Problème de la Semaine

Problèmes et Solutions 2024-2025

Problème A

3e/4e année

Le CENTRE d'ÉDUCATION
en MATHÉMATIQUES et en INFORMATIQUE
cemc@uwaterloo.ca

Table des Matières

Les problèmes dans ce livret sont organisés par thème. Un problème peut apparaître dans plusiers thèmes. Cliquer sur le nom du thème ci-dessous pour sauter à cette section.

Algèbre (A)

Amène-moi à la couverture

Problème de la semaine

Problème A

Les claquettes en cadence

Dans le cadre de leur unité de danse, Yelena, Jackson et Todd ont dû créer une chorégraphie de 30 secondes. Comme tous les trois sont des danseurs de claquettes, ils ont décidé que chacun ferait un solo de 5 secondes, et que les 15 secondes restantes seraient consacrées à une section de claquettes en groupe.

Durant son solo, Yelena tapait à une cadence de 5 frappes par seconde.

Durant son solo, Jackson tapait à une cadence de 4 frappes par seconde.

Durant son solo, Todd tapait à une cadence de 3 frappes par seconde.

Durant la section de claquettes en groupe, ils ont tous tapé à la cadence la plus lente afin que tout le monde puisse suivre.

Combien de frappes chaque danseur a-t-il effectuées au total durant la chorégraphie complète ?

Thème : Algèbre

Problem of the Week
Problem A and Solution
Tapping to Success

Problem

For their dance unit, Yelena, Jackson, and Todd had to come up with a 30 second dance routine. All three of them are tap dancers, so they decided that each of them would do a 5 second solo tap, and the remaining 15 seconds would be a group tap.

During Yelena's solo tap section, she tapped at a rate of 5 taps per second.

During Jackson's solo tap section, he tapped at a rate of 4 taps per second.

During Todd's solo tap section, he tapped at a rate of 3 taps per second.

For their group tap section, they all tapped at the slowest rate so that they could all keep up.

How many taps did each dancer do in total during the full routine?

Solution

We use a table to keep track of how many taps each dancer did during their solo.

Dancer	Total Taps after 1 s	Total Taps after 2 s	Total Taps after 3 s	Total Taps after 4 s	Total Taps after 5 s
Yelena	5	10	15	20	25
Jackson	4	8	12	16	20
Todd	3	6	9	12	15

Since they all tapped at the slowest rate for the group tap, then they all tapped at 3 taps per second, which is the same as Todd's rate in the solo. From the table above, we know that in 5 seconds, Todd tapped 15 times. Then at this rate, in 10 seconds they would tap 30 times, and in 15 seconds they would tap 45 times.

Then we can conclude the following:

- Yelena tapped a total of $25 + 45 = 70$ times during the full routine.
- Jackson tapped a total of $20 + 45 = 65$ times during the full routine.
- Todd tapped a total of $15 + 45 = 60$ times during the full routine.

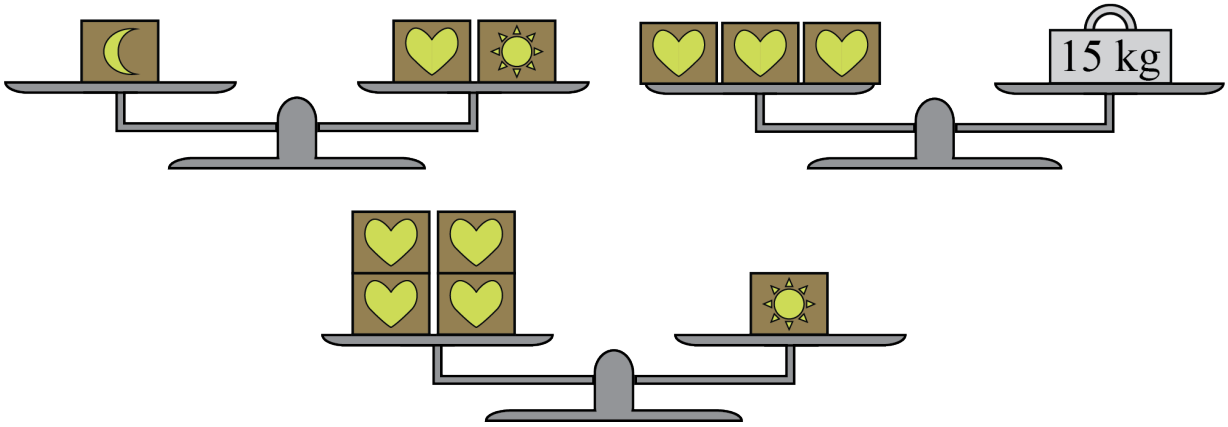
Problème de la semaine

Problème A

L'équilibre des masses mystérieuses

James est responsable de l'expédition de boîtes depuis un centre de distribution. Les contenus des boîtes sont identifiés par des formes étampées dessus : un cœur, une lune ou un soleil. Toutes les boîtes portant la même étampe ont la même masse.

Les diagrammes suivants illustrent ce que James a observé en disposant certaines de ces boîtes et des poids étalons sur une balance.



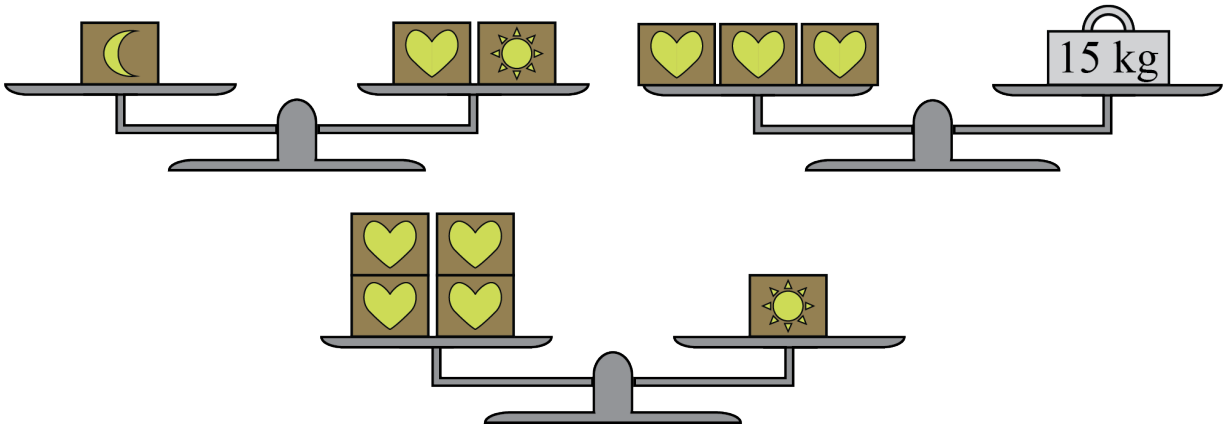
Sachant que chaque balance est en équilibre, détermine la masse de chaque boîte.

Problem of the Week
Problem A and Solution
Balancing Act

Problem

James is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass.

The following diagrams show what James observed when arranging some of the boxes and standard weights on a scale.



Given that each scale is balanced, determine the mass of each box.

Solution

From the diagrams we notice the following.

- One moon box has the same mass as the sum of the mass of a heart box and the mass of a sun box.
- Three heart boxes have a total mass of 15 kg.
- Four heart boxes have the same total mass as one sun box.

Since 3 heart boxes have a total mass of 15 kg, then the mass of 1 heart box must be $\frac{1}{3}$ of 15 kg. Therefore, 1 heart box has a mass of 5 kg.

Since 4 heart boxes have the same mass as 1 sun box, then 1 sun box must have a mass of $5 \times 4 = 20$ kg.

Since 1 moon box has the same mass as the sum of the mass of 1 heart box and the mass of 1 sun box, then 1 moon box must have a mass of $20 + 5 = 25$ kg.

Therefore, 1 moon box has a mass of 25 kg, 1 heart box has a mass of 5 kg, and 1 sun box has a mass of 20 kg.

Problème de la semaine

Problème A

Une logistique de livraison

Soraya et Aydin travaillent tous les deux comme chauffeurs-livreurs pour une entreprise locale. Aujourd'hui, ils doivent livrer 64 colis au total. Soraya a reçu trois fois plus de colis à livrer qu'Aydin.

Ils décident qu'il serait préférable que chacun livre le même nombre de colis. Combien de colis Soraya devrait-elle donner à Aydin pour qu'ils aient le même nombre de colis ?

Thèmes : Algèbre, Sens du nombre

Problem of the Week
Problem A and Solution
Delivery Dilemma

Problem

Soraya and Aydin both work as delivery drivers for a local business. Today they have 64 packages to deliver in total. Soraya was given three times as many packages as Aydin to deliver.

They decide it would be better for each person to deliver the same number of packages. How many packages should Soraya give Aydin so that they have the same number of packages?

Solution

Solution 1

One way to solve the problem is to guess and check to figure out how many packages were given to each driver. We can organize our guesses in a table where we keep track of how many packages Aydin has, how many packages Soraya has, and how many packages they have in total. Let's start by guessing that Aydin has 10 packages until we find a combination that results in a total of 64 packages.

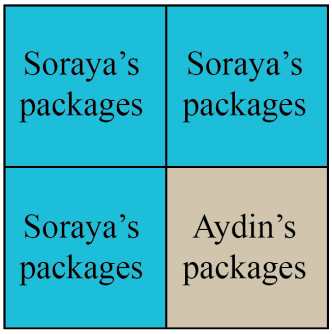
Aydin's Packages	Soraya's Packages	Total Packages
10	$3 \times 10 = 30$	$10 + 30 = 40$
11	$3 \times 11 = 33$	$11 + 33 = 44$
12	$3 \times 12 = 36$	$12 + 36 = 48$
13	$3 \times 12 = 39$	$13 + 39 = 52$
14	$3 \times 14 = 42$	$14 + 42 = 56$
15	$3 \times 15 = 45$	$15 + 45 = 60$
16	$3 \times 16 = 48$	$16 + 48 = 64$

Thus, we see that Aydin started with 16 packages and Soraya started with 48 packages. If they want to deliver the same number of packages, each should take half of the total number of packages. Half of 64 is 32 packages.

So if Soraya gives Aydin $48 - 32 = 16$ packages, then Aydin will have $16 + 16 = 32$ packages and each of them will have the same number to deliver.

Solution 2

Another way to solve this problem is to use fractions. If Soraya was given three times as many packages as Aydin to deliver, then adding Soraya and Aydin's packages together should give us four times as many packages as Aydin has. This means that Aydin has $\frac{1}{4}$ of the total number of packages and Soraya has $\frac{3}{4}$ of the total number of packages. This is shown in the following diagram. The large square represents the total number of packages. The large square is divided into quarters, with three of the quarters representing Soraya's packages and one of the quarters representing Aydin's packages.



In order for Aydin and Soraya to each have the same number of packages, Soraya must give Aydin $\frac{1}{4}$ of the total number of packages, so that they each have $\frac{1}{2}$. Since there are 64 packages in total, $\frac{1}{4}$ of 64 is equal to $64 \div 4 = 16$. Thus, Soraya should give Aydin 16 packages.

Problème de la semaine

Problème A

Calculs circulaires

Lorna utilise les instructions suivantes pour écrire des suites de nombres.

> Étape 1 : Commence avec un nombre entier supérieur à 0.
>
> Étape 2 : Si le nombre est pair, divise-le par 2 pour obtenir le nombre suivant.
> Si le nombre est impair, multiplie-le par 3 et ajoute 1 pour obtenir le nombre suivant.
>
> Étape 3 : Répète l'étape 2 pour continuer la suite.

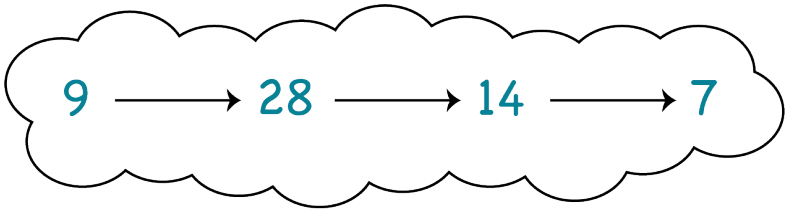
Par exemple, suppose que Lorna commence avec un 9.
Ce nombre est impair, donc le nombre suivant est $9 \times 3 + 1 = 27 + 1 = 28$.
Ce nombre est pair, donc le nombre suivant est $28 \div 2 = 14$.
Ce nombre est pair, donc le nombre suivant est $14 \div 2 = 7$.
Ainsi, les quatre premiers termes de la suite sont 9, 28, 14 et 7.

(a) Suis les instructions en utilisant le nombre de départ donné et écris les 12 premiers termes de chaque suite.

(i) 3 (ii) 13

(b) Que remarques-tu à propos de chaque suite dans la partie (a) ? Que se passerait-il si tu continuais chaque suite ?

(c) Utilise ta réponse à la partie (b) pour prédire le 20^{e} terme de la suite qui commence par 13.



Thèmes : Algèbre, Pensée computationnelle

Problem of the Week
Problem A and Solution
Circular Calculations

Problem

Lorna uses the following instructions to write sequences of numbers.

> Step 1: Start with a whole number greater than 0.
>
> Step 2: If the number is even, divide it by 2 to get the next number.
> If the number is odd, multiply it by 3 and add 1 to get the next number.
>
> Step 3: Repeat Step 2 to continue the sequence.

For example, suppose Lorna starts with 9.
Since this number is odd, the next number is $9 \times 3 + 1 = 27 + 1 = 28$.
Since this number is even, the next number is $28 \div 2 = 14$.
Since this number is even, the next number is $14 \div 2 = 7$.
Thus, the first four numbers in the sequence are 9, 28, 14, and 7.

(a) Follow the instructions using the given starting number and write the first 12 numbers in each sequence.

(i) 3 (ii) 13

(b) What do you notice about each sequence in part (a)? What would happen if you continued each sequence?

(c) Use your answer to part (b) to predict the 20^{th} number in the sequence starting with 13.

Solution

(a) The first 12 numbers in each sequence are as follows.

(i) 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4

(ii) 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2

(b) After each sequence reaches 1, the numbers 4, 2, and 1 repeat over and over.

(c) We can write the first 20 numbers in the sequence starting with 13 by first using part (b) to write out the first 12 numbers in the sequence, and then repeating 4, 2, and 1 until we have 20 numbers.

13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4

Thus, the 20^{th} number is 4.

Teacher's Notes

As an extra challenge, students could attempt this starting with the number 27. Although it takes a long time when you start with the number 27, the sequence will eventually reach 1. It takes 111 numbers to reach 1, as shown.

27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1

Mathematicians believe that starting with any positive integer, the instructions will always lead to a sequence that reaches 1 (or *converges* to 1). This is known as the *Collatz Conjecture*. However, proving this is true for all positive integers is an open problem. There is experimental evidence that shows this is true for very large numbers; however, there is no formal proof that the conjecture holds for all positive integers.

Raisonnement informatiques (C)

Amène-moi à la couverture

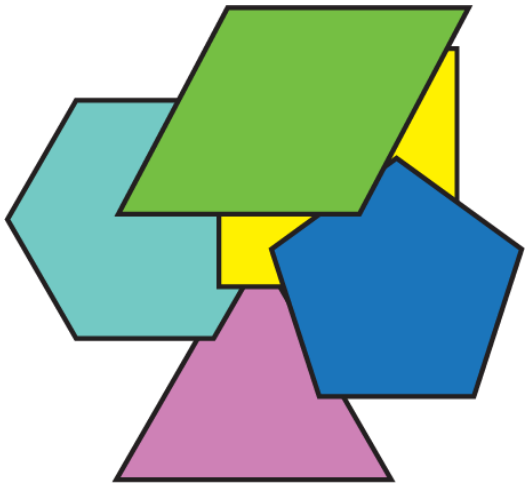
Problème de la semaine

Problème A

Superposer des polygones

Anna a découpé les cinq polygones suivants:

Elle les a ensuite placés sur une table. Voici la vue de dessus:



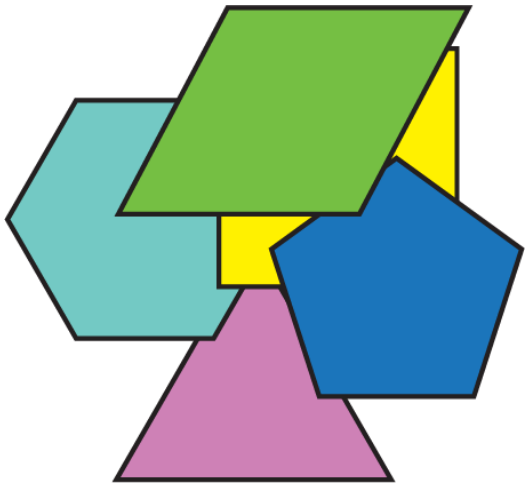
Dans quel ordre a-t-elle placé les polygones sur la table?

Problem of the Week
Problem A and Solution
Stacking Shapes

Problem

Anna cut out the following five polygons.

She then placed them on a table. The top view after doing so is shown.



In what order did she place the polygons on the table?

Solution

First, we recall the names of the five polygons.

Name	Triangle	Square	Parallelogram	Pentagon	Hexagon
Image					

Since the parallelogram has no shapes covering part of it, it must have been the last polygon placed on the table.

The pentagon has the parallelogram covering part of it, so the pentagon must have been placed before the parallelogram.

The square has the pentagon and parallelogram covering part of it, so the square must have been placed before the pentagon and parallelogram.

The hexagon has the square and parallelogram covering part of it, so the hexagon must have been placed before the square and parallelogram.

The triangle has the hexagon, square, and pentagon covering part of it, so the triangle must have been placed before the hexagon, square, and pentagon.

Thus, Anna must have first placed the triangle, then the hexagon, then the square, then the pentagon, then the parallelogram.

Problème de la semaine

Problème A

Quel nombre suis-je ?

Je suis un nombre à 3 chiffres.

La somme de mes chiffres est 11.

Le produit de mes chiffres est 16.

Mes chiffres sont en ordre descendant des centaines jusqu'aux unités.

Aucun chiffre ne se répète.

