

## Problem of the Month Problem 8: Some Surprising Squares

May 2025

## Hint

- 1. There is a solution to  $m^2 8e^2 = 1$  where both m and e are integers between 0 and 5. Can you somehow use your solution for the equation  $m^2 - 8e^2 = 1$  to find solutions to the equations  $m^2 - 8e^2 = 4$  and  $m^2 - 8e^2 = 9$ ?
- 2. There is a pair of integers (m, e) satisfying  $m^2 8e^2 = 49$  where both m and e are single-digit positive integers.
- 3. Try expanding out  $N((a+b\sqrt{8})(c+d\sqrt{8}))$  and  $N(a+b\sqrt{8})N(c+d\sqrt{8})$ .
- 4. Expand out  $N(a + b\sqrt{8})$  and use the result from Question 3.
- 5. You will need to apply Question 3 repeatedly here. Consider your m and e from Question 2. What is  $N((m + e\sqrt{8})^2)$ ? What about  $N((m + e\sqrt{8})^{2^n})$ ?