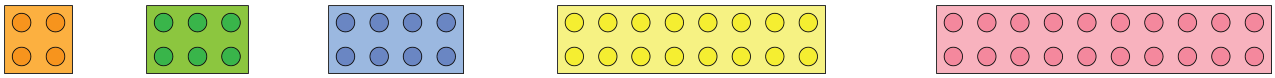
Problem of the Week
Problem B and Solution
Playing with Bricks

Problem

Saskia has the following five sizes of Lego^{TM} bricks.

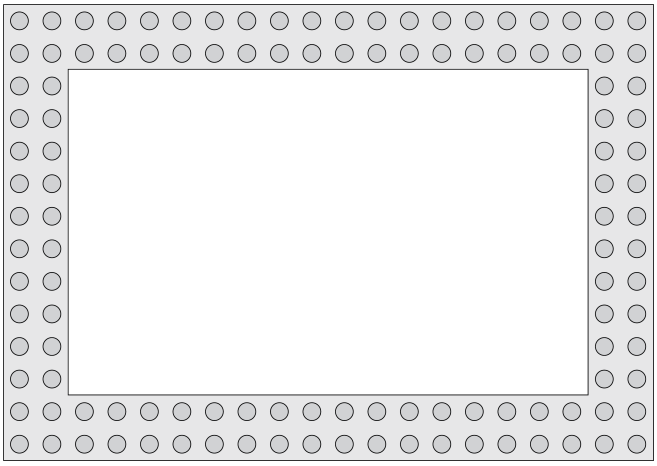


2×2 2×3 2×4 2×8 2×10

For each question below, assume that Saskia will never run out of bricks.

(a) Saskia wants to make a row of bricks that measures 2×16. Which of her brick sizes can she use if all bricks used must be the same size?

(b) Saskia wants to make a rectangular frame of bricks that measures 14×20 on the outside, as shown. Which of her brick sizes can she use if all bricks used must be the same size?



(c) Saskia wants to make a rectangular frame of bricks that measures 320×420 on the outside. What is the largest brick size that she can use if all bricks used must be the same size?

Solution

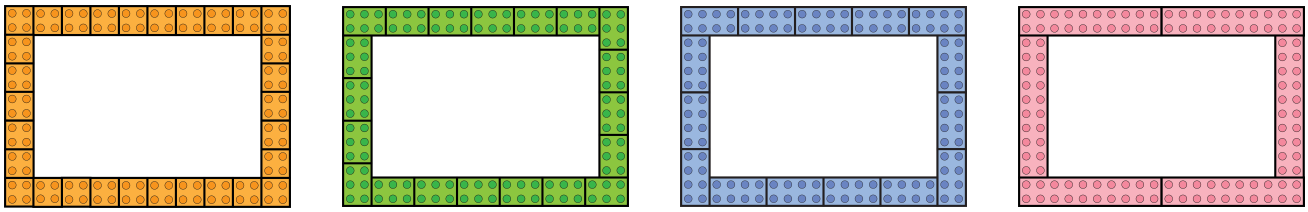
In our solutions, we call the smaller number in the brick size the *width*, and call the larger number the *length*. So a 2×4 brick has a width of 2 and a length of 4.

(a) Saskia can make a row that measures 2×16 by using eight 2×2 bricks, four 2×4 bricks, or two 2×8 bricks.

In general, since the width of the row is the same as the width of each brick, we can focus only on the lengths. If we want to place some number of the same brick in a row and have the total length equal 16, then the possible bricks are ones whose length is a factor of 16. These are 2×2, 2×4, and 2×8.

(b) We can think of the rectangular frame as two 2×20 rows on the top and bottom, and two 2×10 rows on the sides. If we placed these rows end to end, they would create a row with length $20 + 20 + 10 + 10 = 60$. Since 8 is the only length that is not a factor of 60, we know for sure that it is not possible to make the frame using only 2×8 bricks.

For the other brick sizes, we need to show that it is possible to place them around the frame. For each brick size, we start at the top-left corner and place bricks horizontally moving to the right. Once we can no longer place bricks horizontally, we start placing the bricks vertically, moving down. We continue in this clockwise fashion, until all bricks are placed around the frame. This method works for the 2×2, 2×3, 2×4, and 2×10 bricks, as shown.



2×2 2×3 2×4 2×10

Thus, the possible brick sizes are 2×2, 2×3, 2×4, and 2×10.

(c) We can think of the rectangular frame as two 2×320 rows on the sides, and two 2×416 rows on the top and bottom. If we placed these rows end to end, they would create a row with length $320 + 320 + 416 + 416 = 1472$.

Since 10 is not a factor of 1472, we know it is not possible to make the frame using only 2×10 bricks. However, 8 is a factor of 1472, so we will attempt to place the 2×8 bricks around the frame. It helps to notice that 8 is a factor of both 320 and 416. Thus, we can make two 2×320 rows and two 2×416 rows out of 2×8 bricks. Putting these four rows together, with the 2×320 rows on the sides, and the 2×416 rows on the top and bottom gives the desired frame. Thus, the largest brick size that Saskia can use is 2×8.