



# Problem of the Week

## Problem B and Solution

### Letters with Friends

#### Problem

In the game “Scribble”, players place tiles containing individual letters on a board to form words. The board is divided up into squares, and each letter in a word is placed in adjacent squares in a row (reading from left to right) or a column (reading from the top down). Each letter has a point value, and the score for a word is found by adding up the point values for each of its letters. The point values for each letter are shown in the following table.

Letter	Number of Points
A, E, I, L, N, O, R, S, T	1
D, G, U, Y	2
B, C, M, P, W	3
F, H	5
J, K, V	8
Q, X, Z	10

For example, the word POUR would have a score of  $3 + 1 + 2 + 1 = 7$ .

Players alternate turns making words on the board. After the first turn, each new word must build off of one of the letters already on the board. The score for any new word includes the point value for the letter that was already on the board.

Vinny and Theo played Scribble. Their game board is shown after they each had three turns.

- (a) Which word on the game board has the highest score?
- (b) Which word on the game board has the lowest score?
- (c) If Vinny made the first word, what is the maximum total score that he could have for his three words? In order to achieve this score, which word(s) could he have started with?

		K	I	C	K									
			H			B								
		Q	U	E	S	T	I	O	N					
			F			Z								
						A	R	C	A	D	E			
							R							
						O	R	A	N	G	E			
							E							

**EXTENSION:** Suppose Vinny’s next word has a score of 10. Which word could this be? See how many different possible words you can find. You can assume that the game board extends past the size of what is shown above.

#### Solution

- (a) The word KICK has the highest score. Its score is  $8 + 1 + 3 + 8 = 20$ .
- (b) The word ORANGE has the lowest score. Its score is  $1 + 1 + 1 + 1 + 2 + 1 = 7$ .



- (c) The three words with the highest score are KICK, QUESTION, and BIZARRE. Their scores are 20, 18, and 18, respectively. If Vinny made all three of these words, then his total score would be  $20 + 18 + 18 = 56$ .

It is possible for Vinny to place all three of these words. He could have started with QUESTION or BIZARRE, as shown in the tables below.

Turn	Word
Vinny's first turn	QUESTION
Theo's first turn	CHEF
Vinny's second turn	BIZARRE
Theo's second turn	ARCADE
Vinny's third turn	KICK
Theo's third turn	ORANGE

Turn	Word
Vinny's first turn	BIZARRE
Theo's first turn	ARCADE
Vinny's second turn	QUESTION
Theo's second turn	CHEF
Vinny's third turn	KICK
Theo's third turn	ORANGE

However, if Vinny started with KICK, then Theo would have made CHEF next, and then Vinny would have made QUESTION. The only possible next word is BIZARRE, so Theo must have made BIZARRE. Thus, if Vinny started with KICK, then he could not have achieved the maximum total score of 56. Therefore, Vinny could have started with QUESTION or BIZARRE.

### SOLUTION TO EXTENSION:

There are many possible words. It helps by looking at the game board to find the letters that Vinny could build off of. Here are some possible words he could make that have a score of 10.

- Using one of the Ks from KICK he could make ASK.
- Using the N from QUESTION he could make CHIN.
- Using the E from ARCADE he could make SIMPLE.
- Using the N from ORANGE he could make NIGHT.
- Using the G from ORANGE he could make GIFTS.
- Using the E from ORANGE he could make EACH.