Quel nombre suis-je?

Thèmes : Pensée computationnelle, Sens du nombre

Problem of the Week
Problem A and Solution
What Number Am I?

Problem

I am a 3-digit number.

The sum of my digits is 11.

The product of my digits is 16.

My digits are in decreasing order from the hundreds digit to the ones digit.

I have no repeated digits.

What number am I?

Solution

We start by determining the ways to multiply three single-digits to get a product of 16. Here are the possibilities:

$1 \times 4 \times 4$ (in any order)
$1 \times 2 \times 8$ (in any order)
$2 \times 2 \times 4$ (in any order)

Since the number we are looking for does not have any repeated digits, then the digits in the number must be 1, 2, and 8. We can confirm this conclusion by noticing that the sum of these digits is $1 + 2 + 8 = 11$.

Since the digits appear in decreasing order, the number must be 821.

Problème de la semaine

Problème A

Code d'accès

Janet possède un coffre-fort qui peut s'ouvrir avec un code à 4 chiffres. Janet a configuré son coffre-fort pour qu'il s'ouvre avec tout code à 4 chiffres respectant les règles suivantes :

1. Le premier chiffre et le dernier chiffre du code ne peuvent pas être égaux.
2. Le deuxième chiffre du code doit être supérieur au troisième chiffre.
3. Le dernier chiffre du code doit être supérieur soit au troisième chiffre, soit au premier chiffre.
4. Au moins un chiffre du code doit être un nombre pair.

Lesquels des codes suivants ouvriraient le coffre-fort ? Justifie tes réponses.

(a) 1234

(b) 4321

(c) 5313

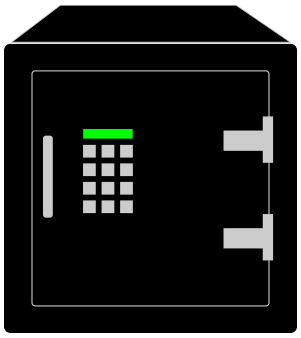
(d) 2644

(e) 3333

(f) 5312

(g) 7437

(h) 5857



Thème : Pensée computationnelle

Problem of the Week
Problem A and Solution
Entry Code

Problem

Janet has a safe that can be opened with a 4-digit code. Janet has set up her safe to open for any 4-digit code that satisfies the following rules:

1. The first digit and the last digit cannot be equal to each other.
2. The second digit must be greater than the third digit.
3. The last digit must be greater than either the third digit or greater than the first digit.
4. At least one digit must be an even number.

Which of the following codes would unlock the safe? Justify your answers.

(a) `1234`

(b) `4321`

(c) `5313`

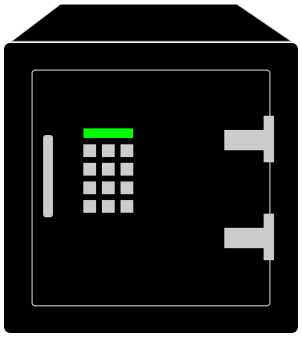
(d) `2644`

(e) `3333`

(f) `5312`

(g) `7437`

(h) `5857`



Solution

We can check each code to see if satisfies the given rules.

(a) `1234` would *not* unlock the safe because the second digit is less than the third digit, so it does not follow rule 2.

(b) `4321` would *not* unlock the safe because the last digit is less than both the first digit and the third digit, so it does not follow rule 3.

(c) `5313` would *not* unlock the safe because there are no even digits, so it does not follow rule 4.

(d) 2644 would unlock the safe.

(e) 3333 would *not* unlock the safe because it breaks all four rules.

(f) 5312 would unlock the safe.

(g) 7437 would *not* unlock the safe because the first and last digit are equal to each other, so it does not follow rule 1.

(h) 5857 would unlock the safe.

Problème de la semaine

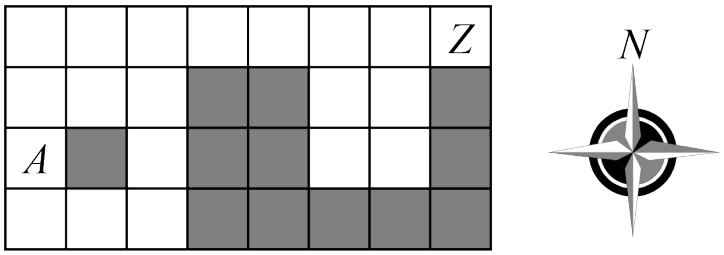
Problème A

Une navigation époustouflante

Juanita et AJ créent des labyrinthes sur du papier quadrillé. Chaque labyrinthe est une grille rectangulaire contenant des cases blanches et des cases grises. Une case blanche est marquée de la lettre A et une autre de la lettre Z.

Pour compléter un labyrinthe, ils doivent partir du A et atteindre le Z en se déplaçant d'une case à la fois dans l'une des directions suivantes : nord (N), est (E), sud (S) ou ouest (O), où le haut de la page est considéré comme le nord. Ils *ne peuvent pas* traverser les cases grises et doivent passer par chacune des cases blanches *exactement une fois*. Autrement dit, ils doivent traverser toutes les cases blanches, mais ils ne peuvent pas passer par l'une d'elles plus d'une fois.

(a) Détermine les directions qu'ils doivent suivre pour compléter avec succès le labyrinthe donné.



(b) AJ crée un autre labyrinthe en modifiant l'emplacement des cases grises dans le labyrinthe de la partie (a). (Les positions du A et du Z demeurent inchangées.) Juanita réussit à compléter ce nouveau labyrinthe en suivant ces directions :

$$E,\ S,\ E,\ E,\ E,\ N,\ E,\ N,\ O,\ O,$$
$$O,\ O,\ N,\ E,\ E,\ E,\ E,\ E,\ E$$

À quoi ressemble le labyrinthe créé par AJ ?

Thèmes : Pensée computationnelle, Géométrie et mesure

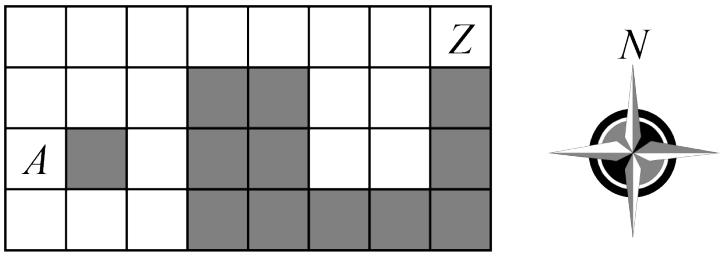
Problem of the Week
Problem A and Solution
Amazing Navigation

Problem

Juanita and AJ create mazes on grid paper. Each maze is a rectangular grid containing white squares and grey squares. One white square is marked A and another is marked Z.

To complete a maze, they start at A and need to reach Z by moving one square at a time in one of the following directions: north (N), east (E), south (S), or west (W), where the top of the page is considered north. They *cannot* go through any of the grey squares and must go through each of the white squares *exactly once*. That is, they must go through all of the white squares but cannot go through any of them more than once.

(a) Determine the directions they need to follow to successfully complete the given maze.



(b) AJ creates another maze by changing where the grey squares are in the maze from part (a). (The locations of A and Z remain unchanged.) Juanita successfully completes this new maze by following these directions:

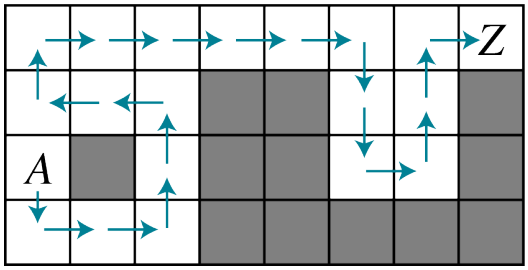
$$E,\ S,\ E,\ E,\ E,\ N,\ E,\ N,\ W,\ W,$$
$$W,\ W,\ N,\ E,\ E,\ E,\ E,\ E,\ E$$

What does AJ's maze look like?

Solution

(a) The first direction must be either N or S, because there is a grey square to the right of A. Suppose they start with N. Then at some point, they must go through the square that is directly below A. However, once they reach that square they will be stuck because they can't go through any white square more than once. So the first direction must be S.

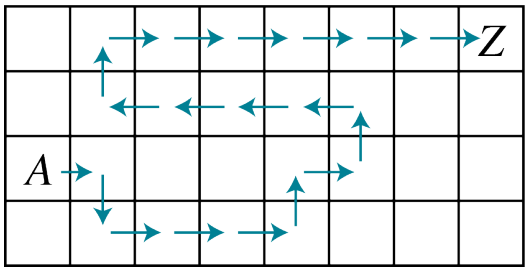
Using similar reasoning, we can complete the maze as shown.



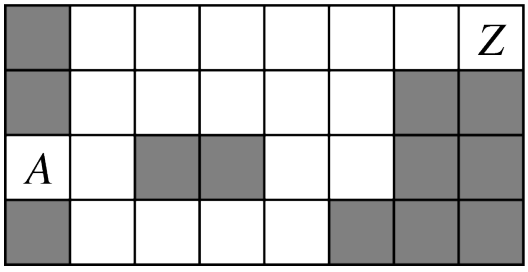
The directions followed are then

$$S,\quad E,\quad E,\quad N,\quad N,\quad W,\quad W,\quad N,\quad E,\quad E,$$
$$E,\quad E,\quad E,\quad S,\quad S,\quad E,\quad N,\quad N,\quad E$$

(b) We first mark the given directions on the maze, as shown.



Since Juanita cannot go through any of the grey squares, and must go through each of the white squares exactly once, the squares that are blank after marking the path must be the grey squares. Thus, AJ's maze is as shown.



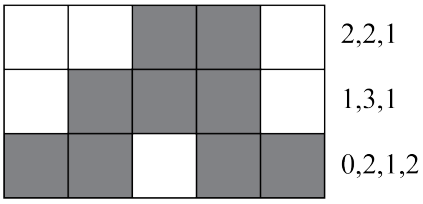
Problème de la semaine

Problème A

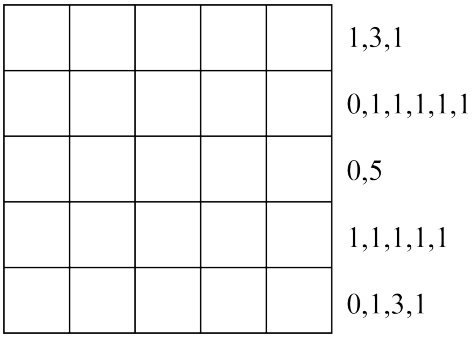
Pixels mystères

Akhil envoie à ses amis des codes leur indiquant comment colorier des cases dans une grille afin de révéler une image secrète. Chaque code comprend une grille vide accompagnée d'une liste de nombres à côté de chaque rangée, utilisée pour colorier la rangée de gauche à droite. Le premier nombre d'une liste représente le nombre de cases vides au début de la rangée. Le deuxième nombre indique le nombre de cases à colorier ensuite. Après cela, les nombres alternent entre le nombre de cases vides et le nombre de cases colorées jusqu'à la fin de la rangée.

Par exemple, si la liste de nombres était 2, 2, 1, alors les 2 premières cases à gauche seraient vides, les 2 cases suivantes seraient colorées et la case suivante serait vide. Une image complétée est montrée.

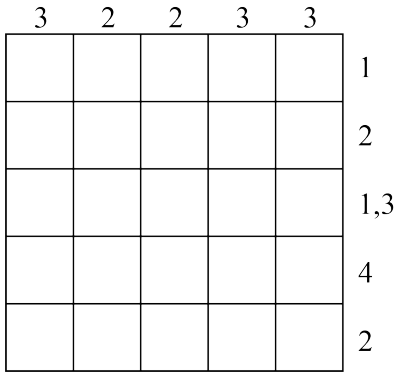


(a) Dessine l'image correspondant au code donné.

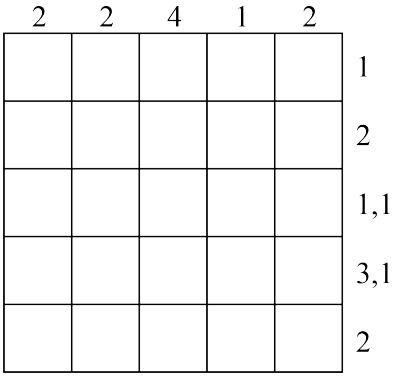


(b) Akhil transforme certains de ses codes en casse-têtes. Pour chaque colonne, il ajoute une liste de nombres représentant le nombre de cases vides et colorées, comme il l'a fait pour les rangées. Ensuite, pour chaque rangée et chaque colonne, il supprime les nombres représentant les cases vides. Deux de ses casse-têtes sont présentés. Essaie de les résoudre.

(i)



(ii)



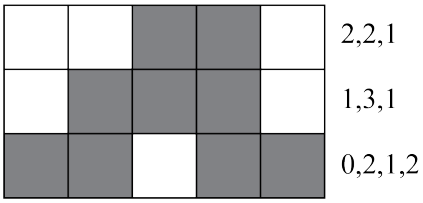
Thème : Pensée computationnelle

Problem of the Week
Problem A and Solution
Picture Pixels

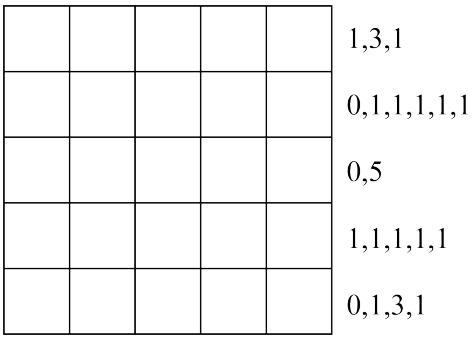
Problem

Akhil sends codes to his friends that tell them how to shade in squares of a grid to reveal a secret picture. Each code contains a blank grid with a list of numbers next to each row, which are used to shade in the row from left to right. The first number in a list represents the number of blank squares at the beginning of the row. The second number represents the number of squares that need to be shaded next. After that, the numbers alternate between the number of blank squares and the number of shaded squares until you reach the end of the row.

For example, if the list of numbers was 2, 2, 1, then the leftmost 2 squares would be blank, the next 2 squares would be shaded, and the next square would be blank. A completed picture is shown.

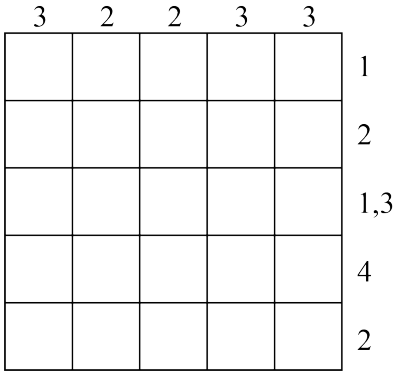


(a) Draw the picture for the given code.

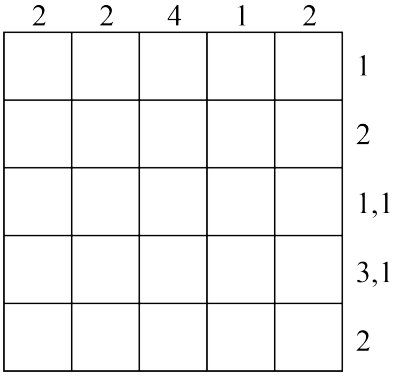


(b) Akhil turns some of his codes into puzzles. For each column, he adds a list of numbers representing the number of blank and shaded squares, as he has for the rows. Then, for each row and column, he removes the numbers representing blank squares. Two of his puzzles are shown. Try to solve them.

(i)

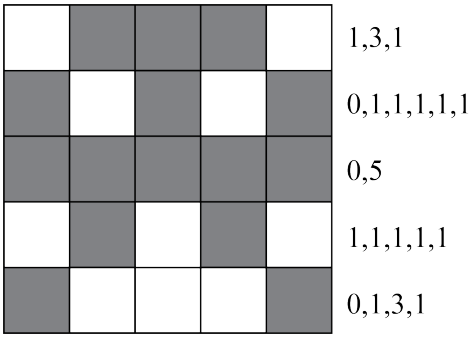


(ii)



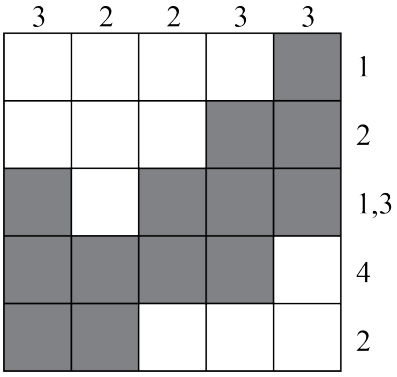
Solution

(a) The completed picture is shown.

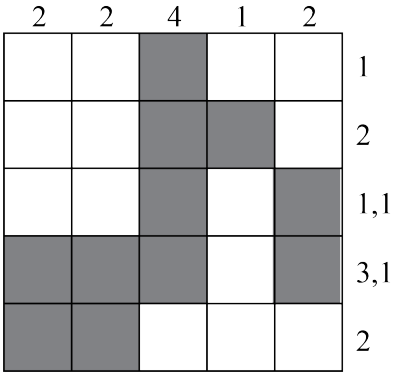


(b) For each puzzle, start by identifying any squares that you know must be either blank or filled. For example, for puzzle (i), the row with list 1, 3 must have at least one blank between the 1 shaded square and the 3 shaded squares. Since there are 5 squares in the row, this means that, from left to right, the row must contain one shaded square, then one blank square, then three shaded squares. It also helps to mark the blank squares with a dot. The completed puzzles are shown. Note that these kind of puzzles are often called *Nonograms*.

(i)



(ii)



Teacher's Notes

The original code in this problem uses *run length encoding* to describe images. Run length encoding is a way to describe large amounts of data in a more compact way; it is an example of a *compression* technique.

This particular compression technique was used by fax machines to send information more efficiently. Since fax images were black and white, rather than sending the image data one pixel at a time, it took less data to identify the image by grouping white pixels and black pixels in each row as is shown in this problem.

