A

Fencepost Problem

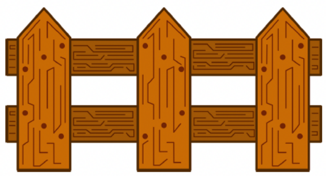
Elyas helps his parents install fence posts in their backyard. They install a row of 7 equally-spaced posts. The posts are all the same size. The distance between the middle of the 2^{nd} post and the middle of the 5^{th} post is four and a half meters.

What is the distance between the middle of the first post and the middle of the last post?

Hint: You might start by drawing a diagram of the fence posts and labelling it with the distance you know.



Themes: Geometry & Measurement, Number Sense



Problem of the Week
Problem A and Solution
Fencepost Problem

Problem

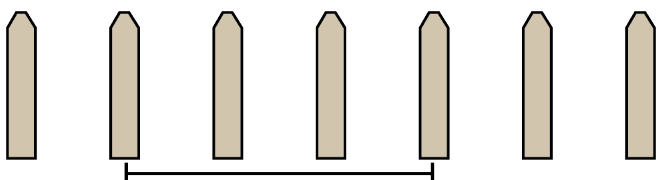
Elyas helps his parents install fence posts in their backyard. They install a row of 7 equally-spaced posts. The posts are all the same size. The distance between the middle of the 2nd post and the middle of the 5th post is four and a half meters.

What is the distance between the middle of the first post and the middle of the last post?

HINT: You might start by drawing a diagram of the fence posts and labelling it with the distance you know.

Solution

There are many ways to calculate the distance. We start with a diagram of the posts, labelled with the distance of $4\frac{1}{2}$ metres between the middle of the 2nd and 5th posts.

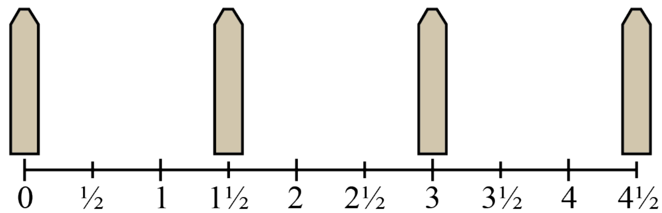


Solution 1

Notice that there are a total of three gaps between the 2nd and 5th posts. There are also a total of three gaps between the 1st and 2nd posts and the 5th and 7th posts. Since the distances between the middle of adjacent posts are all the same, then the total distance between the 1st and 2nd posts and the 5th and 7th posts must also be $4\frac{1}{2}$ m. Since there are six gaps in total, the distance between the middle of the first post and the middle of the last post is $4 + \frac{1}{2} + 4 + \frac{1}{2} = 9$ m.

Solution 2

Another way to solve this problem is to draw a number line from 0 to $4\frac{1}{2}$ m and space the 2^{nd}, 3^{rd}, 4^{th}, and 5^{th} fenceposts evenly along this line.

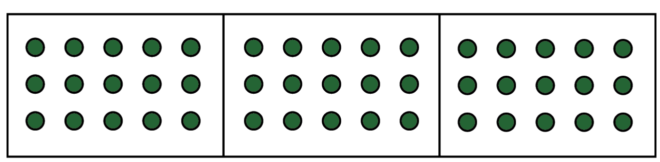


From this, we observe that the distance from the middle of one post to the middle of an adjacent post is $1\frac{1}{2}$ m. Since there are six gaps in total between adjacent posts, the distance between the middle of the first post and the middle of the last post is:

$$1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}=9 \text{ m}$$

Solution 3

First we convert the distance to centimetres: $4\frac{1}{2}$ m $=450$ cm. Then we use a tape diagram. We observe that there are 3 gaps between the 2^{nd} and 5^{th} posts. That means we need to divide 450 cm into three equal distances. To make it easier, since $450=45\times 10$, we let one dot represent 10 cm, and then distribute 45 dots into the three equal pieces of our tape diagram.



