



Problem of the Week

Problem D and Solution

Height Correction

Problem

At a local high school, the basketball coach measured the height of each player. Nine of the ten heights, in cm, were 180, 181, 183, 187, 188, 190, 193, 195, and 196. The height of the tenth player, Richard, was a whole number.

Initially, the coach measured Richard's height incorrectly. After the error was corrected, both the mean and median of the heights increased by 0.5 cm.

Determine all the possible values for the height of Richard.

Solution

We will let Richard's correct height be R and the incorrectly recorded height be r .

To find a relationship between R and r we will look at the averages of the heights. The average of the heights with the incorrect height, r , is

$$\frac{180 + 181 + 183 + 187 + 188 + 190 + 193 + 195 + 196 + r}{10} = \frac{1693 + r}{10}$$

The average with the correct height, R , is

$$\frac{180 + 181 + 183 + 187 + 188 + 190 + 193 + 195 + 196 + R}{10} = \frac{1693 + R}{10}$$

Since the new mean is 0.5 cm more than the old mean, we have:

$$\begin{aligned}\frac{1693 + r}{10} + 0.5 &= \frac{1693 + R}{10} \\ 1693 + r + 5 &= 1693 + R \\ r + 5 &= R\end{aligned}$$

Let's examine the median for different values of r .

Case 1: If $r \leq 182$, then $R \leq 187$ (since $R = r + 5$).

The median with the incorrect height is $\frac{187+188}{2} = 187.5$. Since R and r are both less than 188, the median with the correct height will remain at 187.5. Therefore, this case is not a possible solution.

Case 2: If $r = 183$, then $R = 188$.

The median with the incorrect height is $\frac{187+188}{2} = 187.5$. Since $R = 188$, the median with the correct height is $\frac{188+188}{2} = 188$, which is an increase of 0.5 in the median. Therefore, this case is one possible solution.



Case 3: If $r = 184$, then $R = 189$.

The median with the incorrect height is $\frac{187+188}{2} = 187.5$. Since $R = 189$, the median with the correct height is $\frac{188+189}{2} = 188.5$, which is an increase of 1 in the median. Therefore, this case is not a possible solution.

Case 4: If $185 \leq r \leq 187$, then $190 \leq R \leq 192$.

The median with the incorrect height is $\frac{187+188}{2} = 187.5$. Since $190 \leq R \leq 193$, the median with the correct height is $\frac{188+190}{2} = 189$, which is an increase of 1.5 in the median. Therefore, this case is not a possible solution.

Case 5: If $r = 188$, then $R = 193$.

The median with the incorrect height is $\frac{188+188}{2} = 188$. Since $R = 193$, the median with the correct height is $\frac{188+190}{2} = 189$, which is an increase of 1 in the median. Therefore, this case is not a possible solution.

Case 6: If $r = 189$, then $R = 194$.

The median with the incorrect height is $\frac{188+189}{2} = 188.5$. Since $R = 194$, the median with the correct height is $\frac{188+190}{2} = 189$, which is an increase of 0.5 in the median. Therefore, this case is one possible solution.

Case 7: If $r \geq 190$, then $R \geq 195$.

The median with the incorrect height is $\frac{188+190}{2} = 189$. Since both r and R are greater than or equal to 190, the median with the correct height will remain at $\frac{188+190}{2} = 189$. Therefore, this case is not a possible solution.

Therefore, the possible correct heights for Richard are 188 cm or 194 cm.