Although fax machines are not used much these days, run length encoding is still a technique used by computer scientists to help reduce the size of large data to be more manageable. One place where it is used is to describe information in gene sequences. Genes can be described as long sequences made up of four different proteins: adenine (A), thymine (T), cytosine (C), and guanine (G). The long sequences that describe the building blocks of DNA can be compressed by grouping common protein letters and using the count of how many of the same letter appears consecutively.

Problème de la semaine

Problème A

Calculs circulaires

Lorna utilise les instructions suivantes pour écrire des suites de nombres.

> Étape 1 : Commence avec un nombre entier supérieur à 0.
>
> Étape 2 : Si le nombre est pair, divise-le par 2 pour obtenir le nombre suivant.
> Si le nombre est impair, multiplie-le par 3 et ajoute 1 pour obtenir le nombre suivant.
>
> Étape 3 : Répète l'étape 2 pour continuer la suite.

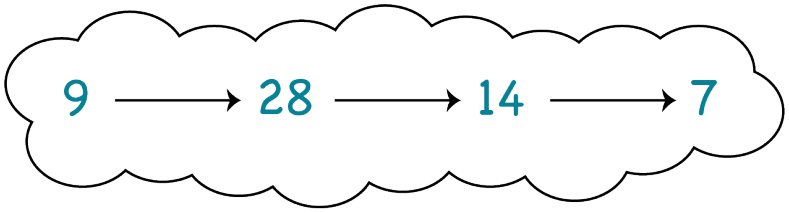
Par exemple, suppose que Lorna commence avec un 9.
Ce nombre est impair, donc le nombre suivant est $9 \times 3 + 1 = 27 + 1 = 28$.
Ce nombre est pair, donc le nombre suivant est $28 \div 2 = 14$.
Ce nombre est pair, donc le nombre suivant est $14 \div 2 = 7$.
Ainsi, les quatre premiers termes de la suite sont 9, 28, 14 et 7.

(a) Suis les instructions en utilisant le nombre de départ donné et écris les 12 premiers termes de chaque suite.

(i) 3 (ii) 13

(b) Que remarques-tu à propos de chaque suite dans la partie (a) ? Que se passerait-il si tu continuais chaque suite ?

(c) Utilise ta réponse à la partie (b) pour prédire le 20^{e} terme de la suite qui commence par 13.



Thèmes : Algèbre, Pensée computationnelle

Problem of the Week
Problem A and Solution
Circular Calculations

Problem

Lorna uses the following instructions to write sequences of numbers.

> Step 1: Start with a whole number greater than 0.
>
> Step 2: If the number is even, divide it by 2 to get the next number.
> If the number is odd, multiply it by 3 and add 1 to get the next number.
>
> Step 3: Repeat Step 2 to continue the sequence.

For example, suppose Lorna starts with 9.
Since this number is odd, the next number is $9 \times 3 + 1 = 27 + 1 = 28$.
Since this number is even, the next number is $28 \div 2 = 14$.
Since this number is even, the next number is $14 \div 2 = 7$.
Thus, the first four numbers in the sequence are 9, 28, 14, and 7.

(a) Follow the instructions using the given starting number and write the first 12 numbers in each sequence.

(i) 3 (ii) 13

(b) What do you notice about each sequence in part (a)? What would happen if you continued each sequence?

(c) Use your answer to part (b) to predict the 20^{th} number in the sequence starting with 13.

Solution

(a) The first 12 numbers in each sequence are as follows.

(i) 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4

(ii) 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2

(b) After each sequence reaches 1, the numbers 4, 2, and 1 repeat over and over.

(c) We can write the first 20 numbers in the sequence starting with 13 by first using part (b) to write out the first 12 numbers in the sequence, and then repeating 4, 2, and 1 until we have 20 numbers.

13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4

Thus, the 20^{th} number is 4.

Teacher's Notes

As an extra challenge, students could attempt this starting with the number 27. Although it takes a long time when you start with the number 27, the sequence will eventually reach 1. It takes 111 numbers to reach 1, as shown.

27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1

Mathematicians believe that starting with any positive integer, the instructions will always lead to a sequence that reaches 1 (or *converges* to 1). This is known as the *Collatz Conjecture*. However, proving this is true for all positive integers is an open problem. There is experimental evidence that shows this is true for very large numbers; however, there is no formal proof that the conjecture holds for all positive integers.

Gestion des données (D)

Amène-moi à la couverture

Problème de la semaine

Problème A

Livres, livres, livres

Dans le pictogramme ci-dessous, on voit le nombre de livres que cinq élèves ont lu pendant ce mois. Chaque  représente un nombre fixe de livres.

Élève	Livres lus
Xuan	
Javya	
Natasha	
Sanan	
Brandon	

(a) Brandon a lu 28 livres ce mois-ci. Combien de livres chaque  représente-t-il dans le pictogramme?

(b) Combien de livres ces élèves ont-ils lus en tout ce mois-ci?

Problem of the Week
Problem A and Solution
Books, Books, Books

Problem

The pictograph below shows how many books five students have each read this month. Each  represents a fixed number of books.

Student	Books Read
Xuan	
Javya	
Natasha	
Sanan	
Brandon	

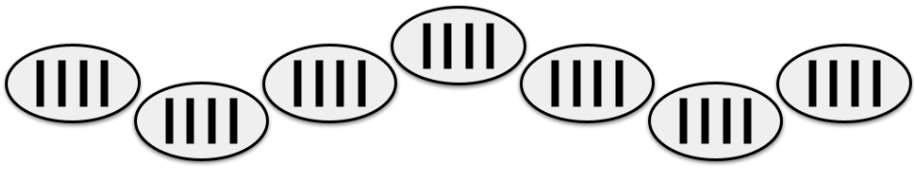
(a) Brandon read 28 books this month. How many books does each  represent in the pictograph?

(b) How many books were read in total by these students this month?

Solution

(a) Since Brandon has 7  , we can skip count by 7s until we get to 28. Doing this gives 7, 14, 21, 28. This means that each  represents 4 books read by a student.

Alternatively, we could use a fair share strategy to determine how many books each  represents. We draw seven ovals, and add a tally to each oval one at a time until 28 tallies have been distributed. Then we end up with 4 tallies in each oval, which means that each  represents 4 books.



(b) Since each  represents 4 books read, we know that Xuan read $5 \times 4 = 20$ books, Javya read $3 \times 4 = 12$ books, Natasha read $10 \times 4 = 40$ books, Sanan read $4 \times 4 = 16$ books, and Brandon read $7 \times 4 = 28$ books.

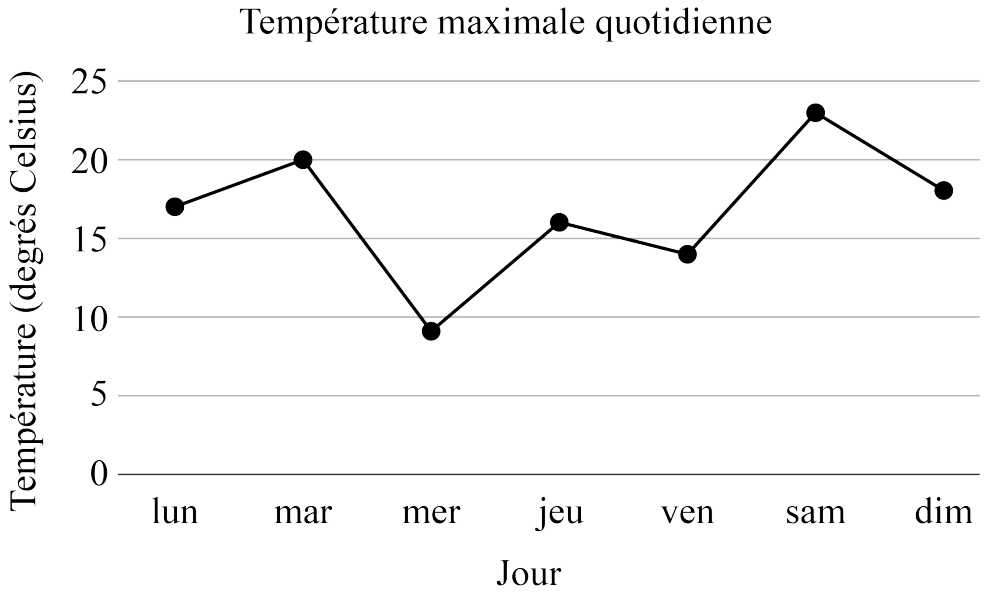
Thus, in total these students read $20 + 12 + 40 + 16 + 28 = 116$ books.

Problème de la semaine

Problème A

Suivi des températures

La station météorologique à l'Université de Waterloo enregistre des informations chaque jour. Le graphique ci-dessous montre la température la plus élevée pour chaque jour de la semaine.



(a) Estime la température la plus élevée enregistrée cette semaine-là.

(b) Estime la température la plus basse enregistrée cette semaine-là.

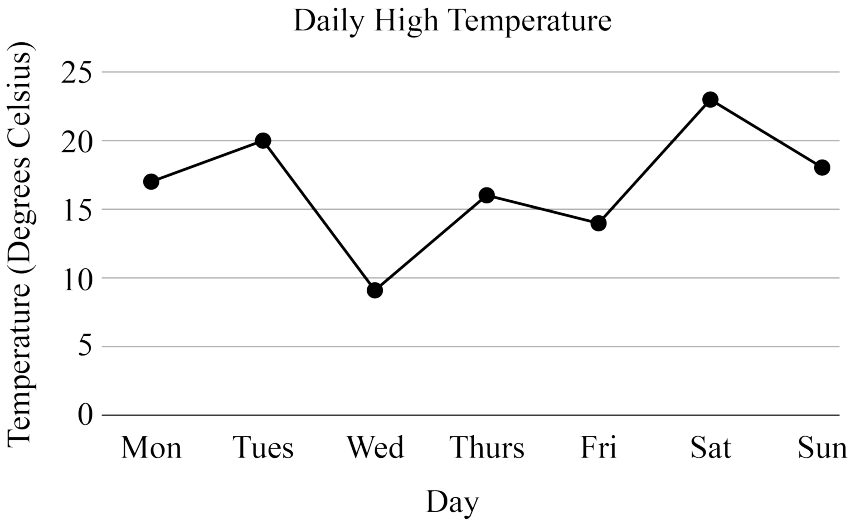
(c) Estime le plus grand changement de température d'un jour à l'autre durant cette semaine-là.

Thèmes : Gestion des données, Sens du nombre

Problem of the Week
Problem A and Solution
Tracking Temperatures

Problem

The weather station at the University of Waterloo records information every day. The graph below shows the high temperature each day for one week.



(a) Estimate the highest temperature recorded during this week.

(b) Estimate lowest temperature recorded during this week.

(c) Estimate the biggest change of temperature from one day to the next during this week.

Solution

(a) The highest temperature occurs on Saturday. It is between 20 degrees and 25 degrees. It appears to be slightly closer to 25 degrees than 20 degrees, so we will estimate the temperature is 23 degrees.

(b) The lowest temperature occurs on Wednesday. It is between 5 degrees and 10 degrees, but is very close to 10 degrees. We will estimate the temperature is 9 degrees.

(c) We estimate the temperature on Monday to be 17 degrees, the temperature on Tuesday to be 20 degrees, the temperature on Thursday to be 16 degrees, the temperature on Friday to be 14 degrees, and the temperature on Sunday to be 18 degrees, along with the estimations in parts (a) and (b).
Thus, from Monday to Tuesday we estimate that it became $20 - 17 = 3$ degrees warmer.
From Tuesday to Wednesday we estimate that it became $20 - 9 = 11$ degrees colder.
From Wednesday to Thursday we estimate that it became $16 - 9 = 7$ degrees warmer.
From Thursday to Friday we estimate that it became $16 - 14 = 2$ degrees colder.
From Friday to Saturday we estimate that it became $23 - 14 = 9$ degrees warmer.
From Saturday to Sunday we estimate that it became $23 - 18 = 5$ degrees colder.
Therefore, we can estimate that the biggest change of temperature from one day to the next is 11 degrees between Tuesday and Wednesday.

Problème de la semaine

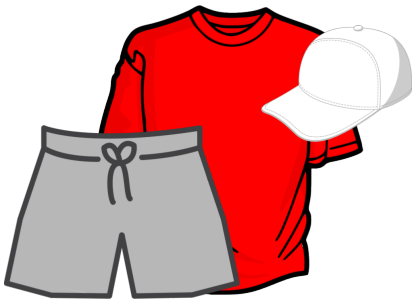
Problème A

Sélections d'été

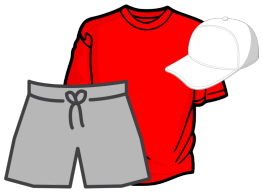
Robbie essaie de simplifier sa garde-robe d'été. Il porte toujours un short, une chemise et un chapeau. Il possède :

- 3 shorts de couleurs rouge, gris et jaune
- 3 chemises de couleurs rouge, blanc et vert
- 3 chapeaux de couleurs jaune, violet et blanc

Combien de combinaisons différentes d'un short, d'une chemise et d'un chapeau peut-il composer de sorte que chaque article soit d'une couleur différente ?



Thème : Gestion des données



Problem of the Week
Problem A and Solution
Summer Selections

Problem

Robbie is trying to simplify his summer wardrobe. He always wears a pair of shorts, a shirt, and a hat. He has:

- 3 pairs of shorts in the colours of red, grey, and yellow
- 3 shirts in the colours of red, white, and green
- 3 hats in the colours of yellow, purple, and white

How many different combinations of a pair of shorts, a shirt, and a hat can he put together so that each item is of a different colour?

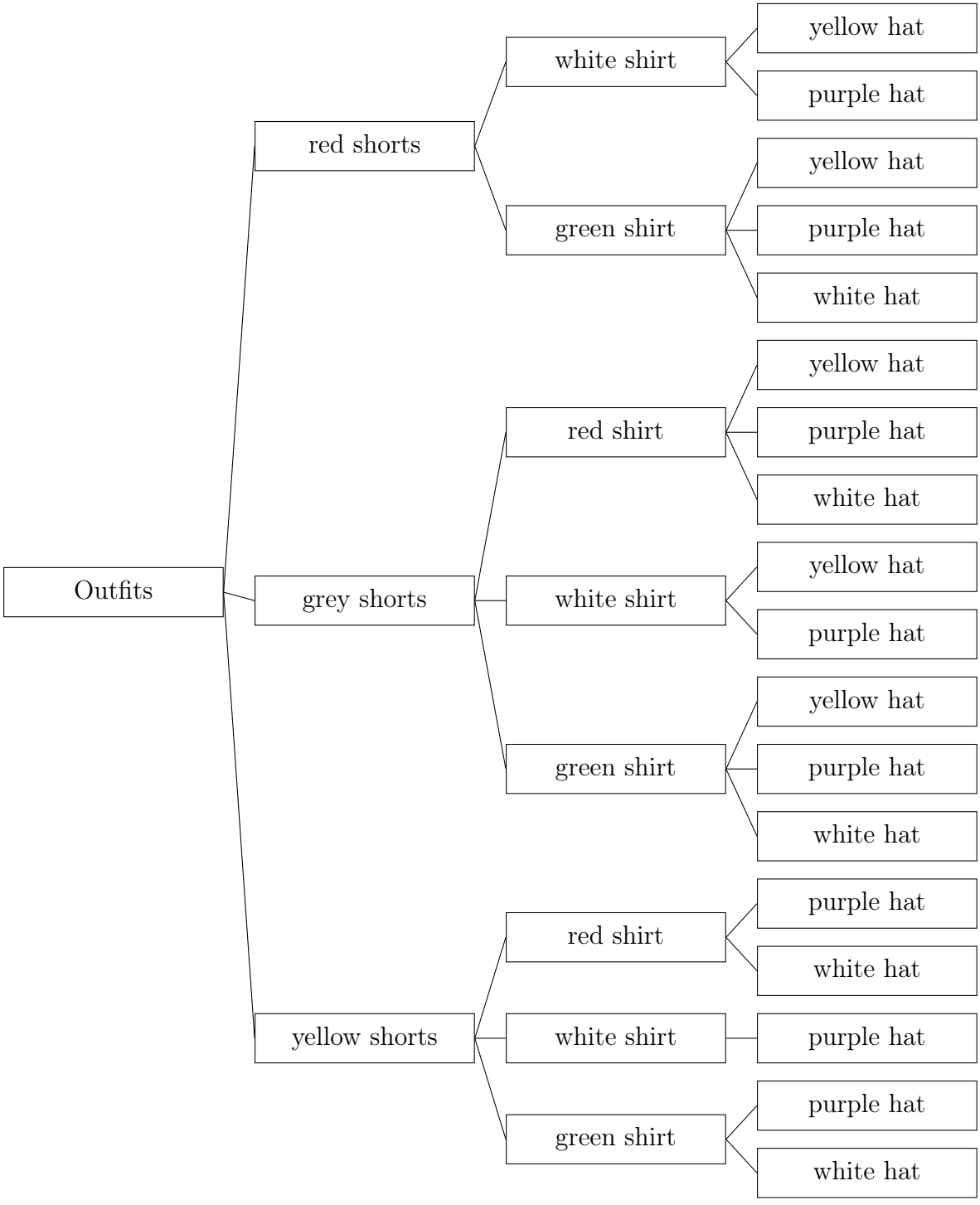
Solution

One way to solve this problem is to create a tree that shows all possible combinations of shorts, shirts, and hats that Robbie can wear. Then we can count the number of leaves of the tree (items in the tree that do not have any branches leading away from them) to determine the number of possible colour combinations.

When creating the tree, it is important that we avoid duplicating colours in each outfit. We can check this by following a path from the root of the tree ("Outfits") to each leaf and ensuring there are no duplicate colours. Each path from the root to a leaf describes a colour combination of a single outfit.

For example **red shorts** $\rightarrow$ **green shirt** $\rightarrow$ **purple hat** is one outfit and **grey shorts** $\rightarrow$ **red shirt** $\rightarrow$ **white hat** is another outfit. There are no duplicate colours in those outfits.

Here is tree showing those combinations:



Since there are 18 leaves in this diagram, there are 18 possible outfits Robbie could wear without duplicating colours.