Each piece has a total of 15 dots. This means the distance between adjacent posts is $15\times 10=150$ cm. Since there are 6 gaps in total between the first post and the last post, the total distance is:

$$150+150+150+150+150+150=900 \text{ cm or } 9 \text{ m}$$

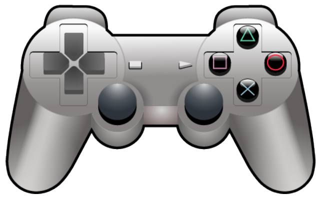
Problem of the Week
Problem A
Savings

Nethra is saving to buy a new gaming system. She needs \$829 in total to buy the console. She has already saved \$117.

Nethra has two part-time jobs. She works at a grocery store every Tuesday and Thursday for 2 hours each day. From the grocery store job she earns \$15 per hour. She works at a restaurant every Saturday for 5 hours. From the restaurant job she earns \$13 per hour.

Nethra gets paid every week on Sunday. That day she pays her parents \$10 for transportation and buys herself \$5 worth of treats. She saves the rest of the money she earned during the week.

How many weeks does Nethra have to work until she has saved enough money to buy the gaming system?



Problem of the Week
Problem A and Solution
Savings

Problem

Nethra is saving to buy a new gaming system. She needs \$829 in total to buy the console. She has already saved \$117.

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How many weeks does Nethra have to work until she has saved enough money to buy the gaming system?

Solution

Since Nethra has already saved \$117, she needs $\$829 - \$117 = \$712$ more to buy the gaming system.

From her job at the grocery store, she earns $4 \times \$15 = \$60$ each week.
From her job at the restaurant, she earns $5 \times \$13 = \$65$ each week.
This is a total of $\$60 + \$65 = \$125$ each week.

Each week she spends $\$10 + \$5 = \$15$.
This means she can save $\$125 - \$15 = \$110$ each week for the gaming system.

Now we can make a table to show how much she saves over multiple weeks:

Week	Total Savings (\$)
1	110
2	220
3	330
4	440
5	550
6	660
7	770

After 7 weeks Nethra has saved more than \$712, so she has enough money to buy the gaming system.

Problem of the Week
Problem A
Delivery Dilemma

Soraya and Aydin both work as delivery drivers for a local business. Today they have 64 packages to deliver in total. Soraya was given three times as many packages as Aydin to deliver.

They decide it would be better for each person to deliver the same number of packages. How many packages should Soraya give Aydin so that they have the same number of packages?

Themes: Algebra, Number Sense

Problem of the Week

Problem A and Solution

Delivery Dilemma

Problem

Soraya and Aydin both work as delivery drivers for a local business. Today they have 64 packages to deliver in total. Soraya was given three times as many packages as Aydin to deliver.

They decide it would be better for each person to deliver the same number of packages. How many packages should Soraya give Aydin so that they have the same number of packages?

Solution

Solution 1

One way to solve the problem is to guess and check to figure out how many packages were given to each driver. We can organize our guesses in a table where we keep track of how many packages Aydin has, how many packages Soraya has, and how many packages they have in total. Let's start by guessing that Aydin has 10 packages until we find a combination that results in a total of 64 packages.

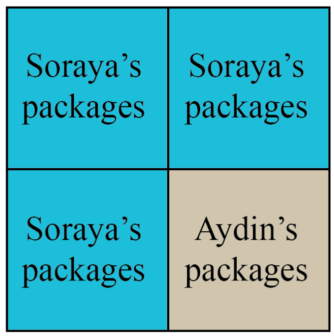
Aydin's Packages	Soraya's Packages	Total Packages
10	$3 \times 10 = 30$	$10 + 30 = 40$
11	$3 \times 11 = 33$	$11 + 33 = 44$
12	$3 \times 12 = 36$	$12 + 36 = 48$
13	$3 \times 12 = 39$	$13 + 39 = 52$
14	$3 \times 14 = 42$	$14 + 42 = 56$
15	$3 \times 15 = 45$	$15 + 45 = 60$
16	$3 \times 16 = 48$	$16 + 48 = 64$

Thus, we see that Aydin started with 16 packages and Soraya started with 48 packages. If they want to deliver the same number of packages, each should take half of the total number of packages. Half of 64 is 32 packages.

So if Soraya gives Aydin $48 - 32 = 16$ packages, then Aydin will have $16 + 16 = 32$ packages and each of them will have the same number to deliver.

