Problem of the Week

Problems and Solutions 2023-2024

Problem B

Grade 5/6

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

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Algebra (A)

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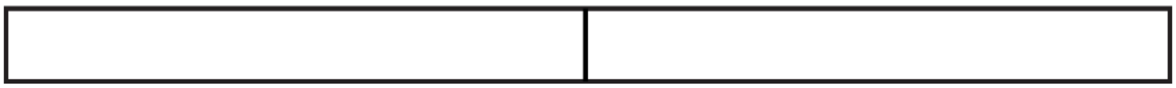
Problem of the Week

Problem B

Wrecked Tangles

Gaby drew a rectangle and called it *Diagram 1*.

She then drew a rectangle divided into two equal parts, and called *Diagram 2*.



She then counted the total number of rectangles in *Diagram 2*. There is 1 rectangle on the left, 1 rectangle on the right, and the original whole rectangle, which makes 3 rectangles in total.

Gaby then drew a rectangle divided into three equal parts, called *Diagram 3*.

Gaby counted a total of 6 rectangles in *Diagram 3*. Can you confirm this?

(a) Gaby continued drawing diagrams by dividing a rectangle into equal parts. *Diagram 4* is divided into four equal parts, *Diagram 5* is divided into five equal parts, and so on. Complete the table by determining the total number of rectangles in each diagram. Draw the diagrams to help you, and then look for a pattern in the total number of rectangles.

Diagram Number	Total Number of Rectangles
1	1
2	3
3	6
4	
5	
6	

(b) Use the pattern you found in part (a) to predict the total number of rectangles in *Diagram 12*.

Themes Algebra, Geometry & Measurement

Problem of the Week

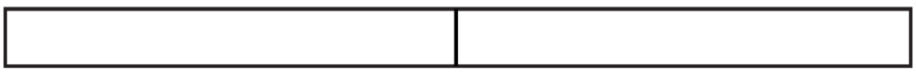
Problem B and Solution

Wrecked Tangles

Problem

Gaby drew a rectangle and called it *Diagram 1*.

She then drew a rectangle divided into two equal parts, and called *Diagram 2*.



She then counted the total number of rectangles in *Diagram 2*. There is 1 rectangle on the left, 1 rectangle on the right, and the original whole rectangle, which makes 3 rectangles in total.

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Gaby counted a total of 6 rectangles in *Diagram 3*. Can you confirm this?

(a) Gaby continued drawing diagrams by dividing a rectangle into equal parts. *Diagram 4* is divided into four equal parts, *Diagram 5* is divided into five equal parts, and so on. Complete the table by determining the total number of rectangles in each diagram. Draw the diagrams to help you, and then look for a pattern in the total number of rectangles.

Diagram Number	Total Number of Rectangles
1	1
2	3
3	6
4	
5	
6	

(b) Use the pattern you found in part (a) to predict the total number of rectangles in *Diagram 12*.

Solution

(a) For each rectangle, we will assign the smallest rectangle a length of one unit.

Diagram 4 is a rectangle divided into 4 equal parts. In this diagram, there are 4 rectangles of length one unit, 3 of length two units, 2 of length three units, and 1 of length four units. This is a total of $4 + 3 + 2 + 1 = 10$ rectangles.

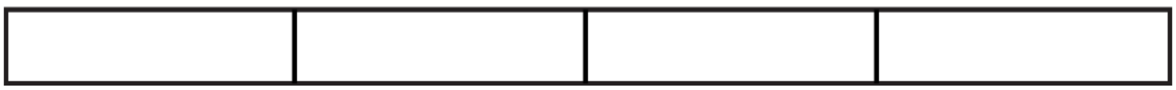
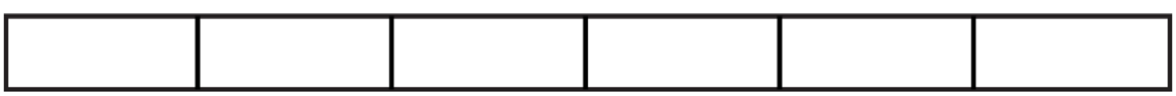


Diagram 5 is a rectangle divided into 5 equal parts. In this diagram, there are 5 rectangles of length one unit, 4 of length two units, 3 of length three units, 2 of length four units, and 1 of length five units. This is a total of $5 + 4 + 3 + 2 + 1 = 15$ rectangles.