Géométrie et mesure (G)

Amène-moi à la couverture

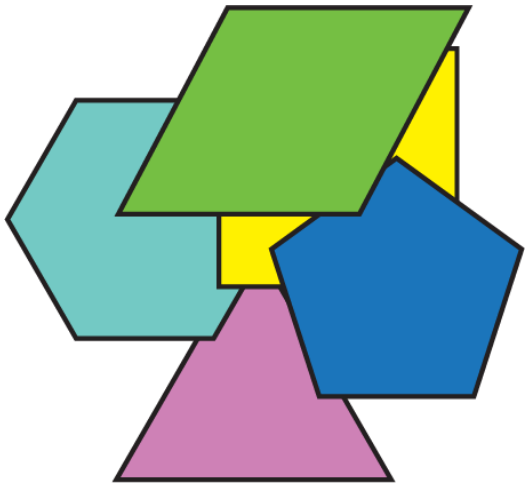
Problème de la semaine

Problème A

Superposer des polygones

Anna a découpé les cinq polygones suivants:

Elle les a ensuite placés sur une table. Voici la vue de dessus:



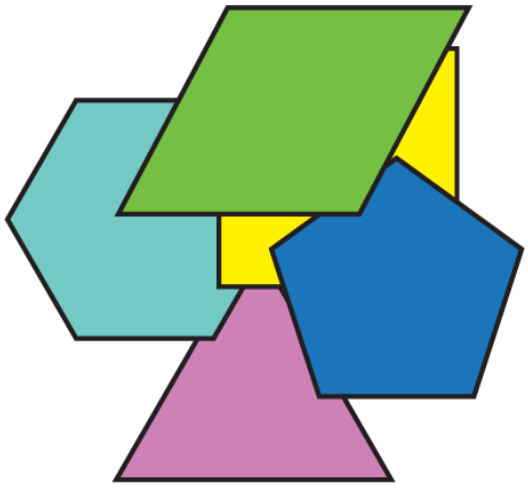
Dans quel ordre a-t-elle placé les polygones sur la table?

Problem of the Week
Problem A and Solution
Stacking Shapes

Problem

Anna cut out the following five polygons.

She then placed them on a table. The top view after doing so is shown.



In what order did she place the polygons on the table?

Solution

First, we recall the names of the five polygons.

Name	Triangle	Square	Parallelogram	Pentagon	Hexagon
Image					

Since the parallelogram has no shapes covering part of it, it must have been the last polygon placed on the table.

The pentagon has the parallelogram covering part of it, so the pentagon must have been placed before the parallelogram.

The square has the pentagon and parallelogram covering part of it, so the square must have been placed before the pentagon and parallelogram.

The hexagon has the square and parallelogram covering part of it, so the hexagon must have been placed before the square and parallelogram.

The triangle has the hexagon, square, and pentagon covering part of it, so the triangle must have been placed before the hexagon, square, and pentagon.

Thus, Anna must have first placed the triangle, then the hexagon, then the square, then the pentagon, then the parallelogram.

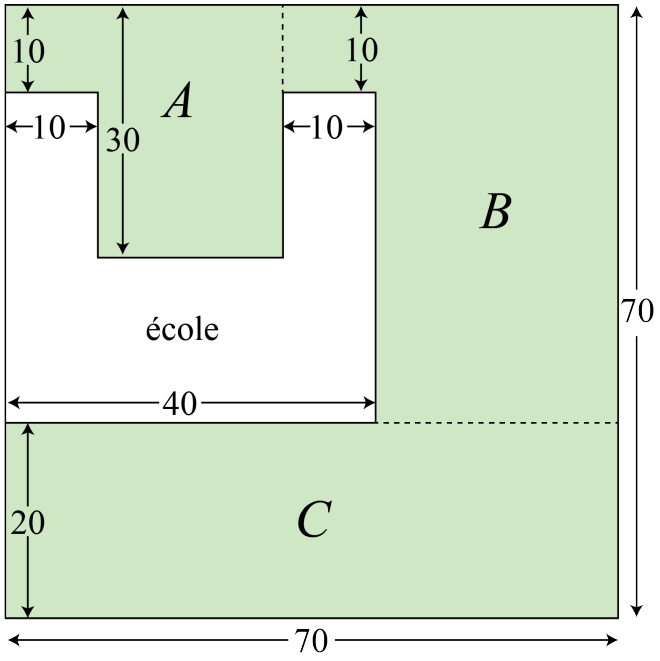
Problème de la semaine

Problème A

Le nettoyage de l'école

Chaque année, les élèves de l'école élémentaire Impeccable participent à un nettoyage extérieur. Tout d'abord, la cour de l'école est divisée en trois sections : A, B et C.

Voici un plan de la cour de l'école, où toutes les longueurs indiquées sont en mètres. Note que la cour de l'école est un carré, que l'école a la forme d'un U rectangulaire et que le schéma n'est pas à l'échelle.



La section ayant la plus petite aire est assignée aux élèves de 1ère et de 2e année, la section ayant la plus grande aire est assignée aux élèves de 5e et de 6e année, tandis que la section qui reste est assignée aux élèves de 3e et de 4e année. Détermine quelle section est assignée à chaque niveau. Justifie ta réponse.

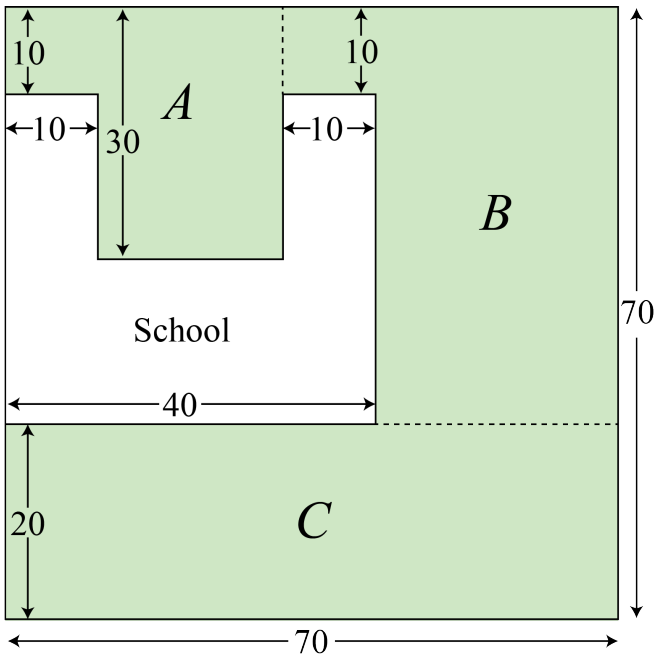
Thème : Géométrie et mesure

Problem of the Week
Problem A and Solution
School Clean Up

Problem

Every year the students at Spotless Elementary do an outdoor cleanup. First, the school yard is divided into three sections: A, B, and C.

A map of the school yard is shown, with all given lengths in metres. Note that the school yard is a square, the school is a rectangular U-shape, and the diagram is not drawn to scale.



The Grade 1 & 2 students are assigned the section with the smallest area, the Grade 5 & 6 students are assigned the section with the largest area, and the Grade 3 & 4 students are assigned the remaining section. Determine which section is assigned to each grade. Justify your answer.

Solution

Solution 1

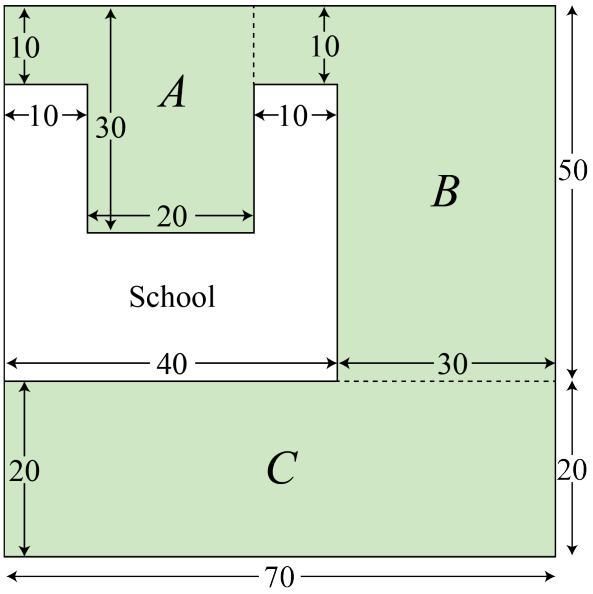
One way to solve this problem is to calculate the area of each section. Since Section C is a rectangle with length 70 m and width 20 m, its area is $70 \times 20 = 1400$ m^2.

We are missing some dimensions needed to calculate the areas of the other two sections. However, we can determine these missing dimensions from the information in the diagram.

Since the bottom side of the school is 40 m and the widths of the ends of the U-shape are each 10 m, then the length in between the ends of the U-shape is $40 - 10 - 10 = 20$ m.

Since the bottom side of the yard is 70 m and the bottom side of the school is 40 m, then the distance from the right side of the school to the right side of the yard must be $70 - 40 = 30$ m.

Since the right side of the yard is 70 m and the right side of Section C is 20 m, then the right side of Section B is $70 - 20 = 50$ m. The following diagram shows all the dimensions.



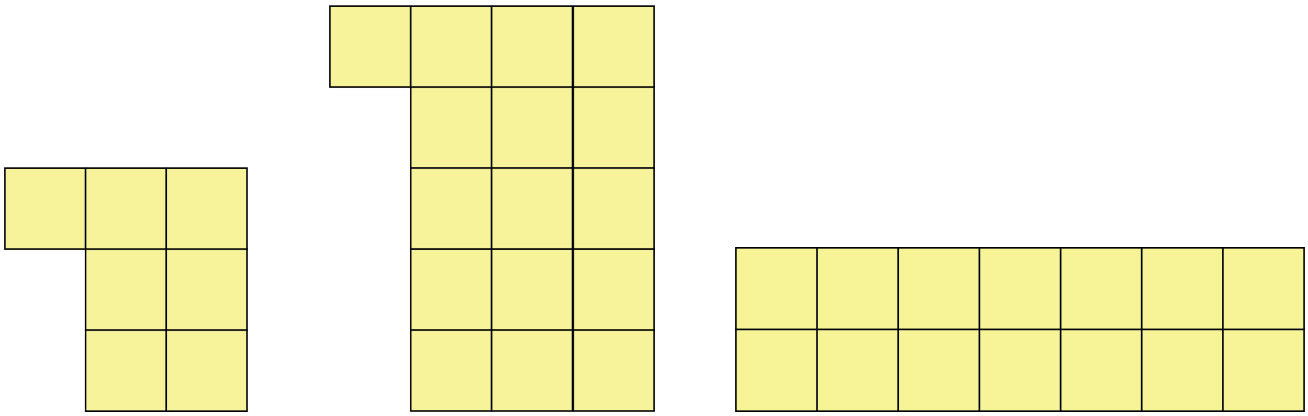
Section A is formed by a square with side length 10 m, and a rectangle with length 30 m and width 20 m. The area of the square is $10 \times 10 = 100$ m^2, and the area of the rectangle is $30 \times 20 = 600$ m^2. The total area of Section A is then $100 + 600 = 700$ m^2.

Similarly, Section B is formed by a square with side length 10 m, and a rectangle with length 50 m and width 30 m. The area of the square is $10 \times 10 = 100$ m^2, and the area of the rectangle is $50 \times 30 = 1500$ m^2. The total area of Section B is then $100 + 1500 = 1600$ m^2.

Thus, the area of Section A is 700 m^2, the area of Section B is 1600 m^2, and the area of Section C is 1400 m^2. Therefore Grades 1 & 2 are assigned to Section A, Grades 5 & 6 are assigned to Section B, and Grades 3 & 4 are assigned to Section C.

Solution 2

Another way to solve this problem is to build each section using 10 m $\times$ 10 m squares, as shown.



Section A Section B Section C

Using this approach, we can count the number of 10×10 squares that form each section. Section A is formed with 7 squares, Section B is formed with 16 squares, and Section C is formed with 14 squares. Therefore, Grades 1 & 2 are assigned to Section A, Grades 5 & 6 are assigned to Section B, and Grades 3 & 4 are assigned to Section C.

Problème de la semaine

Problème A

Le temps qui court

Suppose que l'heure actuelle correspond à celle indiquée sur l'horloge et que c'est le matin.

La journée scolaire se termine dans 5 heures et 6 minutes à partir de l'heure indiquée sur l'horloge.

(a) Combien de minutes reste-t-il avant la fin de la journée scolaire ?

(b) À quelle heure se termine l'école ?

(c) L'école a commencé à 8h55. Combien de temps s'est écoulé, en heures et en minutes, entre le début de la journée scolaire et l'heure indiquée sur l'horloge ?

Problem of the Week
Problem A and Solution
Time After Time

Problem

Assume that the current time matches the time shown on the clock, and the current time is in the morning.

The school day ends in 5 hours and 6 minutes from the time shown on the clock.

(a) How many more minutes are left until the end of the school day?

(b) What time does school end?

(c) School started at 8:55 a.m. How much time has passed, in hours and minutes, between the start of the school day and the time shown on the clock?

Solution

(a) Since each hour has 60 minutes, then 5 hours have a total of $5 \times 60 = 300$ minutes. So the total number of minutes left until the end of the school day is $300 + 6 = 306$ minutes.

(b) The time showing on the clock is 10:09 a.m. We will add 5 hours and 6 minutes to this time. Two hours after 10:09 a.m. it will be 12:09 p.m. Three hours after that it will be 3:09 p.m. Six minutes later it will be 3:15 p.m. Thus, school ends at 3:15 p.m.

(c) We start at 8:55 a.m. One hour after that it will be 9:55 a.m. Five minutes after that it will be 10:00 a.m. Nine minutes after that it will be 10:09 a.m. Thus, a total of 1 hour and $5 + 9 = 14$ minutes have passed between the start of the school day and the time shown on the clock.

Problème de la semaine

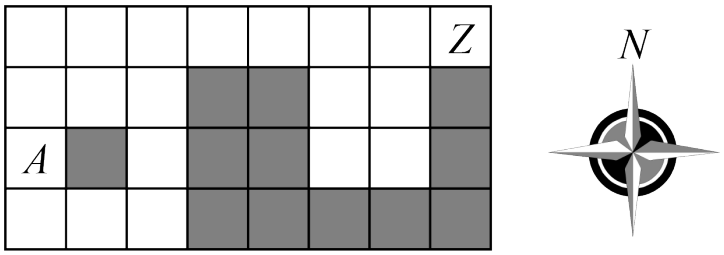
Problème A

Une navigation époustouflante

Juanita et AJ créent des labyrinthes sur du papier quadrillé. Chaque labyrinthe est une grille rectangulaire contenant des cases blanches et des cases grises. Une case blanche est marquée de la lettre A et une autre de la lettre Z.

Pour compléter un labyrinthe, ils doivent partir du A et atteindre le Z en se déplaçant d'une case à la fois dans l'une des directions suivantes : nord (N), est (E), sud (S) ou ouest (O), où le haut de la page est considéré comme le nord. Ils *ne peuvent pas* traverser les cases grises et doivent passer par chacune des cases blanches *exactement une fois*. Autrement dit, ils doivent traverser toutes les cases blanches, mais ils ne peuvent pas passer par l'une d'elles plus d'une fois.

(a) Détermine les directions qu'ils doivent suivre pour compléter avec succès le labyrinthe donné.



(b) AJ crée un autre labyrinthe en modifiant l'emplacement des cases grises dans le labyrinthe de la partie (a). (Les positions du A et du Z demeurent inchangées.) Juanita réussit à compléter ce nouveau labyrinthe en suivant ces directions :

$$E,\ S,\ E,\ E,\ E,\ N,\ E,\ N,\ O,\ O,$$
$$O,\ O,\ N,\ E,\ E,\ E,\ E,\ E,\ E$$

À quoi ressemble le labyrinthe créé par AJ ?

Thèmes : Pensée computationnelle, Géométrie et mesure

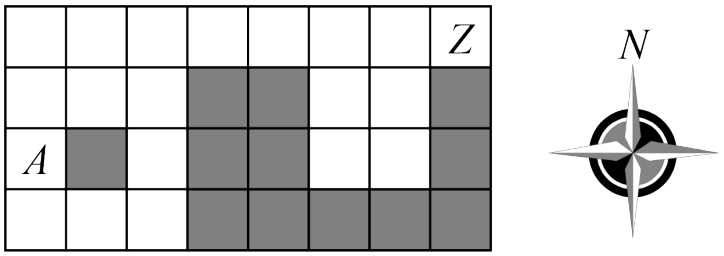
Problem of the Week
Problem A and Solution
Amazing Navigation

Problem

Juanita and AJ create mazes on grid paper. Each maze is a rectangular grid containing white squares and grey squares. One white square is marked A and another is marked Z.

To complete a maze, they start at A and need to reach Z by moving one square at a time in one of the following directions: north (N), east (E), south (S), or west (W), where the top of the page is considered north. They *cannot* go through any of the grey squares and must go through each of the white squares *exactly once*. That is, they must go through all of the white squares but cannot go through any of them more than once.

(a) Determine the directions they need to follow to successfully complete the given maze.



(b) AJ creates another maze by changing where the grey squares are in the maze from part (a). (The locations of A and Z remain unchanged.) Juanita successfully completes this new maze by following these directions:

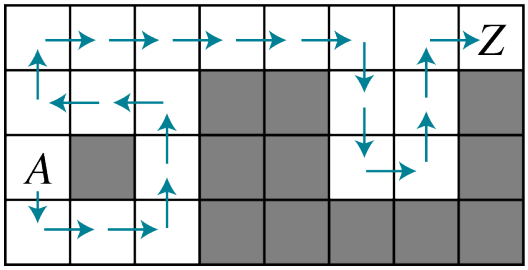
$$E,\ S,\ E,\ E,\ E,\ N,\ E,\ N,\ W,\ W,$$
$$W,\ W,\ N,\ E,\ E,\ E,\ E,\ E,\ E$$

What does AJ's maze look like?

