The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2025 Pascal Contest

(Grade 9)

Wednesday, February 26, 2025
(in North America and South America)

Thursday, February 27, 2025
(outside of North America and South America)

Solutions

1. Calculating, $(2 \times 0) + (2 \times 5) = 0 + 10 = 10$.

   Answer: (C)

2. Numbers to the left of -3 on a number line are less than -3 (and numbers to the right are greater). Exactly 2 of the given numbers, namely -4 and -3.5, are less than -3 and thus would be placed to the left of -3 on the number line.

   Answer: (B)

3. *Solution 1*

   Solving, we get $18 + 18 + 18 = 3x$ or $54 = 3x$, and so $x = \dfrac{54}{3} = 18$.

   Solution 2

   Since $18 + 18 + 18$ is equal to 3×18, then $18 + 18 + 18 = 3x$ can be written as $3 \times 18 = 3x$, and so $x = 18$.

   Solution 3

   Since $3x$ is equal to $x + x + x$, then $18 + 18 + 18 = 3x$ can be written as $18 + 18 + 18 = x + x + x$, and so $x = 18$.

   Answer: (D)

4. Since $5 \times 1 = 5$ and $5 \times 2 = 10$, the smallest multiple of 5 that is greater than 8 is 10.
   Since $5 \times 11 = 55$ and $5 \times 12 = 60$, the largest multiple of 5 that is less than 58 is 55.
   Between 8 and 58, there are $11 - 2 + 1 = 10$ multiples of 5.

   Answer: (E)

5. The fourth point is 1 unit to the right of $(-1, 4)$ and so its x-coordinate is 1 unit greater than -1, which is $-1 + 1 = 0$.
   The fourth point is 2 units above $(-1, 4)$ and so its y-coordinate is 2 units greater than 4, which is $4 + 2 = 6$.
   The fourth point that Cynthia graphs is $(0, 6)$.

   Answer: (E)

6. Since $4^3 = 4 \times 4 \times 4 = 64$ and $8^2 = 8 \times 8 = 64$, then $4^3 = 8^2$, and so $a = 2$.

   Answer: (B)

7. A total of 140 of the students surveyed don't like puzzles. Since 92 of these students were in Grade 8, then $140 - 92 = 48$ Grade 9 students don't like puzzles. Since 68 Grade 9 students do like puzzles and 48 do not, then the number of Grade 9 students surveyed was $68 + 48 = 116$.

   Answer: (C)

8. In total, Rachel, Christophe and Alfonzo are paid \$50. Since Alfonzo is paid \$14, then in total, Rachel and Christophe are paid $\$50 - \$14 = \$36$. Rachel is paid twice what Christophe is paid, and so Rachel is paid $\frac{2}{3}$ of \$36 and Christophe is paid $\frac{1}{3}$ of \$36, or $\dfrac{\$36}{3} = \$12$.

   Answer: (B)

9. Since x is a positive integer and $y = 6x + 3$, then y is 3 more than a positive multiple of 6. Since $45 = 6 \times 7 + 3$, then $y = 45$ when $x = 7$. Of the given answers, 45 is the only number that is 3 more than a positive multiple of 6.

   Answer: (D)

10. The area of the shaded region and the area of the unshaded region are each equal to 18 cm^2. The area of the larger square is equal to the sum of the areas of the shaded and unshaded regions, which is $18\text{ cm}^2 + 18\text{ cm}^2 = 36\text{ cm}^2$.
Thus, the side length of the larger square is $\sqrt{36\text{ cm}^2} = 6$ cm.

ANSWER: (C)

11. $\triangle ABD$ is isosceles with $AB = AD$, and so $\angle ADB = \angle ABD = 80^\circ$. The measure of $\angle BDC$ is 180° since it is a straight angle. Thus, $\angle ADC = 180^\circ - \angle ADB = 180^\circ - 80^\circ = 100^\circ$.
$\triangle ADC$ is isosceles with $AD = DC$, and so $\angle ACD = \angle CAD$. The sum of the three angles in $\triangle ADC$ is 180°, and so $\angle ACD + \angle CAD = 180^\circ - \angle ADC = 180^\circ - 100^\circ = 80^\circ$.
Since $\angle ACD + \angle CAD = 80^\circ$ and $\angle ACD = \angle CAD$, then $\angle ACD = \dfrac{80^\circ}{2} = 40^\circ$.