Diagram 6 is a rectangle divided into 6 equal parts. In this diagram, there are 6 rectangles of length one unit, 5 of length two units, 4 of length three units, 3 of length four units, 2 of length five units, and 1 of length six units. This is a total of $6 + 5 + 4 + 3 + 2 + 1 = 21$ rectangles.



Now we see a pattern. The total number of rectangles for each diagram is equal to the sum of the diagram number and all the whole numbers smaller than it. Alternatively, the total number of rectangles for each diagram is equal to the diagram number plus the previous number of rectangles. So, the total number of rectangles in *Diagram 7* is equal to
$7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$, or $21 + 7 = 28$.

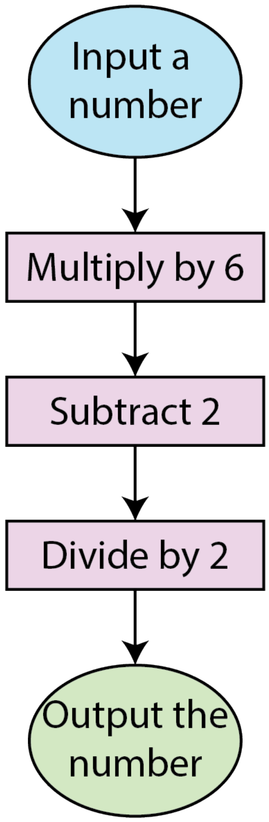
(b) Using the pattern from part (a), the total number of rectangles in *Diagram 12* is equal to $12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 78$, or $28 + 8 + 9 + 10 + 11 + 12 = 78$.

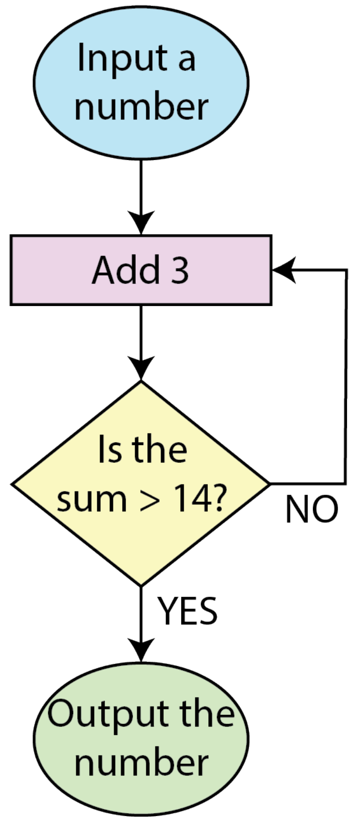
Problem of the Week

Problem B

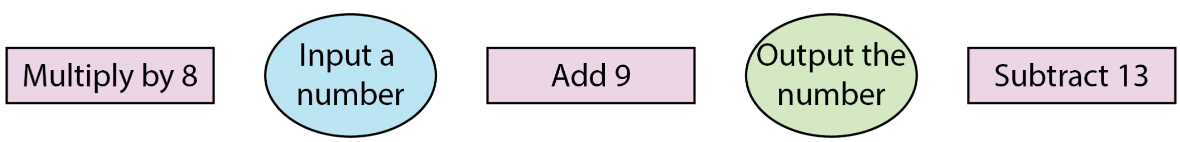
Nice Flow

(a) For each flowchart below, determine the output value when the number 13 is the input number and when the number 10 is the input number.





(b) Using the symbols below, create all possible flowcharts.



