Problem of the Month

Problem 3: Multiplication in two dimensions

December 2025

Hint

1. (a) Expand out the left-hand side, and equate it to the right-hand side of the given equation.

   (b) If it's true for all (c, d) it must be true when $(c, d) = (1, 1)$ (or any other specific point).

2. Compute the first six powers and plot them on the Cartesian plane. Question 1(b) may come in handy here.

3. For both (a) and (b), it may help to plot $(0, 2), (1, 1)$, and $(0, 2) * (1, 1)$ on the Cartesian plane. For (b), try to write the coordinates of D_1 in terms of r_1 and ϕ_1. To do this, begin by assuming D_1 is in the first quadrant of the Cartesian plane and draw out a diagram relating r_1, θ_1, and the coordinates of D_1.

4. Suppose C is a solution to $x^4 = (-119, -120)$. Using 3(b), see if you can figure anything out about $|C|$ and $\theta(C)$.

5. Interpret the conditions given in the question in terms of $|F|$, $|Y|$, $\theta(F)$, and $\theta(Y)$. Then apply 3(b).