**The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING**
cemc.uwaterloo.ca

Cayley Contest
(Grade 10)

Wednesday, February 26, 2025
(in North America and South America)

Thursday, February 27, 2025
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.
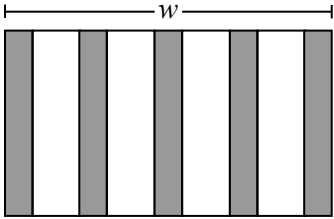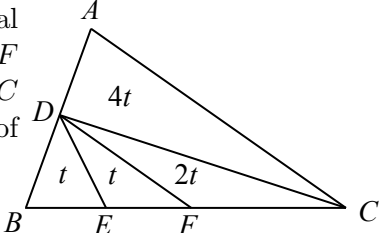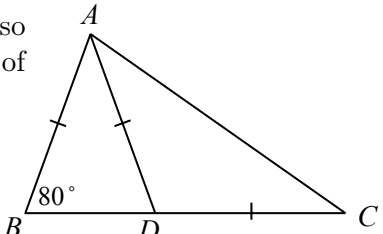
Instructions

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name and city/town in the box in the upper right corner.

5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**

6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").

8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

9. Diagrams are *not* drawn to scale. They are intended as aids only.

10. When your supervisor tells you to begin, you will have 60 minutes of working time.

11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{20 + 25}{25 + 20}$ is

   (A) -1 **(B)** 1   **(C)** 0   **(D)** 2025   **(E)** -2025

2. A mother bear collects 14 fish. She gives 4 fish to each of her 3 bear cubs. How many fish does the mother bear have left over?

   (A) 0   **(B)** 2   **(C)** 3   **(D)** 4   **(E)** 5

3. In the diagram, the large rectangle is divided into 9 smaller rectangles. Each of the 5 smaller shaded rectangles has a width of 4 and each of the 4 smaller unshaded rectangles has a width of 8. The width, w, of the large rectangle is

   (A) 20   **(B)** 32   **(C)** 52

   (D) 56   **(E)** 68

4. If the average of $n - 3$, $n - 1$, $n + 1$, and $n + 3$ is 17, the value of n is

   (A) 13   **(B)** 15   **(C)** 17   **(D)** 19   **(E)** 21

5. A theatre has 600 seats. Exactly 25% of these seats are filled. All of the people in the seats then move to an empty theatre that has 200 seats. What percentage of the seats in the smaller theatre are now filled?

   (A) 50%   **(B)** 40%   **(C)** 60%   **(D)** 75%   **(E)** 55%

6. In the diagram, point D is on side AB of $\triangle ABC$ and points E and F are on BC. For some positive real number t, the area of $\triangle DBE$ is t, the area of $\triangle DEF$ is t, the area of $\triangle DFC$ is $2t$, and the area of $\triangle DAC$ is $4t$. If the area of $\triangle DEC$ is 63, what is the area of $\triangle ABC$?

   (A) 168   **(B)** 105   **(C)** 84

   (D) 189   **(E)** 126

7. If $50 - 2\sqrt{x} = 18$, the value of x is

   (A) 32   **(B)** 16   **(C)** 64   **(D)** 256   **(E)** 8

8. In the diagram, point D lies on side BC of $\triangle ABC$ so that $AB = AD = CD$. If $\angle ABC = 80°$, the measure of $\angle ACD$ is

   (A) 20°   **(B)** 60°   **(C)** 80°

   (D) 100°   **(E)** 40°

9. Teddy has 10 rectangular blocks each of which measures 3 cm by 4 cm by 5 cm. He builds a stack that is exactly 21 cm high, where each block can be stacked on any of its faces. What is the smallest number of blocks that Teddy could use to make this stack?

(A) 4   (B) 5   (C) 6   (D) 7   (E) 8

10. If $x = \frac{1}{4}$, which of the numbers x, $-x$, x^2, $3x$, and $\sqrt{x}$ is the greatest?

(A) x (B) $-x$ (C) x^2 (D) $3x$ (E) $\sqrt{x}$

Part B: Each correct answer is worth 6.

