Problem of the Month

Problem 0: Equations in the integers

September 2025

Hint

1. An exhaustive search is a reasonable approach to this problem. It can be made easier if you notice that x must be a multiple of 8 and that y must be a multiple of 5.
2. Find a positive integer c with the property that $ax+by=c$, $ax+by=c+1$, $ax+by=c+2$, $ax+by=c+3$, and $ax+by=c+4$ all have non-negative solutions.

3., 4., 5. As always, it is good to work out a few small examples to try to guess a pattern. It might be useful to understand the set of *all* integer solutions to $ax+by=c$ for fixed a, b, and c with $\gcd(a,b)=1$. Once you do this, you might consider the integer solution $(x,y)=(u,v)$ with u negative but as close to 0 as possible.