ANSWER: (E)

12. To determine the largest possible integer that can be used in this sum, we let each of the other 9 positive integers be as small as possible. The smallest positive integer is 1, and so we let 9 of the numbers in the sum each be equal to 1. Thus, the largest positive integer that can be used in this sum is $30 - 9 \times 1 = 21$.

ANSWER: (C)

13. In $\triangle ABC$, $AC^2 = AB^2 + BC^2$ by the Pythagorean Theorem.
Substituting, we get $17^2 = 8^2 + BC^2$ or $BC^2 = 289 - 64 = 225$. Since $BCDE$ is a square, then its area is equal to $BC^2 = 225$.
(Alternately, we could have determined that $BC = \sqrt{225} = 15$ and so the area of the square is $15 \times 15 = 225$.)

ANSWER: (C)

14. During the 5 games during which the Eulers averaged 3 goals per game, they scored a total of $5 \times 3 = 15$ goals.
To average 4 goals per game over the 6 games, they need to score a total of $4 \times 6 = 24$ goals.
Thus in the 6th game, the Eulers need to score $24 - 15 = 9$ goals.

ANSWER: (D)

15. Each student answered exactly 2 questions correctly, meaning that each student answered exactly 1 question incorrectly.

Suppose that the correct answer to Question #3 is not 24, and so each student answered Question #3 incorrectly.
In this case, each student must have answered Question #2 correctly (since each answered exactly 1 question incorrectly). However, each student answered Question #2 differently, and so each student cannot have answered Question #2 correctly. This tells us that the correct answer to Question #3 is 24, and each student answered this question correctly.

Next, suppose that the correct answer to Question #1 is not 15. In this case, Students A and C did not answer Question #1 correctly, and so they both must have answered Question #2 correctly (since each answered exactly 1 question incorrectly). However, Students A and C have different answers for Question #2 and thus both cannot have answered Question #2 correctly. This tells us that the correct answer to Question #1 is 15, and so Student B answered Question #1 incorrectly.

Finally, since Student B answered Question #1 incorrectly, then they must have answered Question #2 correctly, and so the correct answer to Question #2 is 38.

The sum of the correct answers to the 3 questions is $15 + 38 + 24 = 77$.

	Question #1	Question #2	Question #3
Student A	15 ✓	36	24 ✓
Student B	20	38 ✓	24 ✓
Student C	15 ✓	54	24 ✓

Answer: (B)

16. *Solution 1*

The number of students to the left of Pedro, 23, added to the number of students to the right of Hwie-Lie, 15, is $23 + 15 = 38$.
In this count, each of the students sitting in between Pedro and Hwie-Lie is counted twice, while all other students (including Pedro and Hwie-Lie) are each counted once.
Since the total number of students sitting in the row is 34, then $38 - 34 = 4$ students are counted twice, and so the number of students seated between Pedro and Hwie-Lie is 4.

Solution 2

Suppose that the number of students seated in between Pedro and Hwie-Lie is x.
Not including Hwie-Lie, there are $23 - 1 = 22$ students seated to Pedro's left.
Since x of these 22 students are seated in between Pedro and Hwie-Lie, then $22 - x$ are seated to the left of Hwie-Lie.
Similarly, not including Pedro, there are $15 - 1 = 14$ students seated to Hwie-Lie's right.
Since x of these 14 students are seated in between Pedro and Hwie-Lie, then $14 - x$ are seated to the right of Pedro.
The total number of students seated in the row, 34, is the number of students seated to the left of Hwie-Lie, plus the number seated to the right of Pedro, plus the number seated in between Pedro and Hwie-Lie, plus 2 (to count Pedro and Hwie-Lie).
Therefore, $34 = (22 - x) + (14 - x) + x + 2$ or $34 = 38 - x$, and so $x = 38 - 34 = 4$.
The number of students seated between Pedro and Hwie-Lie is 4.

