The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Canadian Senior Mathematics Contest

Wednesday, November 12, 2025
(in North America and South America)

Thursday, November 13, 2025
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 2 hours ©2025 University of Waterloo

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Do not open this booklet until instructed to do so.

There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

PART A

1. This part consists of six questions, each worth 5 marks.

2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.

2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Write your name, school name, and question number on any inserted pages.

3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

Canadian Senior Mathematics Contest

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. Wenfei has 7 cookies at the end of Monday. On each of the next four days (Tuesday through Friday), Wenfei eats 2 cookies at lunch and then after school he buys enough cookies to double the total number of cookies he has. How many cookies does Wenfei have after buying cookies on Friday?

2. The equation $10(4x - 3) - k(4x - 3) = 16x - 12$ is satisfied by all real numbers x. What is the value of the real number k?

3. A standard six-sided die will be rolled two times. What is the probability that the outcome of the second roll will be greater than the outcome of the first roll?

4. The integers x and y are perfect squares with $x - y = 35$. What are the possible values of $x + y$?

5. A sector with an angle of θ with $0° < \theta < 180°$ is cut out of a circular piece of paper of radius 1. The two straight edges of this sector are joined to form a cone. A sector with an angle of 2θ is cut out of a different circular piece of paper of radius 1 and then used to form a cone by joining its two straight edges. If the two cones have the same volume, what is the value of θ?

   The volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.

6. All of the 8-digit positive integers are listed, and then every digit equal to 0 is erased. This causes some integers in the list to be replaced by another integer or by several other integers. For example, the integer $89\,160\,000$ is replaced by 8916, the integer $34\,041\,034$ is replaced by the three integers 34, 41, and 34, and the integer $49\,671\,349$ does not change. How many integers are in the list after the 0s have been erased?

PART B

For each question in Part B, your solution must be well-organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. (a) A line has equation $y = px + p^2$ for some real number p. The line passes through the point $(x, y) = (6, -9)$. Determine the value of p.

   (b) The following table of values includes three points (x, y) that are on the parabola with equation $y = ax^2 + bx + c$.

x	y
-1	5
0	4
1	11

   Determine the values of a, b, and c.

   (c) The parabola with equation $y = (x - 1)^2 + q$ passes through the points $(d, 5)$ and $(d + 6, 5)$ for some real number d. Determine the value of q.

2. (a) In the diagram, trapezoid $ABCD$ has AD parallel to BC, $AB = 3$, $BC = 2$, $\angle ABC = 90°$, and $\angle CAD = \theta$. Determine the exact value of $\sin \theta$.
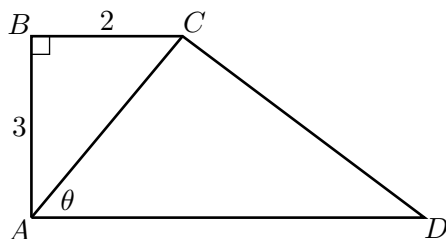
   (b) In the diagram, $\triangle PQR$ is right-angled at Q, $PQ = 4$, and T is on QR such that $QT = 3$ and $\angle TPR = 45°$. Determine the length of TR.
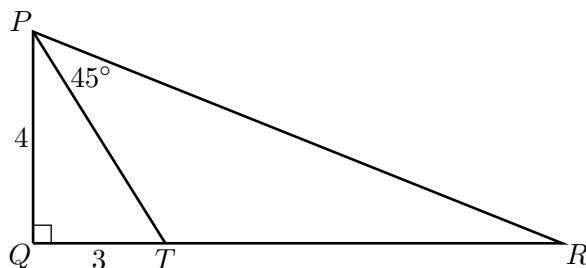
   (c) The points X, Y, and Z have coordinates $X(0, 0)$, $Y(7, 24)$, and $Z(15, 0)$. The point W is on line segment YZ such that $\angle WXZ = 3\angle WXY$. Determine the coordinates of W.

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3. Given a sequence S of real numbers $b_1, b_2, b_3, \ldots$, the *complement* of S is the sequence $c_1, c_2, c_3, \ldots$ where $c_1 = 1$ and for each integer $n \geq 2$, c_n satisfies

$$c_n = b_1 c_{n-1} + b_2 c_{n-2} + \cdots + b_{n-1} c_1$$

That is, c_n is the sum of all terms $b_k c_{n-k}$ where k ranges from 1 to $n-1$.

(a) The sequence S with $b_n = 2^{n-1}$ for each integer $n \geq 1$ has terms $1, 2, 4, 8, 16, \ldots$. Determine the terms c_2, c_3, and c_4 from the complement of S.

(b) Prove that if S is a geometric sequence with complement $c_1, c_2, c_3, \ldots$, then there is a constant t such that $c_n = t c_{n-1}$ for all $n \geq 3$.

(c) Suppose that $b_1, b_2, b_3, \ldots$ is an arithmetic sequence and that its complement satisfies
$$c_{2025} = 3, \quad c_{2026} = 0, \quad c_{2027} = -3, \quad c_{2028} = 3$$

Determine the value of b_{2025}.

A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a non-zero constant, called the common ratio. For example, 3, 6, 12, 24 are the first four terms of a geometric sequence with common ratio 2.

An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant, called the common difference. For example, 2, 5, 8, 11 are the first four terms of an arithmetic sequence with common difference 3.