The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

2025 Canadian Team Mathematics Contest

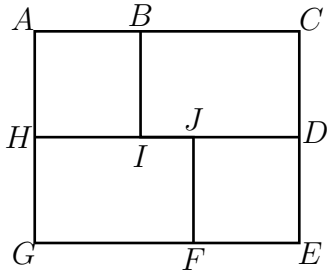
Individual Problems (45 minutes)

IMPORTANT NOTES:

- Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

PROBLEMS:

1. If $m = 4$, what is the value of $(m + 1)(m + 2)$?
2. In the integer $n = 7A6\,65B\,2A7$, the digit A appears twice and the digit B appears once. The sum of the digits of n is 46 and the sum of the rightmost two digits of n is 11. What is $A + B$?
3. A pot of soup is initially $\frac{2}{3}$ full. After 3 L of soup is eaten, the pot is $\frac{1}{2}$ full. What is the capacity of the pot?
4. What is the smallest positive multiple of 6 that can be expressed as the sum of five consecutive positive integers?
5. In the diagram shown, $ACEG$ is a rectangle with area 2420. Also, $\frac{AB}{BC} = \frac{2}{3}$ and $ABIH$ and $DEFJ$ are congruent squares. What is the perimeter of $ACEG$?



6. Ana, Bertrand, and Chi share a collection of stamps.
   - Ana divides the stamps into piles of 9 and notices that there are 0 stamps left over.
   - Bertrand divides the stamps into piles of 5 and notices that there are 2 stamps left over.
   - Chi divides the stamps into piles of 4 and notices that there is 1 stamp left over.

   What is the smallest number of stamps they could have in their collection?

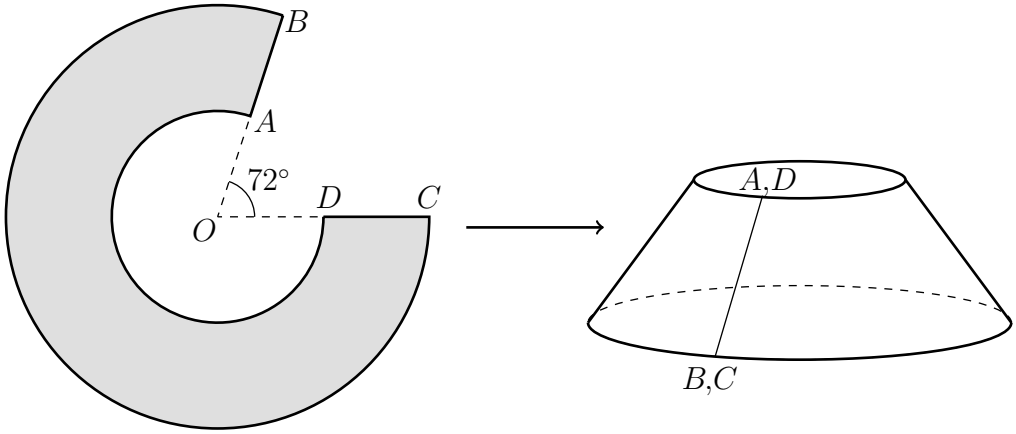
7. Three odd integers between 0 and 10 are selected randomly, with each odd integer equally likely to be selected. (It is possible for an integer to be selected more than once.) What is the probability that the product of the squares of the three integers is equal to 2025?

8. The real numbers x, y, and z simultaneously satisfy the three equations

$$\frac{xy}{4x+y} = \frac{3}{10} \qquad \frac{yz}{y+z} = \frac{3}{10} \qquad \frac{xz}{4x+z} = \frac{1}{14}.$$

What are all triples (x, y, z) of real numbers that satisfy the three equations?

9. A flat, circular piece of paper with centre O and radius 10 has sector BOC with angle $72°$ removed from it. Next, a circle centred at O of radius 5 is removed. (Some of this circle had already been removed when the sector was removed.) The remaining "C-shaped" piece of paper is shown below on the left. A hollow, 3-dimensional figure can be obtained by folding this piece of paper to connect line segment AB to line segment DC, then adding circular faces on the top and bottom. This is shown in the diagram on the right. (This type of figure is called a *frustum* and is also the figure obtained by removing the top part of a cone by cutting it parallel to its base.) What is the volume of the resulting hollow figure?