Answer: (A)

17. In this solution we make use of the fact that $n \times (n-1)! = n!$.
For example, $4 \times 3! = 4 \times 3 \times 2 \times 1 = 4!$.
Manipulating the factors in the given product, we obtain the following equivalent equations:

$$\begin{aligned} n! &= 3! \times 5! \times 7! \\ &= 3 \times 2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1 \times 7! \quad \text{(expanding 3! and 5!)} \\ &= 5 \times 2 \times 3 \times 3 \times 4 \times 2 \times 7! \quad \text{(rearranging the factors)} \\ &= 10 \times 9 \times 8 \times 7! \\ &= 10! \quad \text{(using the fact three times)} \end{aligned}$$

and so $n = 10$.

Answer: (D)

18. For the two arrows to point directly toward each other again, they must each return to their original positions at exactly the same time.
The left dial makes one complete rotation every $\dfrac{360^\circ}{20^\circ} = 18$ seconds, and so it returns to its original position at 18 s, 36 s, 54 s, 72 s, 90 s, and so on. The right dial makes one complete rotation every $\dfrac{360^\circ}{8^\circ} = 45$ seconds, and so it returns to its original position at 45 s, 90 s, and so on.
Comparing the two lists of times, the minimum number of seconds that must pass before the arrows are pointing directly toward each other again is 90.
(Note that 90 is the lowest common multiple of 18 and 45.)

Answer: (A)

19. *Solution 1*

The volume of Cylinder A is $\pi \times 2^2 \times 8 = 32\pi$, and so the volume of water in Cylinder A is $\frac{3}{4} \times 32\pi = 24\pi$.
If all of the water from Cylinder A is poured into Cylinder B, the volume of water in Cylinder B is 24π.
The volume of Cylinder B is $\pi \times 8^2 \times 2 = 128\pi$, and so the fraction of Cylinder B that is full of water is $\dfrac{24\pi}{128\pi} = \dfrac{3}{16}$.

Solution 2

As in Solution 1, the volume of water in Cylinder B would be 24π.
Suppose that the height of the water in Cylinder B is h.
Then, $\pi \times 8^2 \times h = 24\pi$ or $h = \dfrac{24\pi}{64\pi} = \dfrac{3}{8}$. Since the height of Cylinder B is 2 and the height of the water is $\dfrac{3}{8}$, then the fraction of Cylinder B that is full of water is $\dfrac{3/8}{2} = \dfrac{3}{16}$.

Answer: (A)

20. The probability that all 3 socks are not the same colour is equal to 1 minus the probability that all 3 socks are the same colour.
If all 3 socks are the same colour, then they must all be black or they must all be gold (they cannot all be white since there are only 2 white socks).

We begin by determining the probability that all 3 socks are black.
There are $5 + 3 + 2 = 10$ socks in total, and 5 of these are black.
Therefore, the probability that the first sock that Jack removes is black is $\dfrac{5}{10}$.
There are now 9 socks in the drawer and 4 of them are black, and so the probability that the second sock removed is black is $\dfrac{4}{9}$.
The probability that the third sock removed is black is $\dfrac{3}{8}$, and so the probability that all 3 socks are black is $\dfrac{5}{10} \times \dfrac{4}{9} \times \dfrac{3}{8} = \dfrac{60}{720}$.
Similarly, the probability that all 3 socks removed are gold is $\dfrac{3}{10} \times \dfrac{2}{9} \times \dfrac{1}{8} = \dfrac{6}{720}$.
Thus the probability that all 3 socks removed are the same colour is $\dfrac{60}{720} + \dfrac{6}{720} = \dfrac{66}{720} = \dfrac{11}{120}$, and so the probability that all 3 socks removed are not the same colour is $1 - \dfrac{11}{120} = \dfrac{109}{120}$.

