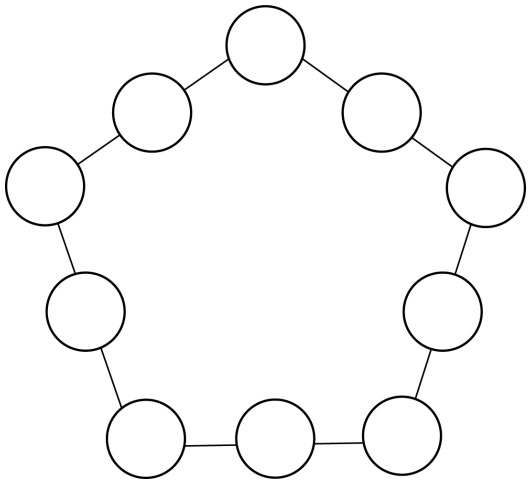
Problem of the Week
Problem B and Solution
A Pentagon Puzzle

Problem

To solve a puzzle, Jonah needs to insert a different number from 1 to 10 into each of the the ten circles in the diagram so that the three numbers on each side of the pentagon have the same sum. He calls this sum the *magic sum*.



(a) What is the least possible sum of any three of the given digits? What is the greatest possible sum of any three of the given digits?

(b) Do you think either of the sums in part (a) could be the magic sum? Explain.

(c) Find a solution to the puzzle.

EXTENSION: Can you find a different solution that is not a rotation or reflection of your solution from (c)?

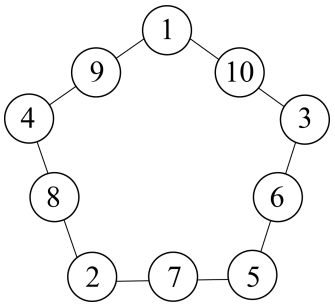
Solution

(a) The least possible sum is $1 + 2 + 3 = 6$. The greatest possible sum is $8 + 9 + 10 = 27$.

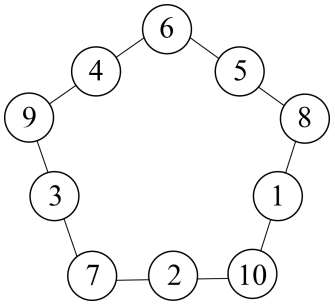
(b) The magic sum cannot be 6, since all other possible sums of three numbers are greater than 6. Similarly, the magic sum cannot be 27, since all other possible sums are less than 27.

(c) When solving this problem, it helps to know what the magic sum is. However, the magic sum will not be the same for every solution, because it depends on where the numbers are placed. For a particular arrangement, if we add up the sum of the numbers on each side of the pentagon, we will have counted the numbers in the corners twice. So the magic sum equals the sum of all the numbers, plus the sum of just the numbers in the corners, divided by five.

We can start by placing the smallest numbers, 1, 2, 3, 4, and 5, in the corners. The sum of all the numbers is $1 + 2 + \cdots + 9 + 10 = 55$. The sum of the numbers in the corners is $1+2+3+4+5 = 15$, and $55+15 = 70$. Then, the magic sum is $70 \div 5 = 14$. This is the smallest possible magic sum because the smallest numbers are in the corners. A solution with a magic sum of 14 is shown.



To find the largest possible magic sum, we will place the largest numbers, 6, 7, 8, 9, and 10, in the corners. The sum of the numbers in the corners is $6 + 7 + 8 + 9 + 10 = 40$, and $55 + 40 = 95$. Then, the magic sum is $95 \div 5 = 19$. A solution with a magic sum of 19 is shown.



Two other solutions, with magic sums of 16 and 17, are shown.