Note: The volume of a cone with height h and a circular base of radius r is $\frac{\pi}{3}r^2h$.

10. 20 different students are lined up in a row. 8 students wear a red hat and 12 students wear a green hat. There are 20! ways to arrange the students. Call a pair of students "complementary" if they are next to each other and have different coloured hats. For example, an arrangement of the students resulting in the following order of hat colours

$$GRRGGRRGGGRGGRGGGGRGGGGR$$

has 11 complementary pairs. Among the 20! ways to arrange the students, what is the average number of complementary pairs of students?

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Team Problems (45 minutes)

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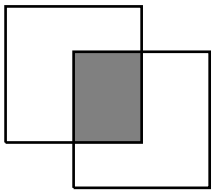
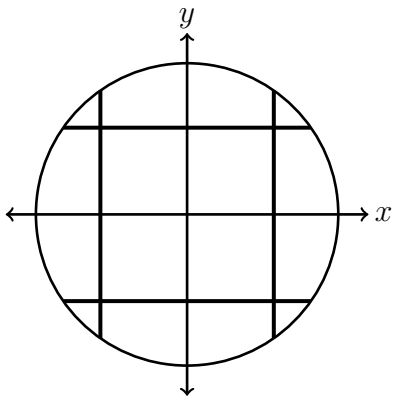
PROBLEMS:

1. What is the integer equal to $\left(\sqrt{25} + \sqrt{49}\right)^2$?
2. How many odd integers are there between $\frac{13}{2}$ and $\frac{37}{2}$?
3. If $3w = x,\ 2x = y,\ 5y = z,$ and $w \neq 0$, what is the value of $\frac{z}{w}$?
4. Define a new operation $\Diamond$ by $a \Diamond b = ab + a + b$. For example, $3 \Diamond 4 = (3)(4) + 3 + 4 = 12 + 7 = 19$. Given that $5 \Diamond x = 41$, what is x?
5. In the diagram below, two identical squares overlap to make an 8-sided figure. The region where the squares overlap is shaded and equal to one third of the area of each square. What fraction of the area of the 8-sided figure is shaded?



6. A car travelled 360 km in total. During the first 180 km, its average speed was 45 km/hr. During the second 180 km, its average speed was 90 km/hr. What was its average speed for the entire 360 km trip?
7. The letters A, B, C, D, and E represent the digits 1, 2, 3, 4, and 5 in some order. AB is a two-digit integer with tens digit A and units digit B, CD is a two-digit integer, and BED is a three-digit integer. Given that $AB \times CD = BED$, what are the digits A, B, C, D, and E?

8. The circle shown has centre $(0, 0)$ and radius 5. Two vertical chords are drawn at $x = -3$ and $x = 3$. Two horizontal chords are drawn at $y = -3$ and $y = 3$. What is the sum of the lengths of the four chords?



9. A bag contains eight chips, each labelled with a different prime number greater than 1 and less than 20. Two different chips are selected from the bag at random and their labels are multiplied. What is the probability that the units digit of the product is 9?

10. What is the smallest five-digit positive integer with the property that the sum of its digits is one less than the product of its digits?

11. At some time in the afternoon, Joni set three clocks to the correct time. Clock A is accurate, Clock F is fast, and Clock S is slow. After 60 minutes had elapsed on Clock A, Clock S showed that 54 minutes had elapsed and Clock F showed that 64 minutes had elapsed. At some point later that same afternoon, Clock S reads 7:20 p.m. and Clock F reads 8:10 p.m. At what time did Joni set the three clocks to the correct time?

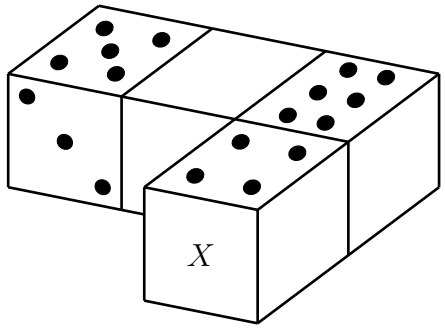
12. In a sequence of seven numbers, each number after the first two is the sum of the two previous numbers. If the seventh (last) number in the sequence is 97 and the sum of the seven numbers in the sequence is 245, what is the third number in the sequence?