Solution 2

Another way to solve this problem is to use fractions. If Soraya was given three times as many packages as Aydin to deliver, then adding Soraya and Aydin's packages together should give us four times as many packages as Aydin has. This means that Aydin has $\frac{1}{4}$ of the total number of packages and Soraya has $\frac{3}{4}$ of the total number of packages. This is shown in the following diagram. The large square represents the total number of packages. The large square is divided into quarters, with three of the quarters representing Soraya's packages and one of the quarters representing Aydin's packages.



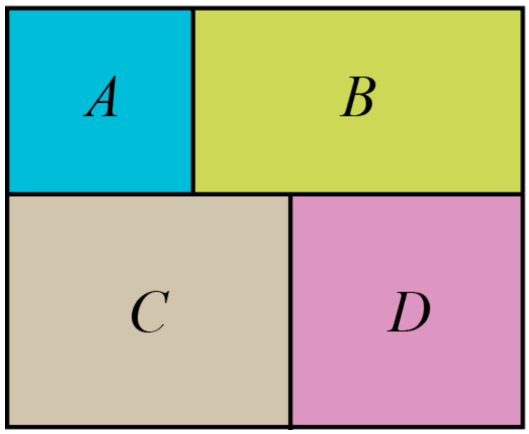
In order for Aydin and Soraya to each have the same number of packages, Soraya must give Aydin $\frac{1}{4}$ of the total number of packages, so that they each have $\frac{1}{2}$. Since there are 64 packages in total, $\frac{1}{4}$ of 64 is equal to $64 \div 4 = 16$. Thus, Soraya should give Aydin 16 packages.

Problem of the Week

Problem A

Quilting Puzzle

The employees of Sew Inspired need to make a tiny quilt for a special project using four different colours of fabric. Their quilt pattern is a rectangle divided into four smaller rectangles, marked A, B, C, and D.



Piece A is a square with area 16 cm^2 and piece D is a square with area 25 cm^2. The horizontal distance between the right side of piece A and the left side of piece D is 2 cm. What is the area of the entire quilt?

Themes: Geometry & Measurement, Number Sense

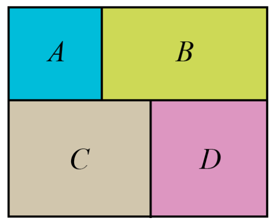
Problem of the Week

Problem A and Solution

Quilting Puzzle

Problem

The employees of Sew Inspired need to make a tiny quilt for a special project using four different colours of fabric. Their quilt pattern is a rectangle divided into four smaller rectangles, marked A, B, C, and D.



Piece A is a square with area 16 cm^2 and piece D is a square with area 25 cm^2. The horizontal distance between the right side of piece A and the left side of piece D is 2 cm. What is the area of the entire quilt?

Solution

To calculate the area of the quilt we will determine the lengths of its sides. We know that piece A is a square with area 16 cm^2. We also know that the lengths of the sides of a square must be the same. So if the length of one side of a square is n, then the area of the square must be $n \times n$. By trial and error, we determine $4 \times 4 = 16$, and so the length of each side of piece A must be 4 cm. Another way to determine the lengths of the sides of piece A is to start with 16 unit squares (using blocks or cut out of paper) and determine how to arrange them into a larger square. The only possible arrangement is a 4×4 square.

Similarly, piece D is a square with area 25 cm^2. Since $5 \times 5 = 25$, the length of each side of piece D must be 5 cm.

The opposite sides of a rectangle must be the same length, so the width of the quilt (i.e. the vertical side) is equal to the sum of the side lengths of pieces A and D. Thus, the width of the quilt is equal to $4 + 5 = 9$ cm. Similarly, we know the bottom of piece A is on the same line as the top of piece D, and the horizontal distance between the two pieces is 2 cm. Therefore the length of the quilt (i.e. the horizontal side) is equal to the sum of the side lengths of pieces A and D, plus 2. Thus, the length of the quilt is equal to $4 + 5 + 2 = 11$ cm.

Now we can calculate the area of the entire quilt. The area of this rectangle is the product of its length and width. So the area of the entire quilt is $9 \times 11 = 99$ cm^2.

