



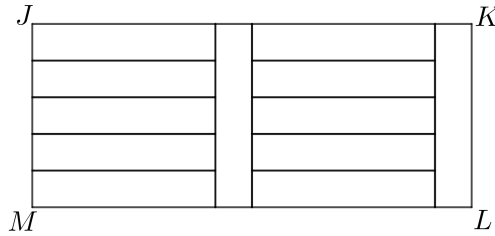
Problem of the Week

Problem C and Solution

A Rectangle of Rectangles

Problem

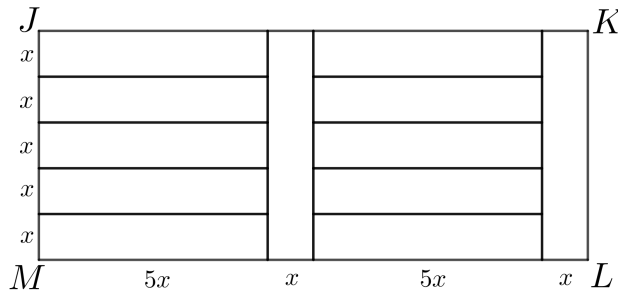
Large rectangle $JKLM$ is formed by twelve identical smaller rectangles, as shown.



If the area of $JKLM$ is 540 cm^2 , then determine the dimensions of the smaller rectangles.

Solution

Let x be the width of one of the smaller identical rectangles, in cm. Five of the smaller rectangles are stacked on top of each other forming JM , so $JM = x + x + x + x + x = 5x$. Since $JKLM$ is a rectangle, $JM = KL = 5x$. Thus, $5x$ is also the length of a smaller rectangle. Therefore, a smaller rectangle is $5x$ cm by x cm.



From here, we proceed with two different solutions.

Solution 1

Since $JKLM$ is formed by twelve identical smaller rectangles, the area of rectangle $JKLM$ is equal to 12 times the area of one of the smaller rectangles.

$$\begin{aligned} \text{Area } JKLM &= 12 \times \text{Area of one smaller rectangle} \\ 540 &= 12 \times 5x \times x \\ 540 &= 60 \times x^2 \end{aligned}$$

Dividing both sides by 60, we obtain $x^2 = 9$. Since x is the width of a smaller rectangle, $x > 0$, and so $x = 3$ follows.

Thus, the width of a smaller rectangle is $x = 3$ cm and the length of a smaller rectangle is $5x = 5(3) = 15$ cm.

Therefore, the smaller rectangles are each 15 cm by 3 cm.



Solution 2

Side length ML is made up of the lengths of two of the smaller rectangles plus the widths of two of the smaller rectangles. Therefore, $ML = 5x + 5x + x + x = 12x$ and rectangle $JKLM$ is $12x$ cm by $5x$ cm.

To find the area of $JKLM$ we multiply the length ML by the width JM .

$$\begin{aligned}\text{Area } JKLM &= ML \times JM \\ 540 &= (12x) \times (5x) \\ 540 &= 12 \times 5 \times x \times x \\ 540 &= 60 \times x^2\end{aligned}$$

Dividing both sides by 60, we obtain $x^2 = 9$. Since x is the width of a smaller rectangle, $x > 0$, and so $x = 3$ follows.

Thus, the width of a smaller rectangle is $x = 3$ cm and the length of a smaller rectangle is $5x = 5(3) = 15$ cm.

Therefore, the smaller rectangles are each 15 cm by 3 cm.