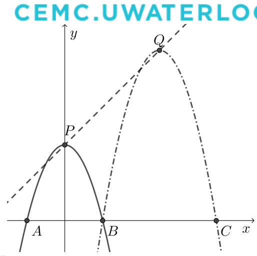

Problem of the Week

Problem E and Solution

Sliding Parabola

Problem

Suppose the parabola with equation $y = 4 - x^2$ has vertex at P and crosses the x-axis at points A and B, with B lying to the right of A on the x-axis.

This parabola is translated so that its vertex moves along the line $y = x + 4$ to the point Q. The new parabola crosses the x-axis at points B and C, with C lying to the right of B on the x-axis.

Determine the coordinates of C.

Solution

For the original parabola $y = -x^2 + 4$, the vertex is $P(0, 4)$ and the x-intercepts are $A(-2, 0)$ and $B(2, 0)$.

Let the vertex of the translated parabola be $Q(q, p)$. Since the new parabola is a translation of the original, the equation of this new parabola is $y = -(x - q)^2 + p$.

Since Q lies on the line $y = x + 4$, we have $p = q + 4$ and the equation of the new parabola is $y = -(x - q)^2 + q + 4$.

Since $B(2, 0)$ lies on the new parabola, we can substitute $(2, 0)$ into this equation:

$$\begin{aligned}
0 &= -(2 - q)^2 + q + 4 \\
0 &= -(q^2 - 4q + 4) + q + 4 \\
0 &= -q^2 + 5q \\
0 &= -q(q - 5)
\end{aligned}$$

Therefore, $q = 0$ or $q = 5$. The value $q = 0$ corresponds to point $P(0, 4)$ in the original parabola. Therefore, $q = 5$. From here we will show two solutions.

Solution 1

Since $q = 5$, the axis of symmetry for the new parabola is $x = 5$. To find C we need to reflect the point $B(2, 0)$ in the axis of symmetry to get $C(8, 0)$.

Solution 2

Since $q = 5$, then the vertex of the new parabola is $(5, 9)$ and the equation of this parabola is $y = -(x - 5)^2 + 9$.

Since C is an x-intercept of this parabola, to determine C we set $y = 0$ in the equation for the parabola and solve for x.

$$\begin{aligned}
0 &= -(x - 5)^2 + 9 \\
(x - 5)^2 &= 9 \\
x - 5 &= \pm 3 \\
x &= 8, 2
\end{aligned}$$

The value $x = 2$ corresponds to point B, and the value $x = 8$ corresponds to point C. Therefore, the coordinates of C are $(8, 0)$.