# Problem of the Month
Problem 5: SET!
February 2025

This month's problem is about the card game SET!. Each card in the game SET! has four properties, and each property has three options as follows:

- **Number**: 1, 2, or 3.

- **Colour**: Red, green, or purple.

- **Shading**: Solid, striped, or open.

- **Shape**: Squiggle, diamond, or oval.

For example, below are some cards from a SET! deck. From left to right we have a card with 1 red open diamond, a card with 2 green solid squiggles, a card with 3 purple striped ovals, and a card with 1 red solid oval.
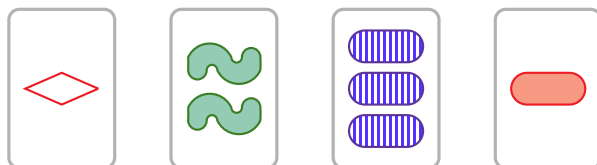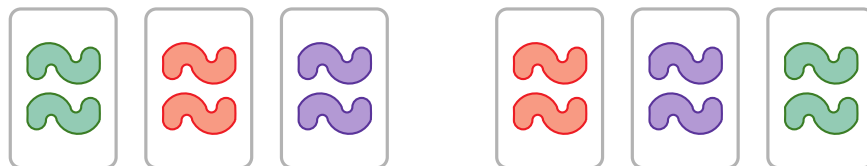

A *set* is a group of three cards such that for each of the four properties, all three cards have the same option, or all three cards have different options. For example, the following four collections of three cards are sets:
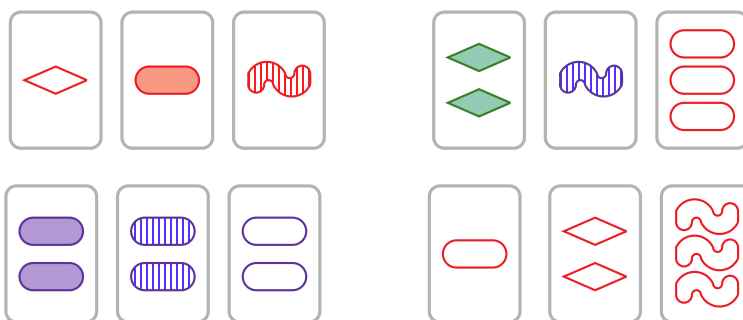
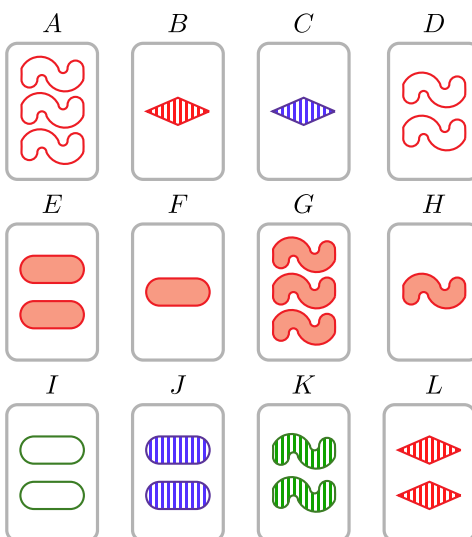However, the following two collections of three cards are not sets:
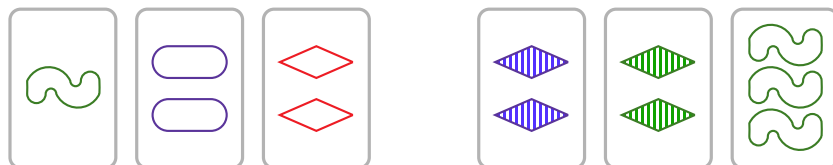
The order of the cards doesn't matter, so the two collections of three cards
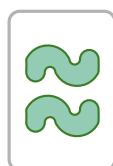
are considered to be the same set. Note that the same card could appear in two distinct sets.

1. Find six distinct sets in the following collection of twelve cards:

2. A SET! deck consists of exactly one of every possible card. How many cards are in a SET! deck?

3. How many distinct sets contain the following card?

4. How many different sets exist in a SET! deck?

5. In a game of SET!, first, a full deck is shuffled. Then twelve cards are dealt on the table, and the players try to find sets. If a player finds a set, they shout "set", and pick up the three cards that form the set. Those three cards are removed from the game (to be counted at the end), and three more cards from the deck are dealt to replace them. The game continues until all the cards from the deck are dealt and there are no more sets among the cards remaining on the table.

   Suppose that during a game of SET!, there are three cards remaining on the table. That is, every card in the deck besides the three on the table has been collected as part of a set. Show that the remaining three cards must form a set.