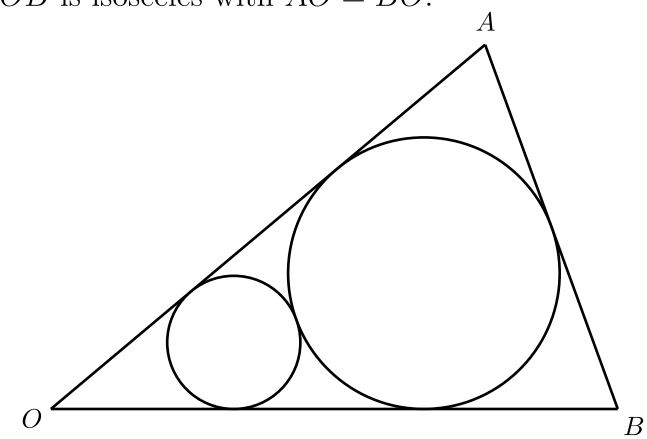
Problem of the Month

Problem 2: Squeezing circles into a triangle

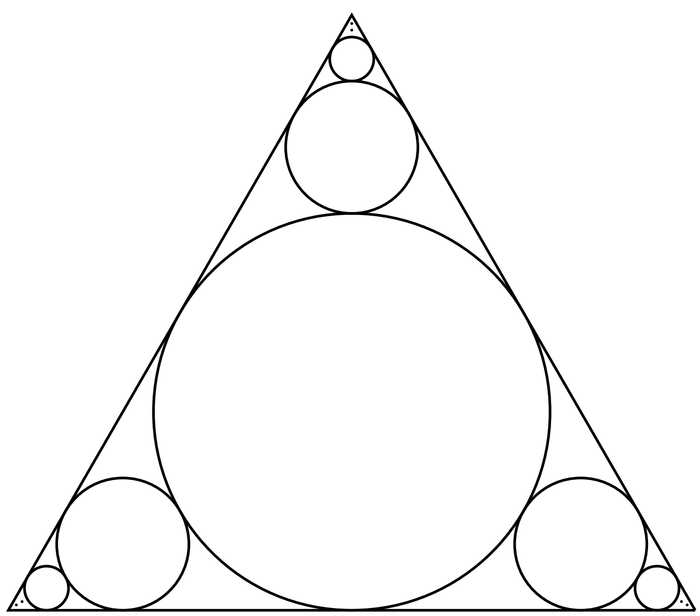
November 2025

1. Let θ be an angle with $0^\circ < \theta < 45^\circ$. In the diagram, points A and B are configured so that $\angle AOB = 2\theta$ and $\triangle AOB$ is isosceles with $AO = BO$.



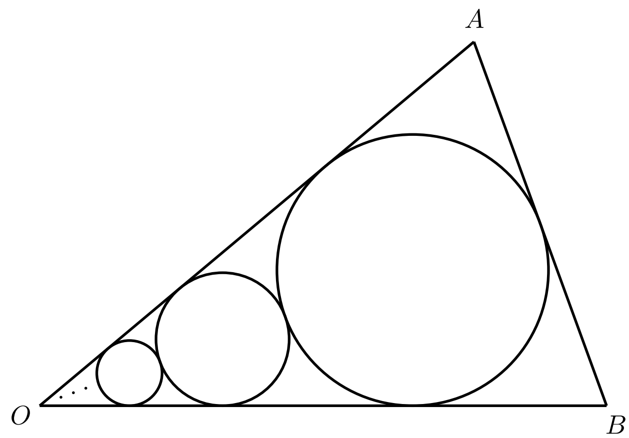
A circle is inscribed in $\triangle AOB$ and another circle is drawn so that it is tangent to the larger circle as well as OA and OB. In terms of θ, find the ratio of the radius of the larger circle to the radius of the smaller circle.

2. Similar to Question 1, an equilateral triangle has a circle inscribed in it. Three circles are then drawn, each tangent to two of the sides of the triangle as well as the larger circle. Another three circles are then drawn, each tangent to two of the three sides of the triangle as well as one of the circles drawn in the previous step.



If this process is continued indefinitely, what fraction of the area of the triangle is covered by circles?

3. Suppose $\triangle AOB$ and θ are as they were defined in Question 1. The process of drawing a circle tangent to OA, OB, and the smallest circle is repeated forever. What fraction of the area of $\triangle AOB$ is covered by circles? Your answer should be in terms of θ.



The result of Question 3 can be applied to solve Question 2. Can you see how?