



12345

Problem of the Week

Problem C and Solution

Five Magnets

Problem

Harlow has five magnets, each with a different number from 1 to 5. They arranged these magnets to create a five digit number $ABCDE$ such that:

- the three-digit number ABC is divisible by 4,
- the three-digit number BCD is divisible by 5, and
- the three-digit number CDE is divisible by 3.

Determine the five-digit number that Harlow created.

Solution

Since ABC is divisible by 4, it follows that C must be even, so $C = 2$ or $C = 4$.

Since BCD is divisible by 5, it follows that $D = 0$ or $D = 5$. However, there is no magnet with a 0, so it follows that $D = 5$.

We also know that CDE is divisible by 3. We can consider the following two cases.

- **Case 1:** $C = 2$

If $C = 2$, then the three-digit number CDE is $25E$. The only possibilities for E are 1, 3, or 4. However, none of 251, 253 and 254 are divisible by 3. It follows that C cannot equal 2.

- **Case 2:** $C = 4$

If $C = 4$ then the three-digit number CDE is $45E$. The only possibilities for E are 1, 2, or 3. Since 451 and 452 are not divisible by 3, but 453 is divisible by 3, it follows that $C = 4$ and $E = 3$.

Thus, the three-digit number ABC is $AB4$. The only magnets not used yet are numbered 1 and 2, so this number is 124 or 214. Since 214 is not divisible by 4, but 124 is divisible by 4, it follows that $A = 1$ and $B = 2$.

Therefore, the five-digit number must be 12453.