Solution

(a) The first direction must be either N or S, because there is a grey square to the right of A. Suppose they start with N. Then at some point, they must go through the square that is directly below A. However, once they reach that square they will be stuck because they can't go through any white square more than once. So the first direction must be S.

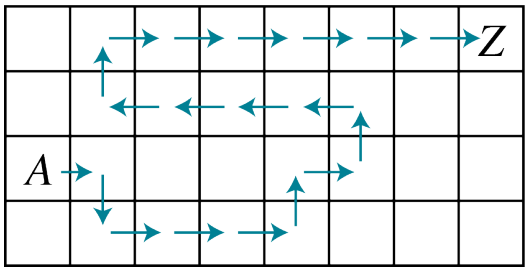
Using similar reasoning, we can complete the maze as shown.



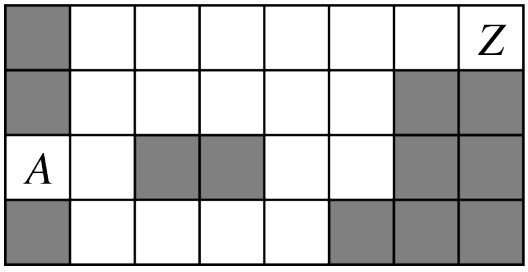
The directions followed are then

$$S,\quad E,\quad E,\quad N,\quad N,\quad W,\quad W,\quad N,\quad E,\quad E,$$
$$E,\quad E,\quad E,\quad S,\quad S,\quad E,\quad N,\quad N,\quad E$$

(b) We first mark the given directions on the maze, as shown.



Since Juanita cannot go through any of the grey squares, and must go through each of the white squares exactly once, the squares that are blank after marking the path must be the grey squares. Thus, AJ's maze is as shown.



Problème de la semaine

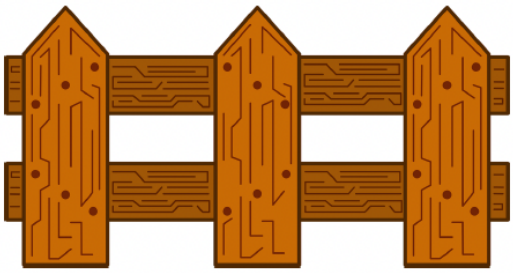
Problème A

Les poteaux de clôture

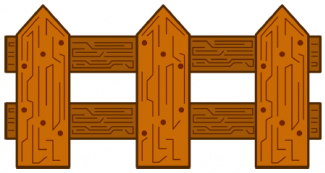
Elyas aide ses parents à installer des poteaux de clôture dans leur cour arrière. Ils installent une rangée de 7 poteaux espacés de manière égale. Les poteaux sont tous de la même taille. La distance entre le milieu du $2^{\text{ème}}$ poteau et le milieu du $5^{\text{ème}}$ poteau est de quatre mètres et demi.

Quelle est la distance entre le milieu du premier poteau et le milieu du dernier poteau ?

INDICE : Tu peux commencer par dessiner un diagramme des poteaux de clôture et y indiquer la distance que tu connais.



THÈMES : GÉOMÉTRIE ET MESURE, SENS DU NOMBRE



Problem of the Week
Problem A and Solution
Fencepost Problem

Problem

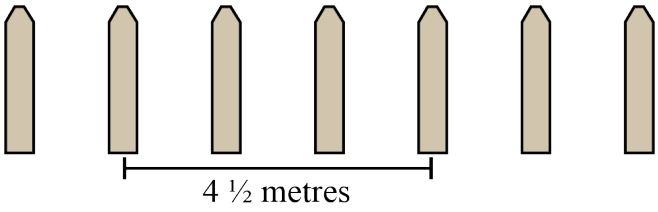
Elyas helps his parents install fence posts in their backyard. They install a row of 7 equally-spaced posts. The posts are all the same size. The distance between the middle of the 2nd post and the middle of the 5th post is four and a half meters.

What is the distance between the middle of the first post and the middle of the last post?

HINT: You might start by drawing a diagram of the fence posts and labelling it with the distance you know.

Solution

There are many ways to calculate the distance. We start with a diagram of the posts, labelled with the distance of $4\frac{1}{2}$ metres between the middle of the 2nd and 5th posts.

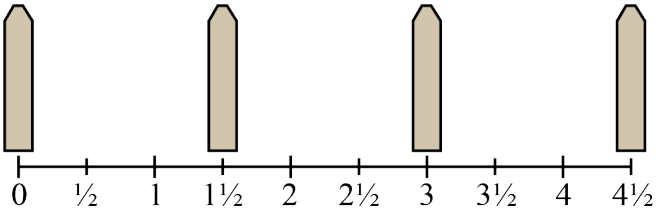


Solution 1

Notice that there are a total of three gaps between the 2nd and 5th posts. There are also a total of three gaps between the 1st and 2nd posts and the 5th and 7th posts. Since the distances between the middle of adjacent posts are all the same, then the total distance between the 1st and 2nd posts and the 5th and 7th posts must also be $4\frac{1}{2}$ m. Since there are six gaps in total, the distance between the middle of the first post and the middle of the last post is $4 + \frac{1}{2} + 4 + \frac{1}{2} = 9$ m.

Solution 2

Another way to solve this problem is to draw a number line from 0 to $4\frac{1}{2}$ m and space the 2^{nd}, 3^{rd}, 4^{th}, and 5^{th} fenceposts evenly along this line.

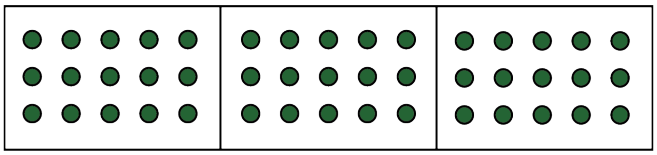


From this, we observe that the distance from the middle of one post to the middle of an adjacent post is $1\frac{1}{2}$ m. Since there are six gaps in total between adjacent posts, the distance between the middle of the first post and the middle of the last post is:

$$1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}=9 \text{ m}$$

Solution 3

First we convert the distance to centimetres: $4\frac{1}{2}$ m $= 450$ cm. Then we use a tape diagram. We observe that there are 3 gaps between the 2^{nd} and 5^{th} posts. That means we need to divide 450 cm into three equal distances. To make it easier, since $450 = 45 \times 10$, we let one dot represent 10 cm, and then distribute 45 dots into the three equal pieces of our tape diagram.



Each piece has a total of 15 dots. This means the distance between adjacent posts is $15 \times 10 = 150$ cm. Since there are 6 gaps in total between the first post and the last post, the total distance is:

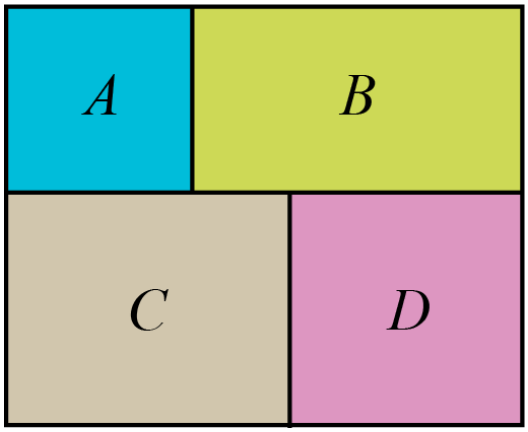
$$150+150+150+150+150+150=900 \text{ cm or } 9 \text{ m}$$

Problème de la semaine

Problème A

Un casse-tête de courtepointe

Les employés de l'entreprise Fil d'Inspiration doivent réaliser une petite courtepointe pour un projet spécial, en utilisant quatre tissus de couleurs différentes. Leur motif de courtepointe est un rectangle divisé en quatre rectangles plus petits, nommés A, B, C et D.

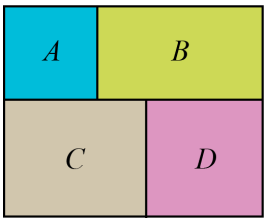


La pièce A est un carré dont l'aire est de 16 cm^2. La pièce D est un carré dont l'aire est de 25 cm^2. La distance horizontale entre le côté droit de la pièce A et le côté gauche de la pièce D est de 2 cm. Quelle est l'aire totale de la courtepointe ?

Problem of the Week
Problem A and Solution
Quilting Puzzle

Problem

The employees of Sew Inspired need to make a tiny quilt for a special project using four different colours of fabric. Their quilt pattern is a rectangle divided into four smaller rectangles, marked A, B, C, and D.



Piece A is a square with area 16 cm^2 and piece D is a square with area 25 cm^2. The horizontal distance between the right side of piece A and the left side of piece D is 2 cm. What is the area of the entire quilt?

Solution

To calculate the area of the quilt we will determine the lengths of its sides. We know that piece A is a square with area 16 cm^2. We also know that the lengths of the sides of a square must be the same. So if the length of one side of a square is n, then the area of the square must be $n \times n$. By trial and error, we determine $4 \times 4 = 16$, and so the length of each side of piece A must be 4 cm. Another way to determine the lengths of the sides of piece A is to start with 16 unit squares (using blocks or cut out of paper) and determine how to arrange them into a larger square. The only possible arrangement is a 4×4 square.

Similarly, piece D is a square with area 25 cm^2. Since $5 \times 5 = 25$, the length of each side of piece D must be 5 cm.

The opposite sides of a rectangle must be the same length, so the width of the quilt (i.e. the vertical side) is equal to the sum of the side lengths of pieces A and D. Thus, the width of the quilt is equal to $4 + 5 = 9$ cm. Similarly, we know the bottom of piece A is on the same line as the top of piece D, and the horizontal distance between the two pieces is 2 cm. Therefore the length of the quilt (i.e. the horizontal side) is equal to the sum of the side lengths of pieces A and D, plus 2. Thus, the length of the quilt is equal to $4 + 5 + 2 = 11$ cm.

Now we can calculate the area of the entire quilt. The area of this rectangle is the product of its length and width. So the area of the entire quilt is $9 \times 11 = 99$ cm^2.

Problème de la semaine

Problème A

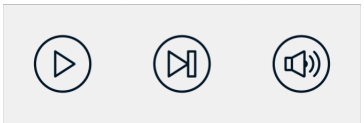
Visionnage de vidéos

Jo aime regarder des vidéos. Parfois, elle les regarde à vitesse normale. Cependant, elle a également la possibilité de les visionner à $\frac{1}{4}$ de la vitesse normale, à $\frac{1}{2}$ de la vitesse normale ou à 2 fois la vitesse normale. Le tableau ci-dessous indique les durées normales de visionnage de quatre vidéos et la vitesse à laquelle Jo regarde chacune d'elles.

	Durée à vitesse normale	**Vitesse de visionnage de Jo**
Vidéo A	50 secondes	$\frac{1}{4}$ de la vitesse
Vidéo B	2 minutes 15 secondes	vitesse normale
Vidéo C	1 minute 40 secondes	$\frac{1}{2}$ de la vitesse
Vidéo D	6 minutes 20 secondes	2 fois la vitesse

Quel est le temps total que Jo a passé à regarder ces quatre vidéos ?

Thèmes : Géométrie et mesure, Sens du nombre



Problem of the Week
Problem A and Solution
Video Viewing

Problem

Jo likes to watch videos. Sometimes she watches them at normal speed. However, she also has the option to watch them at $\frac{1}{4}$ as fast as normal speed, $\frac{1}{2}$ as fast as normal speed, or 2 times as fast as normal speed. The table below lists the normal play times for four videos and the speed at which Jo watches each one.

	Normal Speed Time	**Speed Jo Watches Video At**
Video A	50 seconds	$\frac{1}{4}$ speed
Video B	2 minutes 15 seconds	normal speed
Video C	1 minute 40 seconds	$\frac{1}{2}$ speed
Video D	6 minutes 20 seconds	2 times speed

What is the total time Jo spent watching these four videos?

Solution

If a video is playing at $\frac{1}{4}$ speed, then it will take 4 times as long to play as normal speed. So a 50 second video will take $50 \times 4 = 200$ seconds to watch.

If a video is playing at $\frac{1}{2}$ speed, then it will take 2 times as long to play as normal speed. So a 1 minute and 40 second video will take 2 minutes and $40 \times 2 = 80$ seconds to watch.

If a video is playing at 2 times speed then it will take half as long to play as normal speed. Half of 6 minutes is 3 minutes and half of 20 seconds is 10 seconds.

Now we can add up all the minutes and seconds of playing times of all four videos:

$$2 + 2 + 3 = 7 \text{ minutes and } 200 + 15 + 80 + 10 = 305 \text{ seconds}$$

There are 60 seconds in 1 minute. When we skip count by 60, we get 60, 120, 180, 240, 300. Thus, there are 5 minutes in 305 seconds with $305 - 300 = 5$ seconds left over.

So Jo spent $7 + 5 = 12$ minutes and 5 seconds watching these four videos.

Sens du nombre (N)

Amène-moi à la couverture

Problème de la semaine

Problème A

La magie du cinéma

Comfort et sa famille prévoient d'aller voir un film ce week-end. Comfort a 9 ans. Elle sera accompagnée de ses frères jumeaux, qui ont 5 ans de plus qu'elle, et de ses deux parents. Voici les prix des billets, taxes incluses:

— Adultes: 14 $ chacun
— Enfants entre 12 et 18 ans: 12 $ chacun
— Enfants de moins de 12 ans: 8 $ chacun

La famille dispose d'une carte-cadeau de 25 $ qu'ils prévoient d'utiliser pour payer une partie du prix des billets de cinéma. Quel montant supplémentaire doivent-ils débourser pour l'achat des billets de cinéma pour toute la famille? Justifie ta réponse.

Thème : Sens du nombre

Problem of the Week
Problem A and Solution
Movie Magic

Problem

Comfort and her family are planning to see a movie on the weekend. Comfort is 9 years old. Her twin brothers, who are 5 years older than Comfort, and both her parents will be going to the movie. The prices for tickets, including all taxes, are:

- Adults: $14 each
- Children between 12 and 18: $12 each
- Children under 12: $8 each

They have a gift card worth $25 that they can use to pay for part of the cost of the movie tickets. How much more money do they need to pay for the tickets for the entire family? Justify your answer.

Solution

Since Comfort is 9 years old and her twin brothers are 5 years older, they are both $9 + 5 = 14$ years old. (Note that depending on when their birthdays are exactly, the brothers might actually be 13 or 15 at the time. However, their age would definitely be between 12 and 18.)

So, Comfort's ticket will cost $8.
Comfort's brothers' tickets will cost $12 each.
Comfort's parents' tickets will cost $14 each.

The total cost for the family will then be $\$8 + \$12 + \$12 + \$14 + \$14 = \$60$.

Since they have a $25 gift card, they will need to pay $\$60 - \$25 = \$35$ more for the tickets for the entire family.

Problème de la semaine

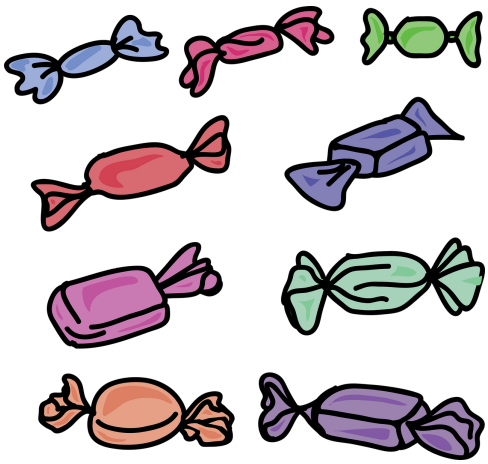
Problème A

Meilleure offre?

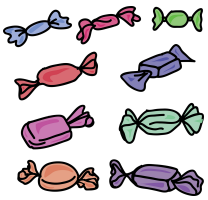
Un distributeur automatique vend 8 bonbons pour 25 cents, tandis qu'un supermarché propose des paquets de 64 bonbons à 2 dollars et 20 cents le paquet.

Si je souhaite acheter 128 bonbons pour ma classe, quelle méthode d'achat me permettrait de dépenser le moins d'argent possible?

Remarque: 1 dollar équivaut à 100 cents.



Thème : Sens du nombre



Problem of the Week
Problem A and Solution
Better Deal?

Problem

From a candy machine, I can buy 8 candies for 25 cents. Alternatively, I can buy 64 candies in a package for 2 dollars and 20 cents.

If I want to buy 128 candies for my class, how should I buy the candies in order to spend the least amount of money?

NOTE: 1 dollar is equal to 100 cents.

Solution

Since 128 is equal to 2×64, two packages of candies will be enough for the class. This would cost $2 \times \$2$ plus 2×20 cents, for a total of \$4 and 40 cents.

We can use skip counting to calculate how much the candies will cost if we buy them from the candy machine. This is summarized in the table below.

Number of Candies	Cost (cents)
8	25
16	50
24	75
32	100
40	125
48	150
56	175
64	200

Thus, 64 candies from the candy machine will cost 200 cents, which is equal to \$2. This is less than the cost of 64 candies in a package.

Therefore, to spend the least amount of money, we should buy all 128 candies from the candy machine. The total cost will then be $2 \times \$2 = \$4$.

Problème de la semaine

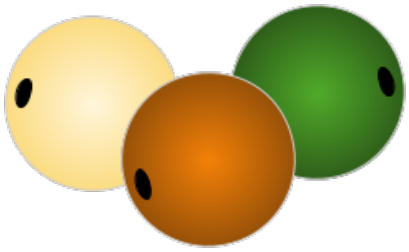
Problème A

Bracelets de perles

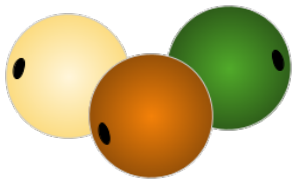
Aditi et Ersal ont conçu un bracelet qui contient 17 perles. Le bracelet est composé de 6 perles rouges, 9 perles vertes et les perles restantes sont jaunes.

(a) Combien y a-t-il de perles jaunes dans le bracelet?

(b) Aditi et Ersal veulent fabriquer 21 bracelets de ce type afin que chacun de leurs camarades en reçoive un. Chaque paquet de 24 perles contient 8 perles rouges, 8 perles vertes et 8 perles jaunes. Combien de paquets de perles leur faut-il ?