(c) Which of the flowcharts in part (b) give an output of 248 for an input of 35?

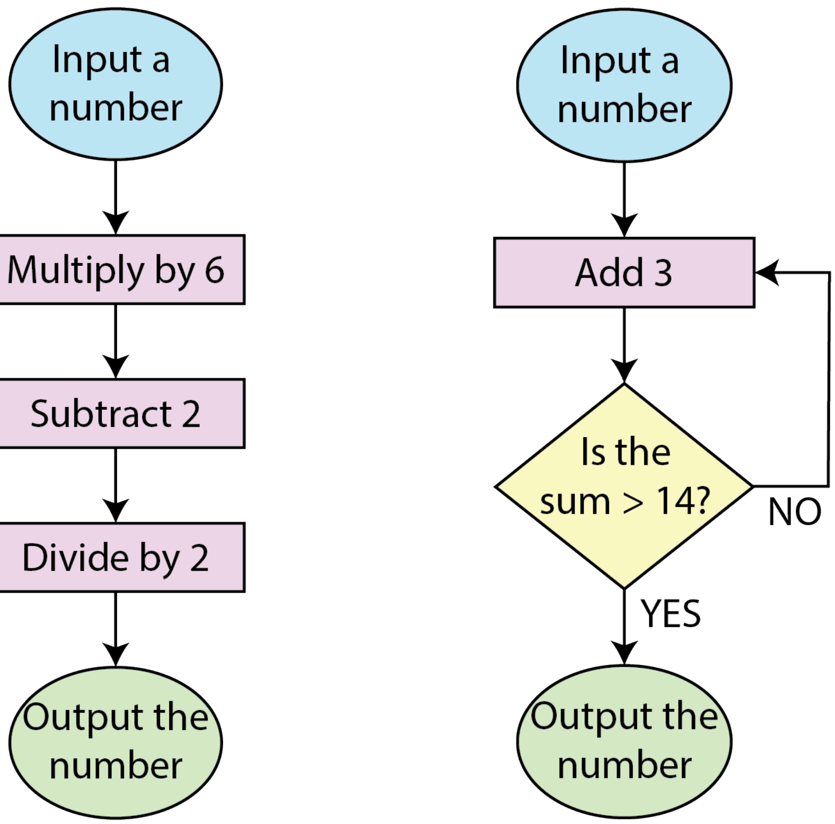
Problem of the Week

Problem B and Solution

Nice Flow

Problem

(a) For each flowchart below, determine the output value when the number 13 is the input number and when the number 10 is the input number.



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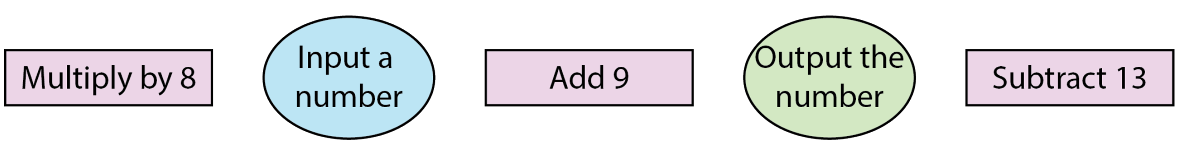


(c) Which of the flowcharts in part (b) give an output of 248 for an input of 35?

Solution

(a) For the flowchart on the left:

When we input 13, we first multiply by 6 to get 78.
Then, we subtract 2 to get 76.
Finally, we divide 76 by 2 to get 38.
Thus, the output is 38.

