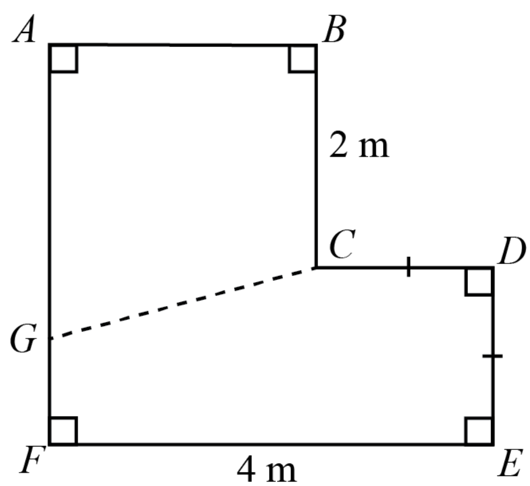
Problem of the Week
Problem D and Solution
Dividing Line

Problem

The Bobsie twins share an L-shaped room. The area of the entire room is 11.2 m^2. The twins are not getting along, so their parents decide to partition the room with tape so that each child has exactly the same area.

The layout of their room is represented by $ABCDEF$ in the diagram. The partitioning tape, indicated by a dashed line, will travel from C to a point G on AF.



Where should G be located on AF in order to split the room into two smaller rooms of equal area?

Solution

Let x represent the length of CD, in metres. Since $DE = CD$, then $DE = x$.

Extend CD to intersect AF at H. This creates two rectangles $ABCH$ and $DEFH$ with $AB \parallel DH \parallel EF$. Also, $AB = EF - CD = 4 - x$.

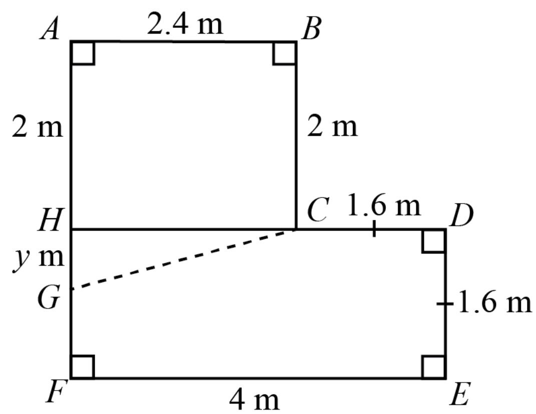
We can now find the value of x using areas.

$$\begin{aligned}\text{Area } ABCDEF &= \text{Area } ABCH + \text{Area } DEFH \\ 11.2 &= (AB \times BC) + (DE \times EF) \\ 11.2 &= (4 - x)(2) + x(4) \\ 11.2 &= 8 - 2x + 4x \\ 3.2 &= 2x \\ 1.6 &= x\end{aligned}$$

Since $x = 1.6$ m, $CD = DE = 1.6$ m, and $AB = 4 - x = 2.4$ m. Also, $AH = BC = 2$ m, and $AF = DE + BC = 1.6 + 2 = 3.6$ m.

Now, the area of $ABCH$ is $2.4 \times 2 = 4.8$ m^2 and the area of area $DEFH$ is $4 \times 1.6 = 6.4$ m^2. Since $6.4 > 4.8$, then G must lie on HF.

Let y represent the length of GH, in metres. A diagram with updated information is below.



$ABCG$ is a trapezoid with opposite parallel sides $BC = 2$ and $AG = 2 + y$. AB is perpendicular to both BC and AG, and $AB = 2.4$ m. We also know that the area of trapezoid $ABCG$ is half the area of $ABCDEF$, so the area of trapezoid $ABCG$ is 5.6 m^2.

Therefore,

$$\begin{aligned}
\text{Area of Trapezoid } ABCG &= \frac{AB \times (BC + AG)}{2} \\
5.6 &= \frac{2.4 \times (2 + 2 + y)}{2} \\
5.6 &= 1.2 \times (4 + y) \\
5.6 &= 4.8 + 1.2y \\
0.8 &= 1.2y
\end{aligned}$$

Thus, $y = \frac{0.8}{1.2} = \frac{8}{12} = \frac{2}{3}$. Since $AG = 2 + y$, we have $AG = 2 + \frac{2}{3} = \frac{8}{3}$ m.

Also, since $GF = AF - AG$, we have $GF = 3.6 - \frac{8}{3} = \frac{18}{5} - \frac{8}{3} = \frac{54 - 40}{15} = \frac{14}{15}$ m.

Therefore, G should be positioned $\frac{14}{15}$ m from F, and $\frac{8}{3}$ m from A.