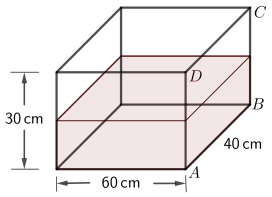


Problem of the Week
Problem D and Solution
Tilt!

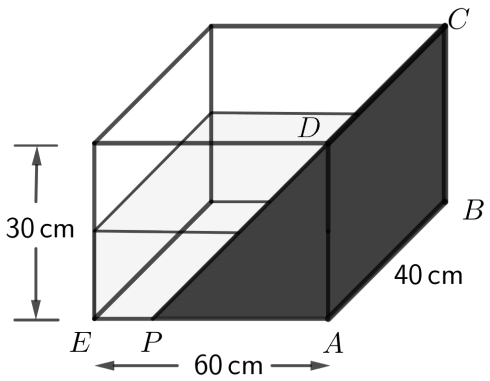
Problem

Troy has a container in the shape of a rectangular prism with a 40 cm by 60 cm base and a height of 30 cm. He labels the vertices of a 40 cm by 30 cm side face A, B, C, and D, with A and B being vertices of the base face too. He then puts some water in the container and tilts the container along AB until the water completely covers face $ABCD$. (He is able to do this so that no water is lost!) At this point, the water still covers $\frac{4}{5}$ of the base area.

Determine the depth of the water, in centimetres, when the container is level.

Solution

Let the other vertex on the bottom face adjacent to A be labelled E. Thus, $EA = 60$ cm. Let P be the point on EA such that $AP = \frac{4}{5}(EA) = \frac{4}{5}(60) = 48$ cm.



When the tank is tilted so that the water completely covers side face $ABCD$, a triangular prism with triangular base ADP and height 40 cm is created. Also, $\triangle ADP$ is a right-angled triangle, so when finding the area of $\triangle ADP$ we can use AP as the base of the triangle and AD as the height of the triangle. That is,

$$\begin{aligned}\text{Volume of triangular prism} &= \text{Area of } \triangle APD \times \text{height of triangular prism} \\ &= \frac{1}{2}(AP)(AD) \times (AB) \\ &= \frac{1}{2}(48)(30) \times (40) \\ &= 28\,800 \text{ cm}^3\end{aligned}$$

Let h represent the height of the water when the tank is level. The volume of the rectangular prism with base 40 cm by 60 cm and height h is the same as the volume of the triangular prism formed when the tank is tilted. That is,

$$\begin{aligned}60 \times 40 \times h &= 28\,800 \\ 2400h &= 28\,800 \\ h &= 12\end{aligned}$$

Therefore, the water is 12 cm deep when the container is level.