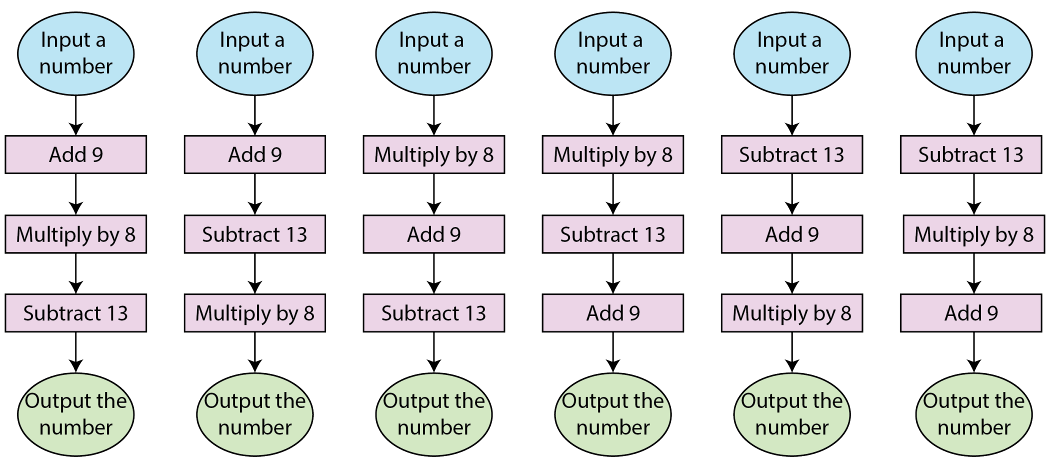
When we input 10, we first multiply by 6 to get 60.
Then, we subtract 2 to get 58.
Finally, we divide 58 by 2 to get 29.
Thus, the output is 29.

For the flowchart on the right:

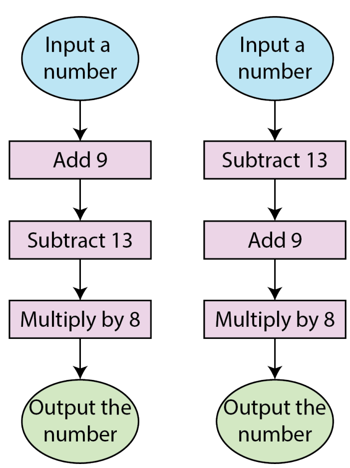
When we input 13, we first add 3 to get 16.
Since $16 > 14$, the output is 16.

When we input 10, we first add 3 to get 13.
Since 13 is not greater than 14, we add 3 again to get 16.
Since $16 > 14$, the output is 16.

(b) Here are the six possible flowcharts:



(c) Here are the two flowcharts that work:



Computational Thinking (C)

Take me to the cover

Problem of the Week
Problem B
Coding Conundrum

Viktoria is writing a program that people can use to guess her favourite number, which is 23.

(a) Use the following blocks to create pseudocode for Viktoria's program. Note that you may not need to use all of the blocks.

say "That's not correct."	say "That's correct."
else	say "Guess my secret number!"
if guess $\neq$ 23	if guess $=$ 23

(b) Using the additional blocks below, modify your pseudocode so that if the user's guess is incorrect, then the program will tell them whether their guess is too high or too low.

say "That's too low."	say "That's too high."
else if guess $<$ 23	else if guess $>$ 23

Not printing this page? You can use our interactive worksheet.

THEME COMPUTATIONAL THINKING

Problem of the Week

Problem B and Solution

Coding Conundrum

Problem

Viktoria is writing a program that people can use to guess her favourite number, which is 23.

(a) Use the following blocks to create pseudocode for Viktoria's program. Note that you may not need to use all of the blocks.

say "That's not correct."	say "That's correct."
else	say "Guess my secret number!"
if guess $\neq 23$	if guess $= 23$

(b) Using the additional blocks below, modify your pseudocode so that if the user's guess is incorrect, then the program will tell them whether their guess is too high or too low.

say "That's too low."	say "That's too high."
else if guess < 23	else if guess > 23

Not printing this page? You can use our interactive worksheet.

Solution

(a) Two possible solutions are shown below.

Solution 1

```
say "Guess my secret number!"
if guess = 23
   say "That's correct!"
else
   say "That's not correct."
```

Solution 2

```
say "Guess my secret number!"
if guess ≠ 23
   say "That's not correct!"
else
   say "That's correct."
```

(b) One possible solution is shown below.