11. The current calendar year, 2025, is a perfect square. In n years from 2025, the calendar year will again be a perfect square. The smallest possible value of n is

(A) 2025   (B) 100   (C) 9   (D) 46   (E) 91

12. In the diagram, a number is to be written at each of the vertices of the three pentagons. Some of the numbers have already been written.
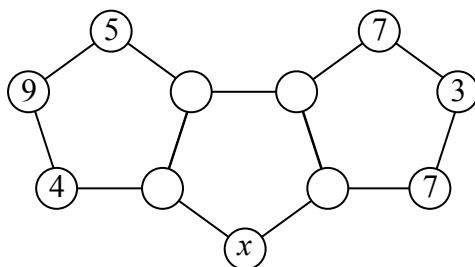
If the sum of the numbers at the vertices of each pentagon is to be 25, what is the value of x?

(A) 6   (B) 7   (C) 8   (D) 9   (E) 10

13. Farhan has a blue hat, a white hat, a blue scarf, a white scarf, and a green scarf. He randomly chooses one hat and one scarf. What is the probability that the hat and the scarf are the same colour?

(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{5}$ (E) $\frac{1}{6}$

14. Yesterday, 200 people bought ice cream at the Cayley Creamery.
A total of 85 people ordered fudge with their ice cream.
A total of 60 people ordered sprinkles with their ice cream.
A total of 32 people ordered both fudge and sprinkles.
How many of the 200 people ordered neither fudge nor sprinkles?

(A) 23   (B) 55   (C) 87   (D) 113   (E) 168

15. In the diagram, $\triangle ABC$ is right-angled at B and the semicircle has diameter BC. If $AB = 20$ and $AC = 40$, what is the area of the semi-circle?

(A) 37.5π (B) 75π (C) 100π

(D) 150π (E) 225π
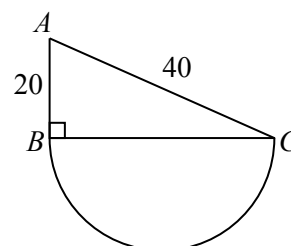
16. A jar contains 600 marbles, each of which is either red, yellow, or green. The ratio of the number of red marbles to the number of yellow marbles to the number of green marbles is $7 : 3 : 5$. If 20 marbles of each colour are removed, the new ratio of the number of red marbles to the number of yellow marbles to number of green marbles is

(A) $5 : 1 : 3$ (B) $15 : 7 : 11$ (C) $13 : 5 : 9$ (D) $9 : 5 : 7$ (E) $209 : 89 : 149$

17. For each non-zero real number a, we define $a^* = \dfrac{5}{a}$.
    The expression $(100^*)^*$ is equal to

(A) 100 (B) $\dfrac{1}{4}$ (C) 20 (D) $\dfrac{1}{20}$ (E) $\dfrac{1}{100}$

18. Lavinia has one bottle with capacity 6 L and another bottle with capacity 5 L. Both bottles are empty. She has a piece of paper with the following steps printed on it:

   (i) Fill the 6 L bottle completely with water.

   (ii) Pour water from the 6 L bottle into the 5 L bottle until the 5 L bottle is full.

   (iii) Empty the 5 L bottle.

   (iv) Pour water from the 6 L bottle into the 5 L bottle until the 5 L bottle is full or the 6 L bottle is empty, whichever happens first. If the 5 L bottle fills, empty the 5 L bottle and return to the beginning of this step.

   Lavinia completes this sequence of four steps in this order a total of 3 times. Lavinia never spills any water. How much water will be in the 5 L bottle after Lavinia is finished?