Thème : Sens du nombre



Problem of the Week
Problem A and Solution
Beaded Bracelets

Problem

Aditi and Ersal have designed a bracelet that has 17 beads. The bracelet has 6 red beads, 9 green beads, and the rest of the beads are yellow.

(a) How many yellow beads are in one bracelet?

(b) They want to make a total of 21 of these bracelets so they have one for everyone in their class. Each package of 24 beads has 8 red, 8 green, and 8 yellow beads in it. How many packages of beads do they need?

Solution

(a) We can calculate the number of yellow beads by subtracting the number of red and green beads from the total, to get $17 - 6 - 9 = 2$ yellow beads.

(b) Since they need more green beads than any other colour, to make 21 bracelets they need to buy enough packages for $21 \times 9 = 189$ green beads. Since each package has 8 green beads, one way to calculate how many packages we need is to divide $189 \div 8 = 23$ with a remainder of 5. This means they need to buy 24 packages to get enough green beads.

Alternatively, we know that 21 packages will contain 8 green beads that can be used to make each bracelet. However, they need one more green bead for each of the 21 bracelets. Now we calculate how many packages are needed for 21 more green beads. We can skip count: 8, 16, 24 to see we need 3 more packages to have at least 21 more green beads. From this we know that Aditi and Ersal need a total of $21 + 3 = 24$ packages of beads to make the bracelets.

Problème de la semaine

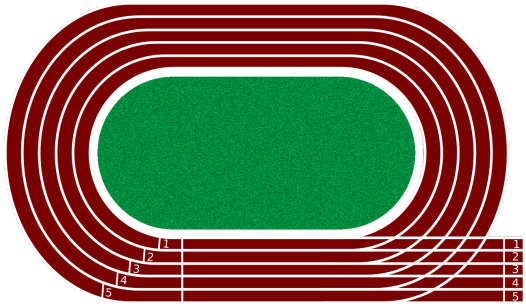
Problème A

Entraînement par intervalles

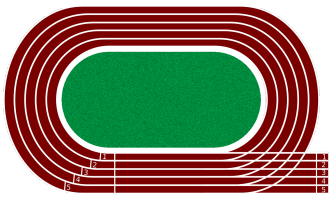
Jelena s'entraîne sur la piste de son école. Elle fait de l'entraînement par intervalles, ce qui consiste à courir sur une distance spécifique puis à s'arrêter pour effectuer différents exercices. Voici son plan d'entraînement:

— Courir la moitié de la piste, s'arrêter et effectuer 10 répulsions.
— Courir les trois quarts de la piste, s'arrêter et effectuer 5 burpees.
— Courir un tour complet et un quart de tour de piste, s'arrêter et effectuer 15 sauts papillons.

Combien de tours de piste Jelena a-t-elle effectués après avoir fait les sauts papillons?



Thème : Sens du nombre



Problem of the Week
Problem A and Solution
Cross Training

Problem

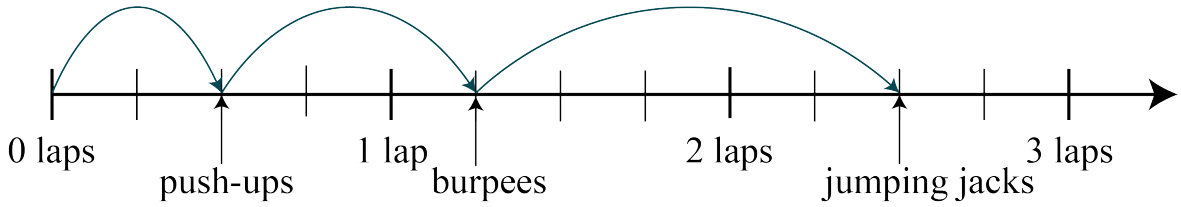
Jelena is training at the track at her school. She does interval training, which means that she runs for some distance then stops to do other exercises. Each time around the track is called a lap. This is her training plan:

- Run half of the way around the track, stop and do 10 push-ups.
- Run three quarters of the way around the track, stop and do 5 burpees.
- Run one and a quarter of the way around the track, stop and do 15 jumping jacks.

How many laps of the track has Jelena completed after doing the jumping jacks?

Solution

One way to solve this problem is to use a timeline broken up into quarter laps.



From this, we determine that after completing the jumping jacks, Jelena has completed 2 and a half laps of the track.

Another way to determine the answer is to add fractions together. To add fractions, we need the fractions written with a common denominator. We know that $\frac{1}{2} = \frac{2}{4}$ and that $1 = \frac{4}{4}$, so we can add to get

$$\frac{2}{4} + \frac{3}{4} + \frac{4}{4} + \frac{1}{4} = \frac{10}{4} = 2\frac{1}{2}$$

Teacher's Notes

Using a *number line* can help with adding fractions, especially when the sum produces a *mixed fraction*.

A number line often includes arrows at one or both ends to indicate that positive and negative numbers continue to increase in magnitude indefinitely in each direction. In particular, an arrow pointing to the right indicates that there are an *infinite* number of positive integers. There are also an infinite number of values between any two marked points on the number line. For example, there are an infinite number of values between two consecutive positive integers. Some of those values are *rational* numbers that can be represented by fractions in the form:

$$\frac{a}{b}, \text{ where } a \text{ and } b \text{ are integers, and } b \text{ is not } 0$$

There are other values, such as $\sqrt{2}$, that cannot be represented in this form. They are known as *irrational* numbers.

Problème de la semaine

Problème A

La joie des Fêtes

Cette année, les élèves de l'école primaire Bienfaisance répandent la joie des Fêtes en recueillant des dons pour les familles dans le besoin. Ils se sont fixé pour objectif de recueillir 2000 dons en une semaine. Heureusement, ils ont non seulement atteint cet objectif, mais ils l'ont également dépassé! Le tableau ci-dessous récapitule le nombre de dons recueillis chaque jour de la semaine.

Jour	Nombre de dons recueillis
Lundi	392
Mardi	46
Mercredi	877
Jeudi	?
Vendredi	229

Malheureusement, quelqu'un a malencontreusement déchiré la feuille de collecte contenant les données du jeudi. Sans cette information, on ne sait pas exactement combien de dons ont été recueillis ce jour-là. Sachant que les élèves de l'école ont dépassé l'objectif de recueillir 2000 dons en une semaine, quel est le plus petit nombre de dons que les élèves auraient pu recueillir jeudi?

Thème : Sens du nombre

Problem of the Week
Problem A and Solution
Holiday Cheer

Problem

Students at Giving Elementary School are spreading holiday cheer this year by collecting donations for families in need within their community. Their goal this year was to collect 2000 items in one week, and they exceeded their goal! The table below shows the number of donations each day during the week.

Day	Number of Items Donated
Monday	392
Tuesday	46
Wednesday	877
Thursday	?
Friday	229

Someone accidentally tore the collection sheet with the data on it, so they don't have the number of items they collected on Thursday. If we know the school exceeded their goal of 2000 items in a week, what is the minimum number of items the students collected on Thursday?

Solution

We start by adding up the number of items for the days we know, and we get $392 + 46 + 877 + 229 = 1544$.

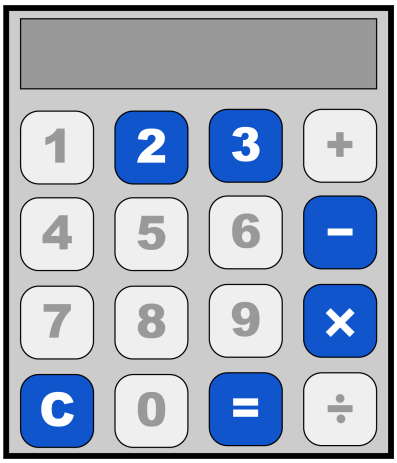
Since we know the students exceeded their goal, then they must have collected at least 2001 items in total. Thus, the minimum number of items they collected on Thursday must be $2001 - 1544 = 457$.

Problème de la semaine

Problème A

Catastrophes de calculatrice

J'ai échappé ma calculatrice et mon chien l'a piétinée. Maintenant, la plupart des touches sont brisées. Les seules touches qui fonctionnent sont celles avec l'arrière-plan foncé, tel que tu peux le voir dans l'image ci-dessous.



J'aimerais que la calculatrice affiche sur son écran chacun des nombres de 1 à 12, en utilisant seulement les touches qui fonctionnent. Les nombres les plus faciles à afficher sont 2 et 3, puisqu'il suffit d'appuyer sur une seule touche. Toutefois, pour tous les autres, tu devras appuyer sur plus d'une touche. Par exemple, pour afficher un 7, je peux utiliser le fait que $7 = 3 \times 3 - 2$.

C'est une calculatrice très simple, je dois donc appuyer sur la touche $\boxed{=}$ après chaque section du calcul. Si je veux compléter le calcul pour $7 = 3 \times 3 - 2$, j'appuierais sur la touche $\boxed{3}$, suivie de la touche $\boxed{\times}$, puis la touche $\boxed{3}$ et enfin la touche $\boxed{=}$ pour obtenir le nombre 9. Ensuite, j'appuierais sur la touche $\boxed{-}$, puis la touche $\boxed{2}$ et finalement la touche $\boxed{=}$ pour obtenir le nombre 7. Ce calcul a demandé que j'appuie sur 7 touches pour que le nombre 7 s'affiche sur l'écran de la calculatrice.

Pour chaque nombre entier de 1 à 12, démontre comment tu peux afficher les nombres en appuyant sur un maximum de 10 fois sur les touches.

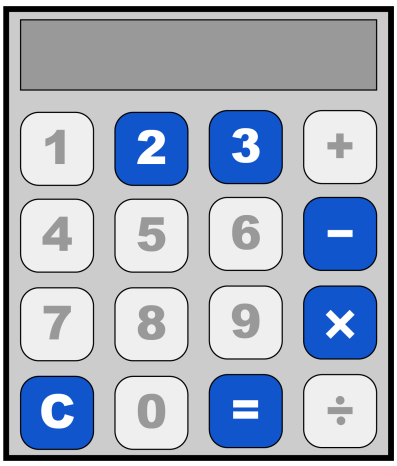
NOTE: Tes calculs peuvent inclure des nombres à 2 chiffres. Par exemple, si tu souhaites utiliser le nombre 32, tu appuierais sur la touche $\boxed{3}$ et ensuite la touche $\boxed{2}$.

THÈME : SENS DU NOMBRE

Problem of the Week
Problem A and Solution
Calculator Catastrophe

Problem

I dropped my calculator and my dog stepped on it. Now, most of the buttons are broken. The only working buttons are the ones with the dark background in the image below:



I want to have the calculator display each of the numbers from 1 to 12, by using only the unbroken buttons. The easiest ones to display are the numbers 2 and 3 since I can just enter them alone. However, all the others will need more buttons pressed. For example, to display a 7, I could use the fact that $7 = 3 \times 3 - 2$.

This is a very simple calculator, so I have to press the $\boxed{=}$ button after each part of the calculation. If I want to complete the calculation for $7 = 3 \times 3 - 2$, I would press the $\boxed{3}$ button, then the $\boxed{\times}$ button, then the $\boxed{3}$ button, and then the $\boxed{=}$ button to get the number 9. Then I would press the $\boxed{-}$ button, then the $\boxed{2}$ button, and then the $\boxed{=}$ button to get the number 7. This calculation takes 7 button presses to get the number 7 to appear in the calculator's display.

For each whole number from 1 to 12, show how you can display the number using at most 10 button presses.

NOTE: Your calculations can include 2-digit numbers. For example, if you wish to enter the number 32 you would press the $\boxed{3}$ button and then the $\boxed{2}$ button.

Solution

Answers will vary. Here is a table summarizing possible solutions for each whole number from 1 to 12.

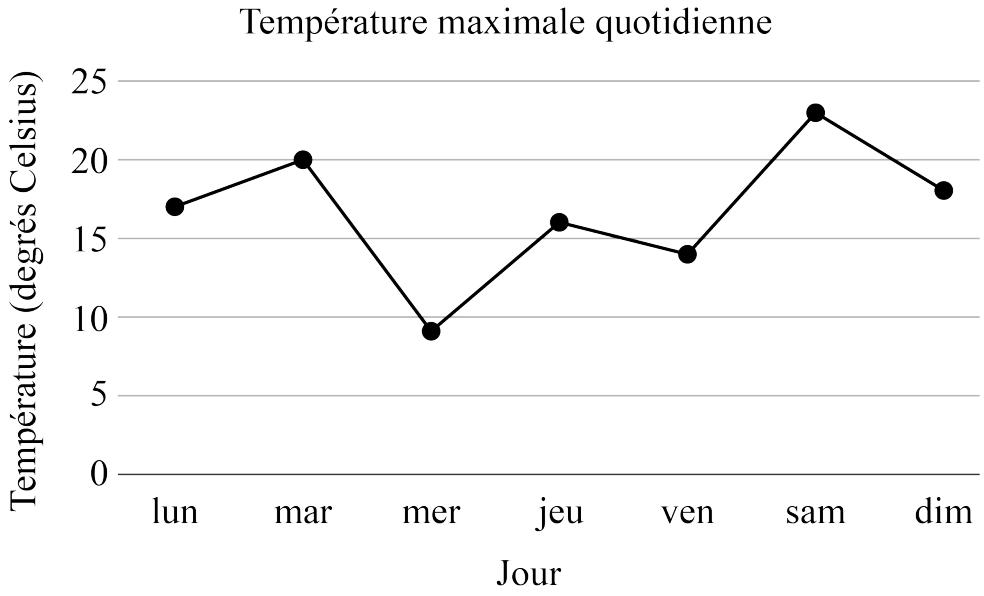
Number	Calculation	Button Presses	Number of Button Presses
1	$3-2$	3 − 2 =	4
2	2	2	1
3	3	3	1
4	2×2	2 × 2 =	4
5	$3\times 3-2-2$	3 × 3 = − 2 = − 2 =	10
6	2×3	2 × 3 =	4
7	$3\times 3-2$	3 × 3 = − 2 =	7
8	$2\times 2\times 2$	2 × 2 = × 2 =	7
9	3×3	3 × 3 =	4
10	$33-23$	3 3 − 2 3 =	6
11	$33-22$	3 3 − 2 2 =	6
12	$3\times 2\times 2$	3 × 2 = × 2 =	7

Problème de la semaine

Problème A

Suivi des températures

La station météorologique à l'Université de Waterloo enregistre des informations chaque jour. Le graphique ci-dessous montre la température la plus élevée pour chaque jour de la semaine.



(a) Estime la température la plus élevée enregistrée cette semaine-là.

(b) Estime la température la plus basse enregistrée cette semaine-là.

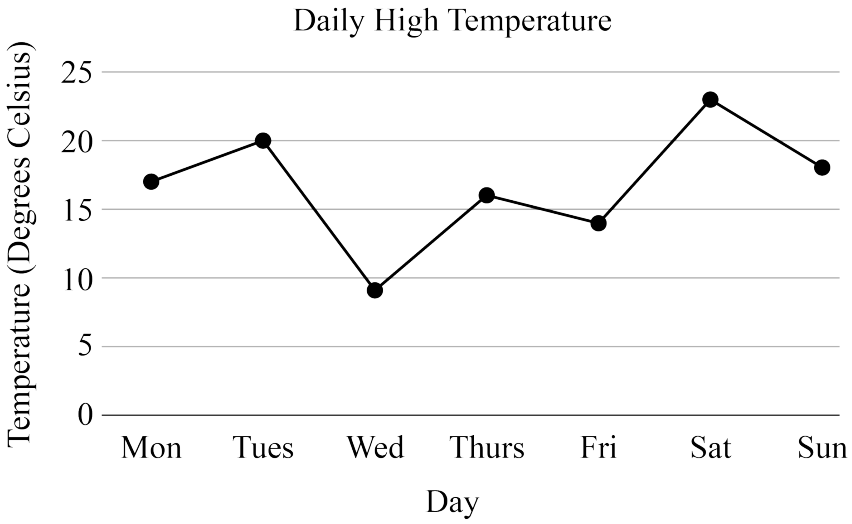
(c) Estime le plus grand changement de température d'un jour à l'autre durant cette semaine-là.

Thèmes : Gestion des données, Sens du nombre

Problem of the Week
Problem A and Solution
Tracking Temperatures

Problem

The weather station at the University of Waterloo records information every day. The graph below shows the high temperature each day for one week.



(a) Estimate the highest temperature recorded during this week.

(b) Estimate lowest temperature recorded during this week.

(c) Estimate the biggest change of temperature from one day to the next during this week.

Solution

(a) The highest temperature occurs on Saturday. It is between 20 degrees and 25 degrees. It appears to be slightly closer to 25 degrees than 20 degrees, so we will estimate the temperature is 23 degrees.

(b) The lowest temperature occurs on Wednesday. It is between 5 degrees and 10 degrees, but is very close to 10 degrees. We will estimate the temperature is 9 degrees.

(c) We estimate the temperature on Monday to be 17 degrees, the temperature on Tuesday to be 20 degrees, the temperature on Thursday to be 16 degrees, the temperature on Friday to be 14 degrees, and the temperature on Sunday to be 18 degrees, along with the estimations in parts (a) and (b).
Thus, from Monday to Tuesday we estimate that it became $20 - 17 = 3$ degrees warmer.
From Tuesday to Wednesday we estimate that it became $20 - 9 = 11$ degrees colder.
From Wednesday to Thursday we estimate that it became $16 - 9 = 7$ degrees warmer.
From Thursday to Friday we estimate that it became $16 - 14 = 2$ degrees colder.
From Friday to Saturday we estimate that it became $23 - 14 = 9$ degrees warmer.
From Saturday to Sunday we estimate that it became $23 - 18 = 5$ degrees colder.
Therefore, we can estimate that the biggest change of temperature from one day to the next is 11 degrees between Tuesday and Wednesday.

Problème de la semaine

Problème A

Quel nombre suis-je ?

Je suis un nombre à 3 chiffres.

La somme de mes chiffres est 11.