13. If $f(x) = x^2 + 9x - n$ for some integer n and $f(n) = -16$, what is the value of $f(-2)$?

14. Four dice each have the following three properties.

    - The number of dots on each face is an integer from 1 through 6, inclusive.
    - No two faces on the same die have the same number of dots on them.
    - Each pair of faces on opposite sides of the same die have a total of 7 dots on them.

    The dice are arranged as shown below so that each pair of faces from different dice that are touching have a total of 8 dots on them. Five of the visible faces, including the one labelled by "X", have had all of their dots hidden. What is the number of dots on the face labelled by X?



15. For a positive integer n, the *Anderson number* of n is the integer obtained be writing the integers from 1 through n in order. For example, the Anderson number of $n = 19$ is

    12345678910111213141516171819

    What is the smallest four-digit integer n for which the Anderson number of n is divisible by 9?

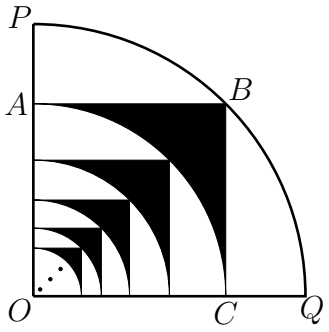
16. Given a positive integer with at least two digits, a new integer can be obtained by removing its leading digit. For example, the leading digit can be removed from 589 to get the integer 89. What is the smallest positive integer n with the property that n is 57 times the integer obtained by removing its leading digit?

17. Some devices display digits using an arrangement of segments. For example, the digit "8" uses seven segments. How many integers from 100 through 999 inclusive can be displayed using exactly 17 segments?

18. In the diagram, OPQ is a quarter circle of radius 1. Square $OABC$ is inscribed in OPQ so that A is on line segment OP, B is on arc PQ, and C is on line segment OQ. Another circular arc is drawn from A to C with centre O, and the region bound by this arc, AB, and BC is shaded.

    Inside the quarter circle OAC, this process can be repeated to inscribe a square then draw a circular arc to obtain a smaller shaded region.

    If this process is continued indefinitely, what is the total area of the infinitely many shaded regions?

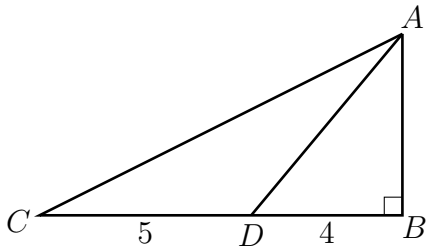


19. The two angles x and y, both between $0°$ and $90°$, satisfy both of the equations

$$\sin^2 x + \cos^2 y = \frac{89}{144}$$
$$\cos^2 x - \sin^2 y = \frac{71}{144}$$

    What is the exact value of $\sin x + \sin y$?

20. In the diagram, $\triangle ABC$ is right-angled at B. Point D is on BC so that $BD = 4$, $CD = 5$, and $\cos \angle CAD = \dfrac{12}{13}$. What is the length of AB?



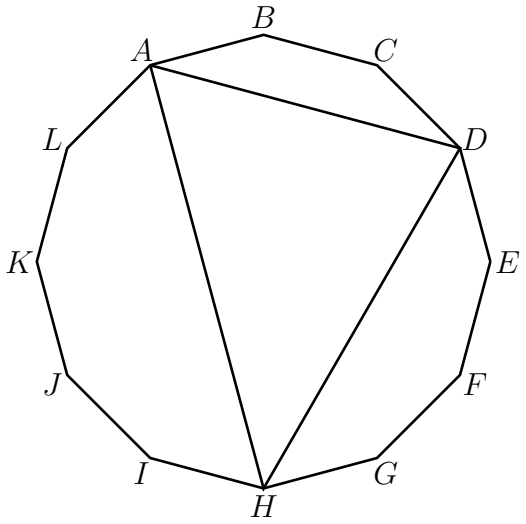
21. The positive real numbers x and y satisfy both of the equations