(A) 0 L   (B) 1 L   (C) 2 L   (D) 3 L   (E) 4 L

19. In the diagram, $ABCD$ is a square with area k. Point E is on side AB with $AE = \frac{1}{3}AB$. Point G is on side BC with $BG = \frac{1}{4}BC$. Point F is on ED so that GF is perpendicular to BC. The area of $\triangle FGC$ is

(A) $\frac{9}{32}k$ (B) $\frac{13}{48}k$ (C) $\frac{3}{14}k$

(D) $\frac{7}{24}k$ (E) $\frac{2}{7}k$
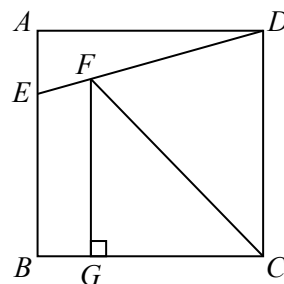
20. In the diagram, line segment CD has length 48 cm. Two ants, Violet and Petunia, are at point C and plan to walk to point D. Violet walks along four congruent semi-circles above CD. Petunia walks along three congruent semi-circles below CD.
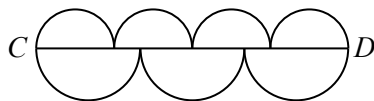
Violet and Petunia start walking at the same time, and they walk at constant, but different, speeds. Petunia arrives at D 12 seconds before Violet and travels 3 times as quickly as Violet. How long does it take Violet to walk from C to D?

(A) 24 seconds (B) 18 seconds (C) 36 seconds (D) 16 seconds (E) 21 seconds

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
   **The integer formed by the rightmost two digits of 6307 is 7, coded 07.**

21. Points $A(-3, 5)$, $B(0, 7)$ and $C(r, t)$ lie along a line. If $BC = 4AB$ and $r > 0$, what is the value of $r + t$?

22. A *Katende number* is a four-digit positive integer where the first two digits and the last two digits, in order, form two integers that are increasing consecutive multiples of some positive integer. For example, 2025 is a Katende number because $20 = 4 \times 5$ and $25 = 5 \times 5$. Also, 2346 and 2324 are Katende numbers. How many Katende numbers are there that are greater than 2400 and less than 2600?

23. Amr, Bai, Cindy, and Derek divide N coins between them so that each receives a whole number of coins. Amr receives $\frac{1}{3}$ of the total number of coins that Bai, Cindy and Derek receive. Bai receives $\frac{1}{5}$ of the total number of coins that Amr, Cindy and Derek receive. Cindy receives $\frac{1}{7}$ of the total number of coins that Amr, Bai and Derek receive. If $N < 100$, what is the largest possible value of N?

24. A circle has area A, a regular octagon has area $\frac{1}{2}A$, and a regular dodecagon has area $\frac{1}{2}A$. The circle has radius 3000. The distance from the centre of the octagon to each of its vertices is x, as shown. Also, the distance from the centre of the dodecagon to each of its vertices is y.
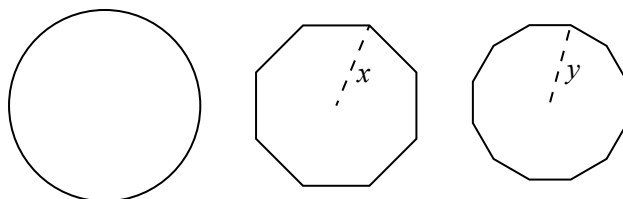
What is the integer closest to $x - y$?

(A regular polygon is a polygon all of whose side lengths are equal and all of whose interior angles are equal. An octagon has 8 sides and a dodecagon has 12 sides.)

25. Rita is colouring a picture of a flower. She has already coloured the centre and the stem of the flower, as shown. Next, she will colour each of the six petals with exactly one of the colours: red, orange, yellow, and blue. No two neighbouring petals can be the same colour and not all four colours need to be used. There are N different-looking ways in which Rita can colour the petals. What are the rightmost two digits of N?
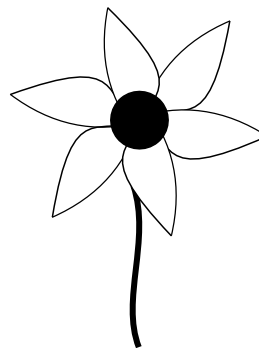
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Thank you for writing the 2025 Cayley Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

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- Free copies of past contests
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