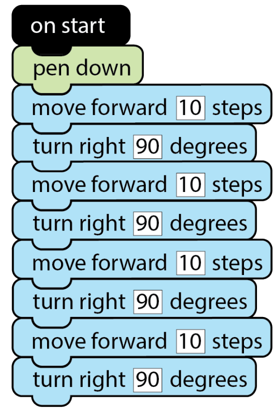
```
say "Guess my secret number!"
if guess = 23
   say "That's correct!"
else if guess < 23
   say "That's too low."
else
   say "That's too high."
```

Problem of the Week

Problem B

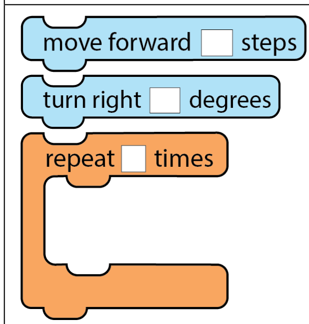
Shelby's Code

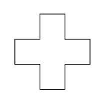
Shelby is using block coding to draw different shapes.



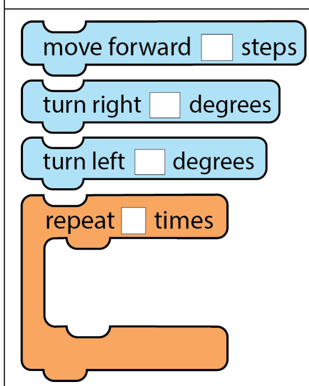
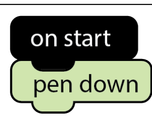
(a) Her first program is shown. What shape will be drawn after running this program?

(b) Using the given blocks, write a program to draw the same shape as Shelby's program, using fewer blocks. Notice that some blocks contain an empty box to be filled with a number.

Blocks	Program
	

(c) Using the given blocks, write a program to draw the following shape: 

You may use a block more than once.

Blocks	Program
	

Themes Computational Thinking, Geometry & Measurement

Problem of the Week

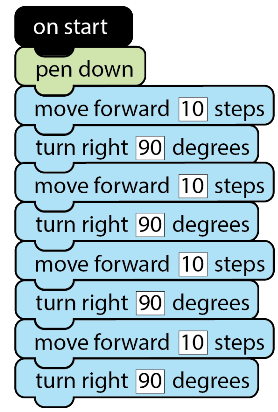
Problem B and Solution

Shelby's Code

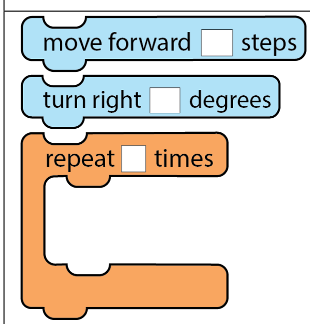
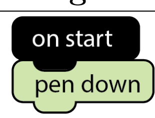
Problem

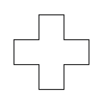
Shelby is using block coding to draw different shapes.

(a) Her first program is shown. What shape will be drawn after running this program?



(b) Using the given blocks, write a program to draw the same shape as Shelby's program, using fewer blocks. Notice that some blocks contain an empty box to be filled with a number.

Blocks	Program
	

(c) Using the given blocks, write a program to draw the following shape: 

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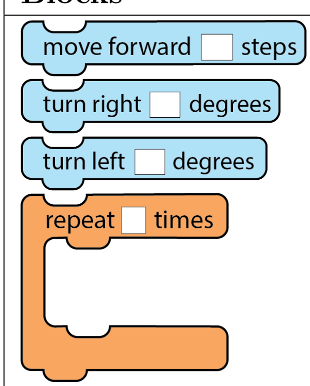
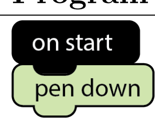
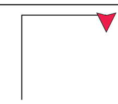
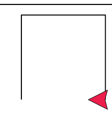
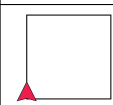
	

Solution

(a) This program will draw a square. The table below shows the drawing progress and position of the pen as we trace through the program.