$$xy = 144$$
$$3\log_y(x) + 3\log_x(y) = 10$$

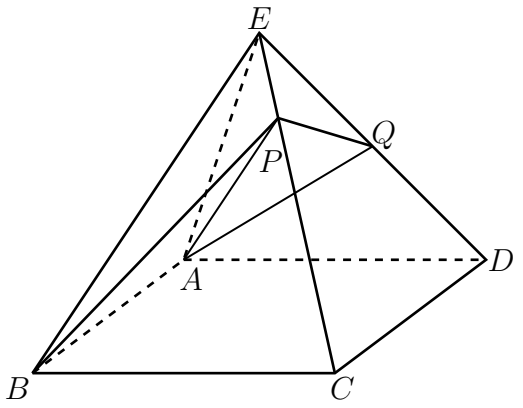
    What is the value of $x + y$?

22. Consider the polynomials $f(x) = x^2 - 2x - p$ and $g(x) = x^3 - 5x^2 - q$. There is an irrational number c such that $f(c) = g(c) = 0$. Given that p and q are rational, what are the three real numbers x that satisfy the equation $x^3 - 5x^2 - q = 0$?

23. In the diagram, $ABCDEFGHIJKL$ is a regular 12-sided polygon (called a dodecagon). What is the area of $\triangle ADH$ divided by the area of the dodecagon?



24. The regular pyramid $EABCD$ has square base $ABCD$ and a volume of 28. The points P and Q are on EC and ED, respectively, such that $\frac{EP}{EC} = \frac{1}{3}$ and $\frac{EQ}{ED} = \frac{1}{2}$. Figure $EABPQ$ has six faces, two of which are $\triangle ABP$ and $\triangle AQP$. What is the volume of $EABPQ$?



25. Willow's job is to write computer programs and Ash's job is to test them. When Willow finishes a computer program, she adds it to a list of programs that are ready to be tested. Ash always tests the program that was most recently added to the list, and once Ash starts testing a program, he always finishes testing that program before starting the next one. Once Ash is finished testing a program, he removes it from the list. On one particular day, Willow and Ash have 7 programs to write and test, respectively. Willow writes Programs 1 through 7 in order. By the time Ash arrives at work, Willow has already been working for some time. At one moment, Ash tells a colleague that he has already tested Program 6. After this moment, how many possibilities are there for the ordered sequence of programs that Ash will test?

Note: It is possible that all programs were already tested when Ash tells a colleague that he has already tested Program 6. All programs will be tested by the end of the day.

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Relay Problem #1 (Seat a)

How many integers between 1 and 25 inclusive are divisible by 3 or 5 (or both)?

Relay Problem #1 (Seat b)

Let t be TNYWR.
The numbers $x, t, y, 26, \ldots$ form an arithmetic sequence. What is the value of $x + y$?

(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant, called the common difference. For example, $3, 5, 7, 9$ are the first four terms of an arithmetic sequence.)

Relay Problem #1 (Seat c)

Let t be TNYWR.
The line with equation $y = 3x + 10$ intersects the parabola with equation $y = x^2 - 6x + t$ twice. What is the smallest value of x where the curves intersect?

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Relay Problem #2 (Seat a)

How many two-digit positive integers are there with two different digits?

Relay Problem #2 (Seat b)

Let t be TNYWR.

The total cost of 5 ounces of coffee and 6 ounces of tea is t cents. The total cost of 11 ounces of coffee and 12 ounces of tea is 171 cents. What is the cost of 1 ounce of tea?

Relay Problem #2 (Seat c)

Let t be TNYWR.

After the parabola with equation $y = x^2 + (m - t)x + 2m - 1$ is translated 2 units to the right, what is its new y-intercept?

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Relay Problem #3 (Seat a)

What is the smallest multiple of 12 that is greater than 2025?

Relay Problem #3 (Seat b)

Let t be TNYWR.

A green box has a square base with side length 10 cm and a height of 13 cm. A black box has a volume of $\frac{t}{4}$ cm^3. If the volume of the black box is equal to $n\%$ of the volume of the green box, what is the value of n?

Relay Problem #3 (Seat c)

Let t be TNYWR.

In the diagram, O is the centre of the circle, and three congruent, isosceles, triangles all have a vertex at O and their other two vertices on the circle. What is the value of x?