Le produit de mes chiffres est 16.

Mes chiffres sont en ordre descendant des centaines jusqu'aux unités.

Aucun chiffre ne se répète.

Quel nombre suis-je?

Thèmes : Pensée computationnelle, Sens du nombre

Problem of the Week
Problem A and Solution
What Number Am I?

Problem

I am a 3-digit number.

The sum of my digits is 11.

The product of my digits is 16.

My digits are in decreasing order from the hundreds digit to the ones digit.

I have no repeated digits.

What number am I?

Solution

We start by determining the ways to multiply three single-digits to get a product of 16. Here are the possibilities:

$1 \times 4 \times 4$ (in any order)
$1 \times 2 \times 8$ (in any order)
$2 \times 2 \times 4$ (in any order)

Since the number we are looking for does not have any repeated digits, then the digits in the number must be 1, 2, and 8. We can confirm this conclusion by noticing that the sum of these digits is $1 + 2 + 8 = 11$.

Since the digits appear in decreasing order, the number must be 821.

Problème de la semaine

Problème A

Le budget du gymnase

Mme Lukezich doit commander de l'équipement sportif pour le gymnase. Un nombre maximal de 40 élèves peut utiliser l'équipement en même temps. Mme Lukezich a besoin des articles suivants :

— un ballon de soccer pour chaque paire d'élèves,
— un parachute pour chaque groupe de 10 élèves,
— trois balles de tennis pour chaque groupe de 4 élèves.

Un ballon de soccer coûte 4 $. Un parachute coûte 25 $. Une balle de tennis coûte 2 $.

Quel est le coût total de l'équipement sportif requis ?

Thème : Sens du nombre

Problem of the Week
Problem A and Solution
Gym Budgets

Problem

Ms Lukezich needs to order sports equipment for the gym. There will be a maximum of 40 students using the equipment at any time. She needs the following equipment:

- one soccer ball for each pair of students
- one parachute for each group of 10 students
- three tennis balls for each group of 4 students

One soccer ball costs \$4. One parachute costs \$25. One tennis ball costs \$2.

How much will it cost to buy all of the required equipment?

Solution

One way to solve this is to figure out the maximum number of groups that could be using each piece of equipment. We will do this by assuming we have 40 students, since that is the maximum number of students using the equipment at any time.

- Since she needs 1 soccer ball for every 2 students, Ms Lukezich needs $40 \div 2 = 20$ soccer balls.

  The soccer balls will cost a total of $20 \times \$4 = \$80$.

- Since she needs 1 parachute for every 10 students, Ms Lukezich needs $40 \div 10 = 4$ parachutes.

  The parachutes will cost a total of $4 \times \$25 = \$100$.

- Since she needs 3 tennis balls for every 4 students, Ms Lukezich needs $40 \div 4 = 10$ sets of 3 tennis balls. Thus, she needs $10 \times 3 = 30$ tennis balls altogether.

  The tennis balls will cost a total of $30 \times \$2 = \$60$.

Therefore, the total cost for the sports equipment is $\$80 + \$100 + \$60 = \$240$.

Alternatively, we could make a table for each piece of equipment to determine how much each will cost. First, we make a table for the soccer balls.

Number of Soccer Balls	Number of Students	Total Cost, in $
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20

We could continue writing out rows in the table until we determine that 20 balls meets the needs of 40 students, for a cost of \$80. Or we might notice at this point that since $40 = 4 \times 10$, then the cost of soccer balls for 40 students is equal to 4 times the cost of soccer balls for 10 students. So the cost for the soccer balls is $4 \times \$20 = \$80$.

Next, we make a table for the parachutes.

Number of Parachutes	Number of Students	Total Cost, in $
1	10	25
2	20	50
3	30	75
4	40	100

So the cost for the parachutes is \$100.

Finally, we make a table for the tennis balls. Note that one set of 3 tennis balls costs $3 \times \$2 = \$6$.

Number of Sets of 3 Tennis Balls	Number of Students	Total Cost, in $
1	4	6
2	8	12
3	12	18
4	16	24
5	20	30

We could continue writing out rows in the table until we determine that 10 sets of tennis balls meets the needs of 40 students for a cost of \$60. Or, we might notice at this point that since $40 = 2 \times 20$, then the cost of the tennis balls for 40 students is equal to 2 times the cost of tennis balls for 20 students. So the cost for the tennis balls is $2 \times \$30 = \$60$.

Once again, we get a total cost of $\$80 + \$100 + \$60 = \$240$ for the sports equipment.

Problème de la semaine

Problème A

La cueillette de pommes

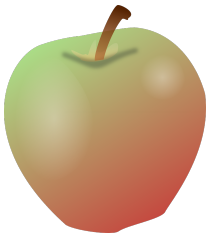
La famille de Caleb est allée cueillir des pommes. Ils ont ramené 24 pommes à la maison.

(a) Ils ont séché un sixième des pommes pour en faire des collations. Combien de pommes ont-ils séchées ?

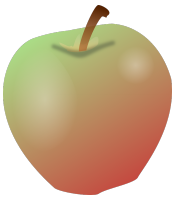
(b) Ils ont utilisé un tiers des pommes pour faire de la compote de pommes. Sachant qu'un pot de compote nécessite deux pommes, combien de pots ont-ils préparés ?

(c) Ils ont utilisé un quart des pommes pour faire deux tartes. Combien de pommes y avait-il dans chaque tarte ?

(d) Ils ont gardé le reste des pommes pour les manger. Combien de pommes leur restait-il à manger ?



Thème : Sens du nombre



Problem of the Week
Problem A and Solution
Apple Picking

Problem

Caleb's family went apple picking. They came home with 24 apples.

(a) They dried one sixth of the apples to make snacks. How many apples did they dry?

(b) They used one third of the apples to make applesauce. It takes two apples to make one jar of applesauce. How many jars did they make?

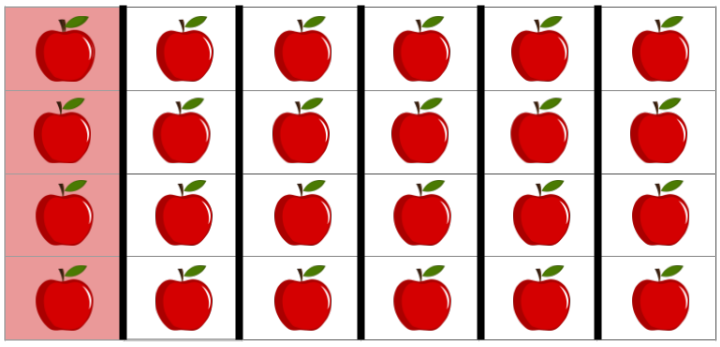
(c) They used one fourth of the apples to make two pies. How many apples were in each pie?

(d) They saved the rest of the apples to eat. How many apples did they have to eat?

Solution

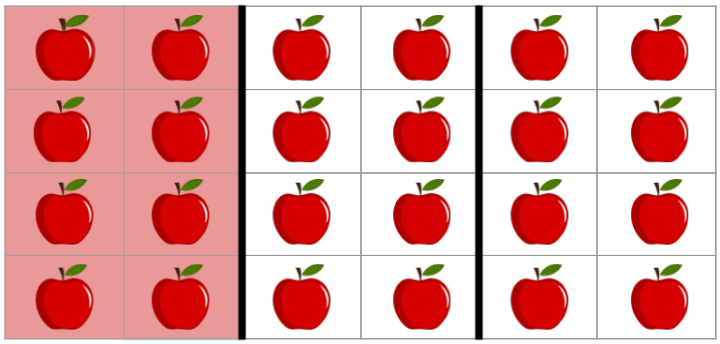
In each part, we will use a grid that has the 24 apples arranged in 4 rows and 6 columns, with 1 apple in each cell, and use this to divide the apples into smaller groups.

(a) We divide the grid of apples into six equal groups by placing a dividing line between each column.



Since each group contains 4 apples, we know that 4 apples were dried for snacks.

(b) We divide the grid of apples into three equal groups by placing a dividing line between the second and third column, and between the fourth and fifth column.

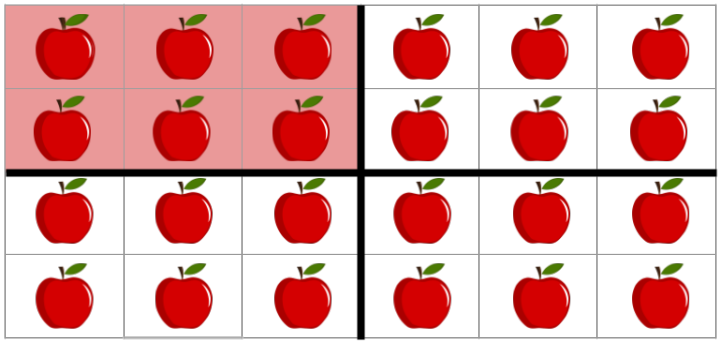


Since each group contains 8 apples, know that one third of the apples is 8 apples. Since it takes 2 apples to make one jar of applesauce, we can skip count by 2s to determine the number of jars they made. This is summarized in the table below.

Number of Apples	2	4	6	8
Number of Jars	1	2	3	4

Therefore, Caleb's family made 4 jars of applesauce.

(c) We divide the grid of apples into four equal groups by placing a dividing line between the third and fourth column, and between the second and third row.



Since each group contains 6 apples, we know that one fourth of the apples is 6 apples. Since they made 2 pies, then half of these apples were used in each pie. So there were 3 apples in each pie.

(d) From the previous three parts, we know that the family used $4 + 8 + 6 = 18$ apples. This means they have $24 - 18 = 6$ apples left for eating.

Problème de la semaine

Problème A

La frénésie de la pêche

L'équipage de huit personnes du bateau **Mordus à l'Hameçon** a pêché 120 poissons en une heure. Les membres de l'équipage ont ensuite vendu chaque poisson à 3 $ et ont divisé le montant total de l'argent gagné entre tous les membres de manière égale.

Combien d'argent chaque membre de l'équipage a-t-il reçu ?

Thème : Sens du nombre

Problem of the Week
Problem A and Solution
Fishing Frenzy

Problem

In one hour, the eight person crew of the **Hook Line and Sinker** boat caught 120 fish. They then sold each fish for \$3 and divided the total amount of money equally between the crew members.

How much money did each crew member receive?

Solution

One way to solve this problem is to divide the number of fish by the number of people in the crew: $120 \div 8 = 15$. Since each person's share of the catch is 15 fish, and each fish is worth \$3, then each person should receive $15 \times \$3 = \$45$ after selling the fish.

Another way to determine each person's share is distribute the money earned into 8 equal piles. However, you probably don't want to do this \$1 at a time.

The total amount earned is $120 \times \$3 = \$360$. You might start by distributing \$10 at a time to each crew member, until you have less than \$80 left. After doing so, each crew member would have received \$10 four times, so \$40 each, and there would be \$40 left to share among the crew.

Now you can distribute that money \$1 or \$2 at a time, or you might try a higher number like \$5. If you distribute \$5 at a time, each crew member would receive \$5 once.

At this point there would be no money left from the original \$360.

Therefore, each person receives $\$40 + \$5 = \$45$ as their share of the catch.

Problème de la semaine

Problème A

Le temps qui court

Suppose que l'heure actuelle correspond à celle indiquée sur l'horloge et que c'est le matin.

La journée scolaire se termine dans 5 heures et 6 minutes à partir de l'heure indiquée sur l'horloge.

(a) Combien de minutes reste-t-il avant la fin de la journée scolaire ?

(b) À quelle heure se termine l'école ?

(c) L'école a commencé à 8h55. Combien de temps s'est écoulé, en heures et en minutes, entre le début de la journée scolaire et l'heure indiquée sur l'horloge ?

Problem of the Week
Problem A and Solution
Time After Time

Problem

Assume that the current time matches the time shown on the clock, and the current time is in the morning.

The school day ends in 5 hours and 6 minutes from the time shown on the clock.

(a) How many more minutes are left until the end of the school day?

(b) What time does school end?

(c) School started at 8:55 a.m. How much time has passed, in hours and minutes, between the start of the school day and the time shown on the clock?

Solution

(a) Since each hour has 60 minutes, then 5 hours have a total of $5 \times 60 = 300$ minutes. So the total number of minutes left until the end of the school day is $300 + 6 = 306$ minutes.

(b) The time showing on the clock is 10:09 a.m. We will add 5 hours and 6 minutes to this time. Two hours after 10:09 a.m. it will be 12:09 p.m. Three hours after that it will be 3:09 p.m. Six minutes later it will be 3:15 p.m. Thus, school ends at 3:15 p.m.

(c) We start at 8:55 a.m. One hour after that it will be 9:55 a.m. Five minutes after that it will be 10:00 a.m. Nine minutes after that it will be 10:09 a.m. Thus, a total of 1 hour and $5 + 9 = 14$ minutes have passed between the start of the school day and the time shown on the clock.

Problème de la semaine

Problème A

Les arrêts d'autobus

Un autobus vide commence son itinéraire quotidien. En chemin, il effectue plusieurs arrêts. À chaque arrêt, des passagers montent dans l'autobus et d'autres en descendent.

Le tableau montre combien de passagers montent et descendent de l'autobus à chaque arrêt.

Numéro de l'arrêt	Nombre de passagers qui montent	Nombre de passagers qui descendent
1	23	0
2	17	12
3	13	3
4	1	9
5	2	8

(a) Combien de passagers montent dans l'autobus au total ?

(b) L'autobus contient des sièges pour 30 passagers. Une fois tous les sièges occupés, les passagers doivent rester debout. Entre le 3^{e} et le 4^{e} arrêt, y a-t-il suffisamment de sièges pour tous les passagers ? Sinon, combien de passagers doivent rester debout ?

(c) L'autobus a un pneu crevé au 6^{e} arrêt, ce qui oblige tous les passagers à descendre. Combien de passagers descendent de l'autobus au 6^{e} arrêt ?

Thème : Sens du nombre

Problem of the Week
Problem A and Solution
Bus Stops

Problem

An empty bus starts its daily route. Along the way it makes several stops. At each stop, some passengers board the bus and some passengers exit the bus.

The table shows how many passengers get on and get off the bus at each stop.

Stop Number	Number Boarding	Number Exiting
1	23	0
2	17	12
3	13	3
4	1	9
5	2	8

(a) How many passengers board the bus in total?

(b) The bus has seats for 30 passengers. Once all the seats are full, passengers must stand. Between the 3^{rd} and 4^{th} stops, are there enough seats for all the passengers? If not, how many passengers must stand?

(c) The bus gets a flat tire at the 6^{th} stop so all the passengers need to exit. How many passengers get off the bus at the 6^{th} stop?

Solution

(a) The total number of passengers who board the bus is: $23 + 17 + 13 + 1 + 2 = 56$.

(b) The table summarizes the number of passengers on the bus after each stop.

Stop Number	Number Boarding	Number Exiting	Total Passengers on the Bus
1	23	0	23
2	17	12	$23 + 17 - 12 = 28$
3	13	3	$28 + 13 - 3 = 38$
4	1	9	$38 + 1 - 9 = 30$
5	2	8	$30 + 2 - 8 = 24$

Thus, between the 3^{rd} and 4^{th} stops, there are 38 passengers on the bus. Since there are only 30 seats, there are not enough seats for all the passengers. The number of passengers who must stand is $38 - 30 = 8$.

(c) From the table in (b), we know that 24 passengers are on the bus when it arrives at the 6^{th} stop. They would all need to exit. Thus, 24 passengers get off the bus at the 6^{th} stop.

Problème de la semaine

Problème A

Des bannières exquises et éclatantes

Un club fabrique une grande bannière colorée à porter lors d'un défilé.

Lorsque tu achètes du tissu, tu dois spécifier la longueur exacte de tissu dont tu as besoin. Le tableau ci-dessous indique la quantité de tissu requise pour chaque couleur dans la conception de la bannière.

Couleur	**Quantité de tissu requise**
rouge	deux fois la quantité de tissu violet
orange	la moitié de la quantité de tissu blanc
violet	un tiers de la quantité de tissu orange
blanc	quatre fois la quantité de tissu vert
vert	un demi-mètre

(a) Quelle est la quantité de tissu requise, en mètres, pour chaque couleur afin de réaliser la bannière ?

(b) Le tissu coûte 6 $ par mètre. Détermine le coût total du tissu requis pour fabriquer la bannière.

Problem of the Week
Problem A and Solution
Bright Beautiful Banners

Problem

A club is making a large, colourful banner to carry in a parade.

When you buy fabric, you specify the exact length of material you need. The table below shows information about the amount of fabric of each colour needed for the banner design.

Colour	Amount of Fabric Needed
red	two times as much fabric as purple
orange	one half as much fabric as white
purple	one third as much fabric as orange
white	four times as much fabric as green
green	half a metre

(a) How many metres of each colour fabric are needed to make the banner?

(b) The fabric costs \$6 per metre. Determine the total cost of the fabric required to make the banner.

Solution

(a) Since $\frac{1}{2}$ m of green fabric is needed, and the club needs 4 times as much white fabric as green, then they must need $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$ m of white fabric.

Since they need $\frac{1}{2}$ as much orange fabric as white, then they must need 1 m of orange fabric.

Since they need $\frac{1}{3}$ as much purple fabric as orange, then they must need $\frac{1}{3}$ m of purple fabric.

Since they need 2 times as much red fabric as purple, then they must need $\frac{2}{3}$ m of red fabric.

(b) The club needs $2 + 1 + \frac{1}{3} + \frac{2}{3} + \frac{1}{2} = 4\frac{1}{2}$ m of fabric in total.

Since the fabric costs \$6 per metre, the total cost of 4 m of fabric is $4 \times \$6 = \$24$. The cost of $\frac{1}{2}$ m of fabric is $\frac{1}{2} \times \$6 = \$3$. So the total cost of the fabric required to make the banner is $\$24 + \$3 = \$27$.

