# Problem of the Week
Problem C and Solution
Selling Sketches

Problem

Elouise sketches pictures and sells them in the park during a summer festival. She uses three colours of paper: white, grey, and black. First she sketches on white paper and puts it on the table for display. Then she sketches on grey paper, and then she sketches on black paper. After she has finished a sketch, she places it on top of the display pile. She continues in this pattern of white, grey, black, so that after she has completed seven sketches, her display pile is as shown above. Elouise continues sketching and adding pictures to the display pile throughout the festival. If someone wants to buy one of her sketches, they can only buy the sketch that is currently on the top of the pile. At the end of the festival, Elouise notes that she has sketched more pictures than she has sold, and the remaining sketches in her display pile are as shown.
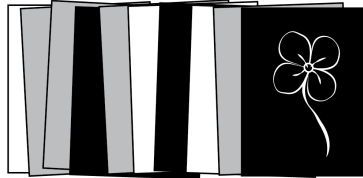
What is the fewest number of sketches that Elouise could have sold during the festival?

This problem was inspired by a past Beaver Computing Challenge (BCC) problem.

Solution

Let W represent a sketch on white paper, G represent a sketch on grey paper, and B represent a sketch on black paper. Then the sequence in the display pile at the end of the festival is $WGGBGWBWGB$, where the leftmost sketches represent those on the bottom of the pile.

If no sketches were sold at all, the sequence would be $WGBWGBWGB\ldots$.

We will place a _ in our display pile sequence at the places where we know some sketches must have been sold, because it does not follow the sequence in which they were made. This gives $WG\_GB\_G\_W\_BWGB$. Moving from left to right, the fewest number of sketches that could fill the first blank is 2, namely BW. The fewest number of sketches that could fill the second blank is 1, namely W. The fewest number of sketches that could fill the third blank is 1, namely B. The fewest number of sketches that could fill the last blank is 1, namely G. Filling these in gives $WG\underline{BW}GB\underline{W}G\underline{B}W\underline{G}BWGB$, as desired.

Therefore, the minimum number of sketches that Elouise could have sold is $2 + 1 + 1 + 1 = 5$.