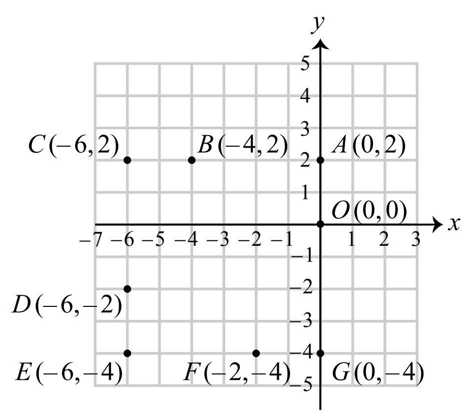
Answer: (E)

21. Suppose that the rectangle has width w and length ℓ, both of which are positive integers with $w \leq \ell$.
The perimeter of the rectangle is $2(w + \ell)$, and thus the perimeter is even (as a result of multiplication by 2).
The perimeter is also a multiple of 7, and so the perimeter is an even multiple of 7.
The smallest even multiple of 7 is 14.
If the perimeter is 14, then $2(w + \ell) = 14$ or $w + \ell = 7$, and so the possible side lengths, (w, ℓ), are $(1, 6)$, $(2, 5)$ and $(3, 4)$.
The areas of the rectangles are $6 \times 1 = 6$, $2 \times 5 = 10$, and 3×4, respectively, none of which is a multiple of 9, and so 14 is not the smallest possible perimeter.

The next smallest even multiple of 7 is 28.
If the perimeter is 28, then $2(w + \ell) = 28$ or $w + \ell = 14$, and so the possible side lengths, (w, ℓ), are $(1, 13)$, $(2, 12)$, $(3, 11)$, $(4, 10)$, $(5, 9)$, $(6, 8)$, and $(7, 7)$.
The areas of the rectangles are 13, 24, 33, 40, 45, 48, and 49, respectively, and 45 is a multiple of 9.
The rectangle with width 5 and length 9 has an area that is a multiple of 9, a perimeter that is a multiple of 28, and so the smallest possible perimeter is 28.
Note that we could also approach this problem by first finding the side lengths for which the area is a multiple of 9, and then determining which of these gives the smallest perimeter that is a multiple of 7.

Answer: 28

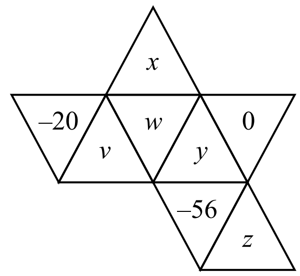
22. Consider placing the robot's movement onto the xy plane.
Suppose that its starting point is $O(0, 0)$, north is in the positive y direction, and 1 unit in the plane represents 1 m travelled by the robot.
The robot begins facing north and so in Step 1 it travels 2 m to the point $A(0, 2)$, as shown.
After turning 90° left (to face west) and moving 4 m in the direction that it is facing (Step 2 and Step 3), the robot is at $B(-4, 2)$.
The robot then repeats these 3 steps, moving 2 m forward to $C(-6, 2)$, turning 90° (to face south), and moving 4 m in the direction that it is facing to $D(-6, -2)$.



Repeating these 3 steps a 3rd time, the robot moves to $E(-6, -4)$ and then to $F(-2, -4)$.
Repeating these 3 steps a 4th time, the robot moves to $G(0, -4)$ and then to its starting point $O(0, 0)$, and is facing north.
Therefore, each time the robot completes this sequence of 3 steps a total of 4 times, it ends at $O(0, 0)$ and is facing north.
Thus, if the robot completes this sequence of 3 steps $4 \times 6 = 24$ times, it ends at $(0, 0)$ and is facing north.
Completing the sequence of 3 steps a 25th time, the robot moves to $B(-4, 2)$ and is facing west (facing point C).
Finally after completing the sequence a 26th time, the robot ends at $D(-6, -2)$, and so its distance from its starting point $O(0, 0)$ is $x = \sqrt{(0 - (-6))^2 + (0 - (-2))^2} = \sqrt{36 + 4} = \sqrt{40}$, and so $x^2 = 40$.