Problem of the Week

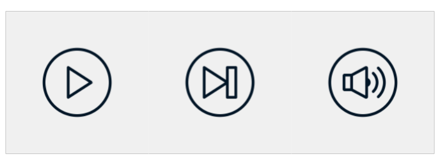
Problem A

Video Viewing

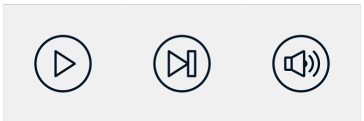
Jo likes to watch videos. Sometimes she watches them at normal speed. However, she also has the option to watch them at $\frac{1}{4}$ as fast as normal speed, $\frac{1}{2}$ as fast as normal speed, or 2 times as fast as normal speed. The table below lists the normal play times for four videos and the speed at which Jo watches each one.

	Normal Speed Time	**Speed Jo Watches Video At**
Video A	50 seconds	$\frac{1}{4}$ speed
Video B	2 minutes 15 seconds	normal speed
Video C	1 minute 40 seconds	$\frac{1}{2}$ speed
Video D	6 minutes 20 seconds	2 times speed

What is the total time Jo spent watching these four videos?



Themes: Geometry & Measurement, Number Sense



Problem of the Week

Problem A and Solution

Video Viewing

Problem

Jo likes to watch videos. Sometimes she watches them at normal speed. However, she also has the option to watch them at $\frac{1}{4}$ as fast as normal speed, $\frac{1}{2}$ as fast as normal speed, or 2 times as fast as normal speed. The table below lists the normal play times for four videos and the speed at which Jo watches each one.

	Normal Speed Time	**Speed Jo Watches Video At**
Video A	50 seconds	$\frac{1}{4}$ speed
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Video D	6 minutes 20 seconds	2 times speed

What is the total time Jo spent watching these four videos?

Solution

If a video is playing at $\frac{1}{4}$ speed, then it will take 4 times as long to play as normal speed. So a 50 second video will take $50 \times 4 = 200$ seconds to watch.

If a video is playing at $\frac{1}{2}$ speed, then it will take 2 times as long to play as normal speed. So a 1 minute and 40 second video will take 2 minutes and $40 \times 2 = 80$ seconds to watch.

If a video is playing at 2 times speed then it will take half as long to play as normal speed. Half of 6 minutes is 3 minutes and half of 20 seconds is 10 seconds.

Now we can add up all the minutes and seconds of playing times of all four videos:

$$2 + 2 + 3 = 7 \text{ minutes and } 200 + 15 + 80 + 10 = 305 \text{ seconds}$$

There are 60 seconds in 1 minute. When we skip count by 60, we get 60, 120, 180, 240, 300. Thus, there are 5 minutes in 305 seconds with $305 - 300 = 5$ seconds left over.

So Jo spent $7 + 5 = 12$ minutes and 5 seconds watching these four videos.

Problem of the Week
Problem A
Can You Kayak?

At Bo's Boat Shop, all kayaks have the same number of seats and all rowboats have the same number of seats. A kayak has fewer seats than a rowboat. Two kayaks and two rowboats have a total of 10 seats. One kayak and three rowboats have a total of 11 seats.

How many seats are in one kayak?

Theme: Number Sense

Problem of the Week
Problem A and Solution
Can You Kayak?

Problem

At Bo's Boat Shop, all kayaks have the same number of seats and all rowboats have the same number of seats. A kayak has fewer seats than a rowboat. Two kayaks and two rowboats have a total of 10 seats. One kayak and three rowboats have a total of 11 seats.

How many seats are in one kayak?

Solution

Since two kayaks and two rowboats have a total of 10 seats, then half as many kayaks and rowboats will have half as many seats. That is, one kayak and one rowboat will have a total of $10 \div 2 = 5$ seats.

Since a kayak has less seats than a rowboat, then it must be the case that one kayak has 1 seat and one rowboat has 4 seats, or that one kayak has 2 seats and one rowboat has 3 seats.

If one kayak has 1 seat and one rowboat has 4 seats, then one kayak and three rowboats have $1 + 4 + 4 + 4 = 13$ seats. Since we're told that one kayak and three rowboats have 11 seats, this is not a possible solution.

If one kayak has 2 seats and one rowboat has 3 seats, then one kayak and three rowboats have $2 + 3 + 3 + 3 = 11$ seats. Since we're told that one kayak and three rowboats have 11 seats, this is a possible solution.

Thus, 2 seats are in one kayak.