Problème de la semaine

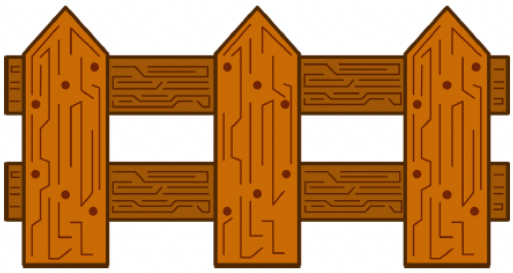
Problème A

Les poteaux de clôture

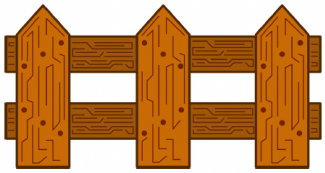
Elyas aide ses parents à installer des poteaux de clôture dans leur cour arrière. Ils installent une rangée de 7 poteaux espacés de manière égale. Les poteaux sont tous de la même taille. La distance entre le milieu du $2^{\text{ème}}$ poteau et le milieu du $5^{\text{ème}}$ poteau est de quatre mètres et demi.

Quelle est la distance entre le milieu du premier poteau et le milieu du dernier poteau ?

INDICE : Tu peux commencer par dessiner un diagramme des poteaux de clôture et y indiquer la distance que tu connais.



THÈMES : GÉOMÉTRIE ET MESURE, SENS DU NOMBRE



Problem of the Week
Problem A and Solution
Fencepost Problem

Problem

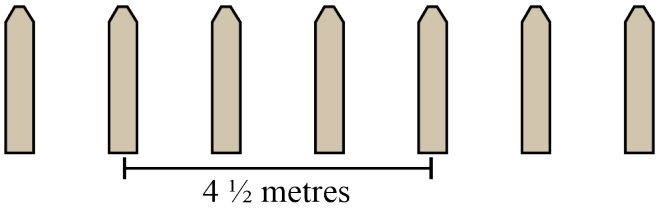
Elyas helps his parents install fence posts in their backyard. They install a row of 7 equally-spaced posts. The posts are all the same size. The distance between the middle of the 2nd post and the middle of the 5th post is four and a half meters.

What is the distance between the middle of the first post and the middle of the last post?

HINT: You might start by drawing a diagram of the fence posts and labelling it with the distance you know.

Solution

There are many ways to calculate the distance. We start with a diagram of the posts, labelled with the distance of $4\frac{1}{2}$ metres between the middle of the 2nd and 5th posts.

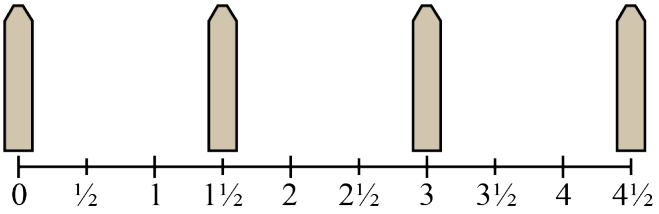


Solution 1

Notice that there are a total of three gaps between the 2nd and 5th posts. There are also a total of three gaps between the 1st and 2nd posts and the 5th and 7th posts. Since the distances between the middle of adjacent posts are all the same, then the total distance between the 1st and 2nd posts and the 5th and 7th posts must also be $4\frac{1}{2}$ m. Since there are six gaps in total, the distance between the middle of the first post and the middle of the last post is $4 + \frac{1}{2} + 4 + \frac{1}{2} = 9$ m.

Solution 2

Another way to solve this problem is to draw a number line from 0 to $4\frac{1}{2}$ m and space the 2^{nd}, 3^{rd}, 4^{th}, and 5^{th} fenceposts evenly along this line.

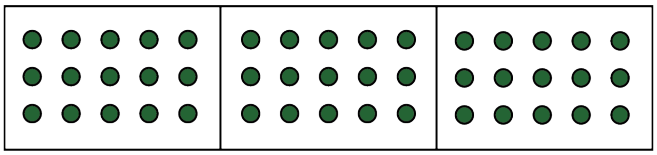


From this, we observe that the distance from the middle of one post to the middle of an adjacent post is $1\frac{1}{2}$ m. Since there are six gaps in total between adjacent posts, the distance between the middle of the first post and the middle of the last post is:

$$1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2}=9 \text{ m}$$

Solution 3

First we convert the distance to centimetres: $4\frac{1}{2}$ m $= 450$ cm. Then we use a tape diagram. We observe that there are 3 gaps between the 2^{nd} and 5^{th} posts. That means we need to divide 450 cm into three equal distances. To make it easier, since $450 = 45 \times 10$, we let one dot represent 10 cm, and then distribute 45 dots into the three equal pieces of our tape diagram.



Each piece has a total of 15 dots. This means the distance between adjacent posts is $15 \times 10 = 150$ cm. Since there are 6 gaps in total between the first post and the last post, the total distance is:

$$150+150+150+150+150+150=900 \text{ cm or } 9 \text{ m}$$

Problème de la semaine

Problème A

Les épargnes

Nethra économise pour acheter une nouvelle console de jeux. Elle a besoin de 829 $ au total pour acheter la console. Elle a déjà économisé 117 $.

Nethra occupe deux emplois à temps partiel. Elle travaille dans une épicerie chaque mardi et jeudi pendant 2 heures par jour. À l'épicerie, elle gagne 15 $ par heure. Elle travaille dans un restaurant chaque samedi pendant 5 heures. Au restaurant, elle gagne 13 $ par heure.

Nethra est payée le dimanche de chaque semaine. Ce jour-là, elle donne 10 $ à ses parents pour le transport et s'achète des friandises à 5 $. Elle épargne le reste de l'argent gagné durant la semaine.

Combien de semaines Nethra doit-elle travailler pour économiser suffisamment d'argent afin d'acheter la console de jeux ?

Thème : Sens du nombre

Problem of the Week
Problem A and Solution
Savings

Problem

Nethra is saving to buy a new gaming system. She needs $829 in total to buy the console. She has already saved $117.

Nethra has two part-time jobs. She works at a grocery store every Tuesday and Thursday for 2 hours each day. From the grocery store job she earns $15 per hour. She works at a restaurant every Saturday for 5 hours. From the restaurant job she earns $13 per hour.

Nethra gets paid every week on Sunday. That day she pays her parents $10 for transportation and buys herself $5 worth of treats. She saves the rest of the money she earned during the week.

How many weeks does Nethra have to work until she has saved enough money to buy the gaming system?

Solution

Since Nethra has already saved $117, she needs $\$829 - \$117 = \$712$ more to buy the gaming system.

From her job at the grocery store, she earns $4 \times \$15 = \$60$ each week.
From her job at the restaurant, she earns $5 \times \$13 = \$65$ each week.
This is a total of $\$60 + \$65 = \$125$ each week.

Each week she spends $\$10 + \$5 = \$15$.
This means she can save $\$125 - \$15 = \$110$ each week for the gaming system.

Now we can make a table to show how much she saves over multiple weeks:

Week	Total Savings ($)
1	110
2	220
3	330
4	440
5	550
6	660
7	770

After 7 weeks Nethra has saved more than $712, so she has enough money to buy the gaming system.

Problème de la semaine

Problème A

Une logistique de livraison

Soraya et Aydin travaillent tous les deux comme chauffeurs-livreurs pour une entreprise locale. Aujourd'hui, ils doivent livrer 64 colis au total. Soraya a reçu trois fois plus de colis à livrer qu'Aydin.

Ils décident qu'il serait préférable que chacun livre le même nombre de colis. Combien de colis Soraya devrait-elle donner à Aydin pour qu'ils aient le même nombre de colis ?

Thèmes : Algèbre, Sens du nombre

Problem of the Week
Problem A and Solution
Delivery Dilemma

Problem

Soraya and Aydin both work as delivery drivers for a local business. Today they have 64 packages to deliver in total. Soraya was given three times as many packages as Aydin to deliver.

They decide it would be better for each person to deliver the same number of packages. How many packages should Soraya give Aydin so that they have the same number of packages?

Solution

Solution 1

One way to solve the problem is to guess and check to figure out how many packages were given to each driver. We can organize our guesses in a table where we keep track of how many packages Aydin has, how many packages Soraya has, and how many packages they have in total. Let's start by guessing that Aydin has 10 packages until we find a combination that results in a total of 64 packages.

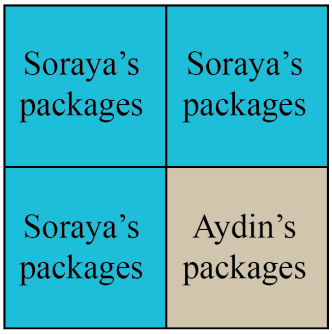
Aydin's Packages	Soraya's Packages	Total Packages
10	$3 \times 10 = 30$	$10 + 30 = 40$
11	$3 \times 11 = 33$	$11 + 33 = 44$
12	$3 \times 12 = 36$	$12 + 36 = 48$
13	$3 \times 12 = 39$	$13 + 39 = 52$
14	$3 \times 14 = 42$	$14 + 42 = 56$
15	$3 \times 15 = 45$	$15 + 45 = 60$
16	$3 \times 16 = 48$	$16 + 48 = 64$

Thus, we see that Aydin started with 16 packages and Soraya started with 48 packages. If they want to deliver the same number of packages, each should take half of the total number of packages. Half of 64 is 32 packages.

So if Soraya gives Aydin $48 - 32 = 16$ packages, then Aydin will have $16 + 16 = 32$ packages and each of them will have the same number to deliver.

Solution 2

Another way to solve this problem is to use fractions. If Soraya was given three times as many packages as Aydin to deliver, then adding Soraya and Aydin's packages together should give us four times as many packages as Aydin has. This means that Aydin has $\frac{1}{4}$ of the total number of packages and Soraya has $\frac{3}{4}$ of the total number of packages. This is shown in the following diagram. The large square represents the total number of packages. The large square is divided into quarters, with three of the quarters representing Soraya's packages and one of the quarters representing Aydin's packages.



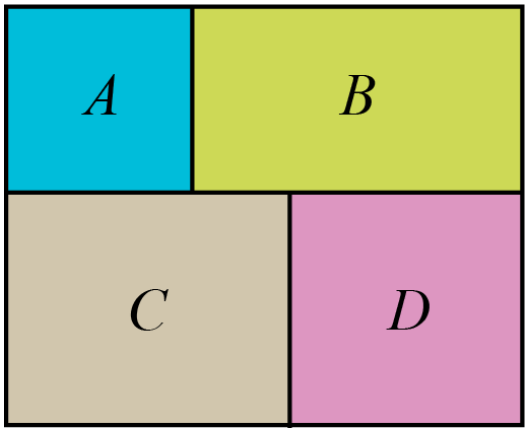
In order for Aydin and Soraya to each have the same number of packages, Soraya must give Aydin $\frac{1}{4}$ of the total number of packages, so that they each have $\frac{1}{2}$. Since there are 64 packages in total, $\frac{1}{4}$ of 64 is equal to $64 \div 4 = 16$. Thus, Soraya should give Aydin 16 packages.

Problème de la semaine

Problème A

Un casse-tête de courtepointe

Les employés de l'entreprise Fil d'Inspiration doivent réaliser une petite courtepointe pour un projet spécial, en utilisant quatre tissus de couleurs différentes. Leur motif de courtepointe est un rectangle divisé en quatre rectangles plus petits, nommés A, B, C et D.

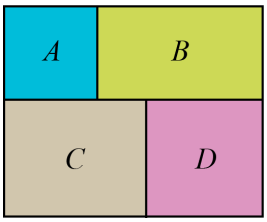


La pièce A est un carré dont l'aire est de 16 cm^2. La pièce D est un carré dont l'aire est de 25 cm^2. La distance horizontale entre le côté droit de la pièce A et le côté gauche de la pièce D est de 2 cm. Quelle est l'aire totale de la courtepointe ?

Problem of the Week
Problem A and Solution
Quilting Puzzle

Problem

The employees of Sew Inspired need to make a tiny quilt for a special project using four different colours of fabric. Their quilt pattern is a rectangle divided into four smaller rectangles, marked A, B, C, and D.



Piece A is a square with area 16 cm^2 and piece D is a square with area 25 cm^2. The horizontal distance between the right side of piece A and the left side of piece D is 2 cm. What is the area of the entire quilt?

Solution

To calculate the area of the quilt we will determine the lengths of its sides. We know that piece A is a square with area 16 cm^2. We also know that the lengths of the sides of a square must be the same. So if the length of one side of a square is n, then the area of the square must be $n \times n$. By trial and error, we determine $4 \times 4 = 16$, and so the length of each side of piece A must be 4 cm. Another way to determine the lengths of the sides of piece A is to start with 16 unit squares (using blocks or cut out of paper) and determine how to arrange them into a larger square. The only possible arrangement is a 4×4 square.

Similarly, piece D is a square with area 25 cm^2. Since $5 \times 5 = 25$, the length of each side of piece D must be 5 cm.

The opposite sides of a rectangle must be the same length, so the width of the quilt (i.e. the vertical side) is equal to the sum of the side lengths of pieces A and D. Thus, the width of the quilt is equal to $4 + 5 = 9$ cm. Similarly, we know the bottom of piece A is on the same line as the top of piece D, and the horizontal distance between the two pieces is 2 cm. Therefore the length of the quilt (i.e. the horizontal side) is equal to the sum of the side lengths of pieces A and D, plus 2. Thus, the length of the quilt is equal to $4 + 5 + 2 = 11$ cm.

Now we can calculate the area of the entire quilt. The area of this rectangle is the product of its length and width. So the area of the entire quilt is $9 \times 11 = 99$ cm^2.

Problème de la semaine

Problème A

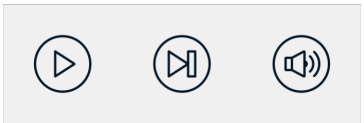
Visionnage de vidéos

Jo aime regarder des vidéos. Parfois, elle les regarde à vitesse normale. Cependant, elle a également la possibilité de les visionner à $\frac{1}{4}$ de la vitesse normale, à $\frac{1}{2}$ de la vitesse normale ou à 2 fois la vitesse normale. Le tableau ci-dessous indique les durées normales de visionnage de quatre vidéos et la vitesse à laquelle Jo regarde chacune d'elles.

	Durée à vitesse normale	**Vitesse de visionnage de Jo**
Vidéo A	50 secondes	$\frac{1}{4}$ de la vitesse
Vidéo B	2 minutes 15 secondes	vitesse normale
Vidéo C	1 minute 40 secondes	$\frac{1}{2}$ de la vitesse
Vidéo D	6 minutes 20 secondes	2 fois la vitesse

Quel est le temps total que Jo a passé à regarder ces quatre vidéos ?

Thèmes : Géométrie et mesure, Sens du nombre



Problem of the Week
Problem A and Solution
Video Viewing

Problem

Jo likes to watch videos. Sometimes she watches them at normal speed. However, she also has the option to watch them at $\frac{1}{4}$ as fast as normal speed, $\frac{1}{2}$ as fast as normal speed, or 2 times as fast as normal speed. The table below lists the normal play times for four videos and the speed at which Jo watches each one.

	Normal Speed Time	**Speed Jo Watches Video At**
Video A	50 seconds	$\frac{1}{4}$ speed
Video B	2 minutes 15 seconds	normal speed
Video C	1 minute 40 seconds	$\frac{1}{2}$ speed
Video D	6 minutes 20 seconds	2 times speed

What is the total time Jo spent watching these four videos?

Solution

If a video is playing at $\frac{1}{4}$ speed, then it will take 4 times as long to play as normal speed. So a 50 second video will take $50 \times 4 = 200$ seconds to watch.

If a video is playing at $\frac{1}{2}$ speed, then it will take 2 times as long to play as normal speed. So a 1 minute and 40 second video will take 2 minutes and $40 \times 2 = 80$ seconds to watch.

If a video is playing at 2 times speed then it will take half as long to play as normal speed. Half of 6 minutes is 3 minutes and half of 20 seconds is 10 seconds.

Now we can add up all the minutes and seconds of playing times of all four videos:

$$2 + 2 + 3 = 7 \text{ minutes and } 200 + 15 + 80 + 10 = 305 \text{ seconds}$$

There are 60 seconds in 1 minute. When we skip count by 60, we get 60, 120, 180, 240, 300. Thus, there are 5 minutes in 305 seconds with $305 - 300 = 5$ seconds left over.

So Jo spent $7 + 5 = 12$ minutes and 5 seconds watching these four videos.

Problème de la semaine

Problème A

Peux-tu faire du kayak ?

À la boutique les Bateaux de Bo, tous les kayaks ont le même nombre de places pour s'asseoir et toutes les chaloupes ont le même nombre de places pour s'asseoir. Un kayak a moins de places pour s'asseoir qu'une chaloupe. Deux kayaks et deux chaloupes peuvent avoir un total de 10 places pour s'asseoir. Un kayak et trois chaloupes ont un total de 11 places pour s'asseoir.

Combien de places pour s'asseoir y a-t-il dans un kayak ?

Thème : Sens du nombre

Problem of the Week
Problem A and Solution
Can You Kayak?

Problem

At Bo's Boat Shop, all kayaks have the same number of seats and all rowboats have the same number of seats. A kayak has fewer seats than a rowboat. Two kayaks and two rowboats have a total of 10 seats. One kayak and three rowboats have a total of 11 seats.

How many seats are in one kayak?

Solution

Since two kayaks and two rowboats have a total of 10 seats, then half as many kayaks and rowboats will have half as many seats. That is, one kayak and one rowboat will have a total of $10 \div 2 = 5$ seats.

Since a kayak has less seats than a rowboat, then it must be the case that one kayak has 1 seat and one rowboat has 4 seats, or that one kayak has 2 seats and one rowboat has 3 seats.

If one kayak has 1 seat and one rowboat has 4 seats, then one kayak and three rowboats have $1 + 4 + 4 + 4 = 13$ seats. Since we're told that one kayak and three rowboats have 11 seats, this is not a possible solution.

If one kayak has 2 seats and one rowboat has 3 seats, then one kayak and three rowboats have $2 + 3 + 3 + 3 = 11$ seats. Since we're told that one kayak and three rowboats have 11 seats, this is a possible solution.

Thus, 2 seats are in one kayak.