Answer: 40

23. Label the remaining faces using the variables v, w, y, and z, as shown in the diagram below.



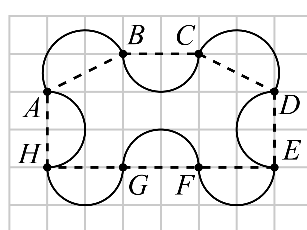
The faces that share an edge with the face labelled by z have values of -20, -56, and 0.
By the condition on the integers labelling the faces, we get $z = -20 - 56 + 0 = -76$.
Now consider the face labelled by -56. The faces with which it shares an edge have labels y, z, and v, and so $-56 = y + z + v$.
Since $z = -76$, we get the equation $-56 = y - 76 + v$ or $v + y = 20$.
Next, consider the face labelled with 0. The faces with which it shares an edge have labels x, y, and z. Therefore, $0 = x + y + z$, and since $z = -76$, we get $x + y = 76$.
Finally, consider the face labelled with -20. The faces with which it shares an edge have labels v, x, and z. Therefore, $-20 = v + x + z$, and since $z = -76$, we get $v + x = 56$.
We have now derived the following three equations:

$$\begin{aligned} v \quad\quad + y &= 20 \\ x + y &= 76 \\ v + x \quad\quad &= 56 \end{aligned}$$

Adding these three equations gives $(v+y)+(x+y)+(v+x) = 20+76+56$ or $2(v+x+y) = 152$.
Dividing by two, we get $v + x + y = 76$.
Using that $v+y = 20$, we can subtract from $v+x+y = 76$ to get $(v+x+y)-(v+y) = 76-20$ or $x = 56$.
We will stop here since the question only asked for the value of x, but it is possible show that $v = 0$, $w = 76$, $y = 20$, and $z = -76$.

ANSWER: 56

24. Draw line segments from A to B, B to C, C to D, D to E, E to F, F to G, G to H, and H to A, as shown.



The line segments BC, DE, EF, FG, GH, and HA each have a length of 2 units.
Hence, the radii of the semicircles with these diameters are all 1, and the areas of the circles with these diameters are all $\frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$.
The line segment AB is the hypotenuse of a triangle with legs of length 1 and 2.
By the Pythagorean Theorem, the length of AB is $\sqrt{1^2 + 2^2} = \sqrt{5}$.

The radius of the semicircle with diameter AB is $\frac{\sqrt{5}}{2}$, so its area is $\frac{1}{2}\pi\left(\frac{\sqrt{5}}{2}\right)^2 = \frac{5\pi}{8}$.

By similar reasoning, the area of the semicircle with diameter CD is also $\frac{5\pi}{8}$.

The area of the figure can be computed as the area of hexagon $ABCDEH$ plus the areas of the semicircles with diameters AB, CD, EF, and GH, minus the areas of the semicircles with diameters BC, DE, FG, and AH.

We have already computed the areas of the semicircles, so we now need to compute the area of hexagon $ABCDEH$.

This hexagon can be viewed as a 3×6 rectangle with two "corners" removed. These "corners" are right-angled triangles with hypotenuses AB and CD.

The legs of these two triangles have length 1 and 2, so their areas are each $\frac{1}{2} \times 1 \times 2 = 1$.

Thus, the area of hexagon $ABCDEH$ is $3 \times 6 - 2 \times 1 = 16$.

Using the areas of the semicircles computed earlier, we can now compute the area of the figure as

$$16 + \frac{5\pi}{8} + \frac{5\pi}{8} + \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} = 16 + \frac{\pi}{4} \approx 16.78539$$

Thus, $x \approx 16.78539$, so $100x \approx 1678.539$. Rounding to the nearest integer, we get $y = 1679$, so the answer is $1 + 6 + 7 + 9 = 23$.

