Hypatia Contest

(Grade 11)

Thursday, April 3, 2025
(in North America and South America)

Friday, April 4, 2025
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes   ©2025 University of Waterloo

Do not open this booklet until instructed to do so.

Number of questions: 4   **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 💡

    - worth 2 or 3 marks each
    - full marks given for a correct answer which is placed in the box
    - **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by ✎

    - worth the remainder of the 10 marks for the question
    - **must be written in the appropriate location** in the answer booklet
    - marks awarded for completeness, clarity, and style of presentation
    - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

   - Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
   - Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.

3. For questions marked 💡, place your answer in the appropriate box in the answer booklet and **show your work**.

4. For questions marked 📐, provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. To begin, tokens are distributed *unequally* between two people. Every minute following this, each person receives 2 more tokens.

    (a) Tokens are distributed between Abi and Brody with Abi receiving 30 tokens and Brody receiving 10 tokens. How many tokens does Abi have in total after 7 minutes?

    (b) A total of 80 tokens are distributed between Carl and Desiree. After 12 minutes, Desiree has 37 tokens. How many tokens did Carl start with?

    (c) A total of 100 tokens are distributed between Essi and Francis, with Essi receiving 12 tokens. After t minutes, Francis has 3 times as many tokens as Essi. Determine the value of t.

2. (a) A rectangle, $\mathcal{R}_1$, has length 5 cm and width 4 cm. The length of the rectangle is increased by 10% and its width remains unchanged. What is the area of the resulting rectangle?

    (b) The area of a square is 100 cm^2. When its length is increased by 30% and its width is decreased by 30%, the area of the resulting rectangle is less than 100 cm^2. Determine the percentage by which the area decreased.

    (c) The length of a rectangle, $\mathcal{R}_2$, is increased by x% and its width is decreased by 20%. If the area of the resulting rectangle is equal to the area of the original rectangle, determine the value of x.

3. A parabola with equation $y = ax^2 + bx + c$, where $a = 1$ or $a = -1$, and a line intersect at points $P(p, q)$ and $R(r, s)$ for some real numbers p, q, r, and s. In this case, the area enclosed by the parabola and the line, as shaded in the diagram, is equal to $\dfrac{(p - r)^3}{6}$, where $p > r$.
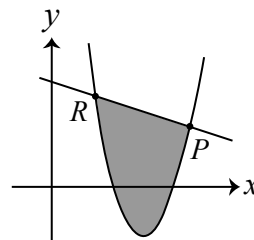

(a) What is the area enclosed by the parabola with equation $y = x^2 + 3x - 12$ and the line with equation $y = 2x$?

(b) For some values of m, the line with equation $y = mx - 6$ intersects the parabola with equation $y = -x^2 + 7x - 90$ at distinct points V and W whose x-coordinates are integers. There are two such values of m for which the area enclosed by the line and the parabola is as small as possible. Determine these two values of m.

(c) Suppose that g and h are real numbers for which the parabolas $y = x^2 + (g + h)x + 9$ and $y = -x^2 + gx + h$ intersect at distinct points T and U, as shown. Determine all possible values of h so that the area enclosed by the parabolas is $\frac{3087}{8}$.
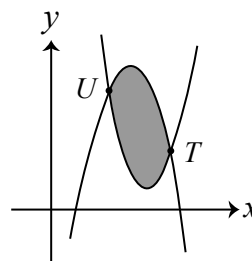
4. A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a non-zero constant, called the *common ratio*. For example, $5, 10, 20$ is a geometric sequence with three terms and a common ratio of 2.

(a) What is the sum of the first three terms of a geometric sequence with a common ratio of $\frac{3}{5}$ and whose second term is 45?

(b) Determine all pairs of positive integers (x, y) so that $x, 12, y$ is a geometric sequence and $x + y = 25$.

(c) Determine all quadruples of integers (a, b, c, d) so that a, b, c, d is a geometric sequence and $a + b + c + d = 65$.

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For students...

Thank you for writing the 2025 Hypatia Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2025.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

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- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
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- Find your school's contest results