



Problem of the Month

Problem 1: Displacing Permutations

October 2025

Hint

1. Explicitly compute $\mathbf{D}(\sigma)$ for all permutations in X_3 . Then try it for X_4 . What do you notice about the permutations satisfying $\mathbf{D}(\sigma) = 2$?
 2. See if you can argue that at most four integers in $\{1, 2, 3, \dots, n\}$ can satisfy $\sigma(i) \neq i$. How must those integers interact with each other?
 3. It may be helpful here to consider the path a single integer takes under repeated application of the permutation. For example, in the permutation σ denoted by 58123764, we can start with 1 and repeatedly apply the permutation. We get, $\sigma(1) = 5$, $\sigma(5) = 3$, $\sigma(3) = 1$, and then the pattern repeats. We can denote this by $1 \rightarrow 5 \rightarrow 3 \rightarrow 1$. Similarly, the path 2 takes is $2 \rightarrow 8 \rightarrow 4 \rightarrow 2$.
 - 4., 5. For a permutation σ in X_{2k+1} , think about how a σ that maximizes $\mathbf{D}(\sigma)$ must behave on the following three sets: $\{1, 2, \dots, k\}$, $\{k+1\}$, $\{k+2, \dots, 2k, 2k+1\}$.
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