Answer: 23

25. Set $n = 7A6\,B5C + 2B9\,C5A + 7C1\,A6B$. We can rearrange as follows

$$\begin{aligned}
n &= (706\,050 + A0\,B0C) + (209\,050 + B0\,C0A) + (701\,060 + C0\,A0B) \\
&= (706\,050 + 209\,050 + 701\,060) + (A0\,B0C + B0\,C0A + C0\,A0B) \\
&= 1\,616\,160 + (A + B + C)(10\,000) + (B + C + A)(100) + (C + A + B) \\
&= 1\,616\,160 + (A + B + C)(10\,101) \\
&= (10101)(160 + A + B + C)
\end{aligned}$$

and so $n = (10101)(160 + A + B + C)$.

We need to determine for which values of A, B, and C the integer n is divisible by 36.

Observe that $36 = 4 \times 9$, and since 4 and 9 have no prime factors in common, n will be divisible by 36 exactly when it is divisible by both 4 and 9.

Since 10101 is odd, it is not a multiple of 4, and so n is a multiple of 4 exactly when $160 + A + B + C$ is a multiple of 4.

Factoring 10101 gives $10101 = 3 \times 3367$, and since 3367 is not a multiple of 3, we get that 10101 has a factor of 3 but not a factor of 9.

Therefore, n is a multiple of 9 exactly when $160 + A + B + C$ is a multiple of 3.

We now have that n is a multiple of 36 exactly when $160 + A + B + C$ is a multiple of 3 and a multiple of 4, or equivalently, when $160 + A + B + C$ is a multiple of 12.

Since 156 is a multiple of 12, $160 + A + B + C = 156 + 4 + A + B + C$ is a multiple of 12 exactly when $4 + A + B + C$ is a multiple of 12.

The digits A, B, and C are each between 0 and 9 inclusive, so $4 + A + B + C$ is at least 4 and at most $4 + 3 \times 9 = 31$. The only multiples of 12 between 4 and 31 are 12 and 24, so we must have that $4 + A + B + C = 12$ or $4 + A + B + C = 24$.

To answer the question, we count the number of triples (A, B, C) of integers, each between 0 and 9 inclusive, for which $A + B + C = 8$ or $A + B + C = 20$.

First, suppose $A + B + C = 8$.
If $A = 0$, then $B + C = 8$. In this case, we can have $B = 0$ and $C = 8$, or $B = 1$ and $C = 7$, or $B = 2$ and $C = 6$, and so on to $B = 8$ and $C = 0$. This gives a total of 9 triples.
If $A = 1$, then $B + C = 7$. In this case, we can have $B = 0$ and $C = 7$, $B = 1$ and $C = 6$, and so on to $B = 7$ and $C = 0$ for a total of 8 triples.
Continuing in this way, there are 7 triples when $A = 2$, 6 triples when $A = 3$, 5 triples when $A = 4$, 4 triples when $A = 5$, 3 triples when $A = 6$, 2 triples when $A = 7$, and there is one triple when $A = 8$.
The number of triples (A, B, C) when $A + B + C = 8$ is $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$.

Now assume $A + B + C = 20$.
If $A = 0$, then $B + C = 20$, which is impossible given that $B \leq 9$ and $C \leq 9$. There are no triples with $A = 0$.
Similarly, if $A = 1$, then there are no triples.
If $A = 2$, then $B + C = 18$, which means $B = 9$ and $C = 9$, so there is only one triple.
If $A = 3$, then $B + C = 17$, so either $B = 9$ and $C = 8$ or $B = 8$ and $C = 9$. There are 2 triples in this case.
If $A = 4$, then $B + C = 16$, and there are 3 triples.
Continuing in this way, when $A = 5$, there are 4 triples, when $A = 6$, there are 5 triples, when $A = 7$, there are 6 triples, when $A = 8$, there are 7 triples, and when $A = 9$, there are 8 triples.
The number of triples when $A + B + C = 20$ is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$.
The number of triples (A, B, C) is $45 + 36 = 81$.

ANSWER: 81