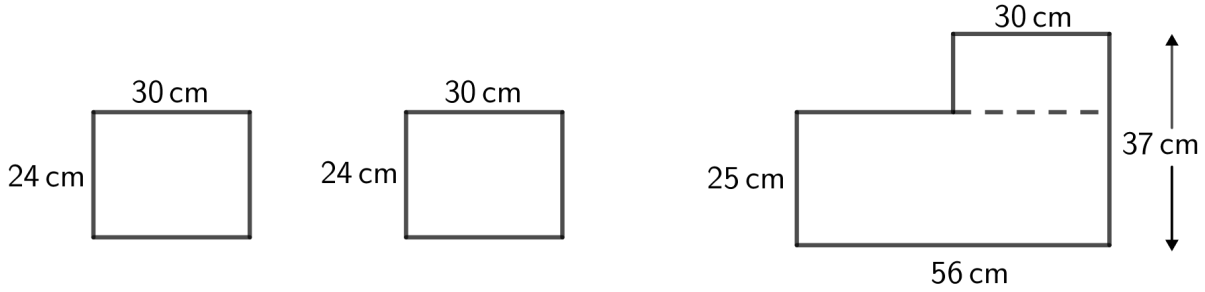
Problem of the Week
Problem B and Solution
Painting Stones

Problem

Iwa wishes to add three stepping stones to her garden. Two of the stepping stones are identical rectangular prisms and the third is a prism made up of two rectangular prisms. The dimensions of the top face of each of the three stepping stones are shown in the diagram below.



(a) Iwa wants to paint the top faces of the three stones. What is the area that she will paint?

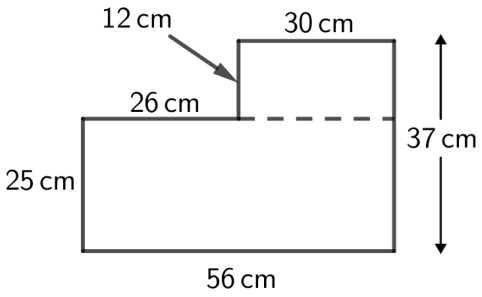
(b) Iwa realizes that the side faces of the prisms will also be seen and, therefore, she wants to paint these as well. If these stones all have height of 6 cm, what is the total area that she will actually paint?

(c) She wants to put edging tightly around each of the stepping stones. What length of edging will she need?

Solution

(a) The two identical stones each have a top face with area $24 \times 30 = 720$ cm^2.

To determine the area of the top face of the third stone, we consider the two rectangles formed by the dashed line. The width of the smaller rectangle is equal to $37 - 25 = 12$ cm. The portion of the length of the larger rectangle which is not joined to the smaller rectangle is equal to $56 - 30 = 26$ cm.



Thus, the smaller rectangle has a length of 30 cm and a width of 12 cm, and so has area $30 \times 12 = 360$ cm^2.

The larger rectangle has a length of 56 cm and a width of 25 cm, and so has area $56 \times 25 = 1400$ cm^2.

The area of the top face of the third stepping stone is the sum of these two areas or $360 + 1400 = 1760$ cm^2.

Therefore, the total area painted is $720 + 720 + 1760 = 3200$ cm^2.

(b) The two stones that are rectangular prisms each have four side faces with dimensions 24 cm by 6 cm, 30 cm by 6 cm, 24 cm by 6 cm, and 30 cm by 6 cm.

Thus, the areas of the side faces are $24 \times 6 = 144$ cm^2, $30 \times 6 = 180$ cm^2, $24 \times 6 = 144$ cm^2, and $30 \times 6 = 180$ cm^2, respectively.

Therefore, the total area of the four side faces of one of these stones is $144 + 180 + 144 + 180 = 648$ cm^2.

The third stone composed of two rectangular prisms has six side faces with dimensions 30 cm by 6 cm, 37 cm by 6 cm, 56 cm by 6 cm, 25 cm by 6 cm, 26 cm by 6 cm, and 12 cm by 6 cm.

Thus, the areas of the side faces are $30 \times 6 = 180$ cm^2, $37 \times 6 = 222$ cm^2, $56 \times 6 = 336$ cm^2, $25 \times 6 = 150$ cm^2, $26 \times 6 = 156$ cm^2, and $12 \times 6 = 72$ cm^2, respectively.

Therefore, the total area of the six side faces of this stone is $180 + 222 + 336 + 150 + 156 + 72 = 1116$ cm^2.

Therefore, the total area that she will now paint is $3200 + 648 + 648 + 1116 = 5612$ cm^2.

(c) The length of edging needed is equal to the total perimeter of the top (or bottom) faces of the stepping stones.

The perimeter of the top face of one of the identical stones is equal to $24 + 30 + 24 + 30 = 108$ cm.

The perimeter of the top face of the third stone is equal to $30 + 37 + 56 + 25 + 26 + 12 = 186$ cm.

Therefore, the amount of edging needed is equal to $108 + 108 + 186 = 402$ cm.