Program Section	Drawing Progress
on start pen down	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	

(b) By using the repeat block, we can use fewer blocks in the program, as shown.

```
on start
pen down
repeat 4 times
    move forward 10 steps
    turn right 90 degrees
```

(c) There are several possible programs, depending on where the pen starts, and how many repeat blocks are used. Two programs are shown.

```
on start
pen down
repeat 4 times
    move forward 10 steps
    turn right 90 degrees
    move forward 10 steps
    turn right 90 degrees
    move forward 10 steps
    turn left 90 degrees
```

```
on start
pen down
repeat 4 times
    repeat 2 times
        move forward 10 steps
        turn right 90 degrees
    move forward 10 steps
    turn left 90 degrees
```

Problem of the Week

Problem B

It's a Race

Manish, Diana, Isebel, Ris, and Ji-Yeong are the five runners in a 400 m race. Their friend cheered them on and took a photo partway through the race. The photo shows the following:

- Isebel is in the lead.
- Ji-Yeong has run farther than Ris, but not as far as Manish.
- Diana has two people ahead of her and two people behind her.

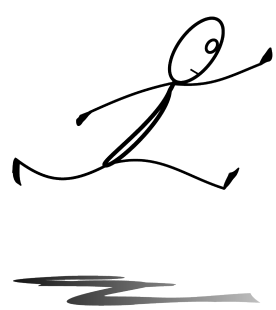
(a) Using the information given, determine the order of the runners in the photo. Fill in the blanks in the list shown below.

START ________, ________, ________, ________, ________ FINISH

(b) The following five fractions represent the fraction of the course that each runner had completed in the photo.

$$\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{3}, \frac{1}{4}$$

Which runner completed each fraction of the course? Show your work using diagrams or equivalent fractions.



THEMES COMPUTATIONAL THINKING, NUMBER SENSE

Problem of the Week

Problem B and Solution

It's a Race

Problem

Manish, Diana, Isebel, Ris, and Ji-Yeong are the five runners in a 400 m race. Their friend cheered them on and took a photo partway through the race. The photo shows the following:

- Isebel is in the lead.
- Ji-Yeong has run farther than Ris, but not as far as Manish.
- Diana has two people ahead of her and two people behind her.

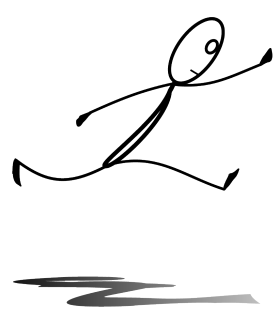
(a) Using the information given, determine the order of the runners in the photo. Fill in the blanks in the list shown below.

START _________, _________, _________, _________, _________ FINISH

(b) The following five fractions represent the fraction of the course that each runner had completed in the photo.

$$\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{3}, \frac{1}{4}$$

Which runner completed each fraction of the course? Show your work using diagrams or equivalent fractions.



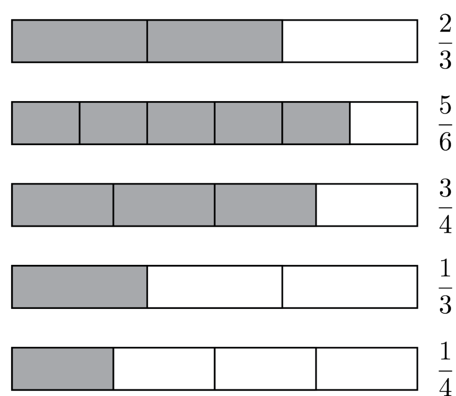
Solution

(a) We will number the positions from 1 to 5, starting on the left. Since Isebel is in the lead, she must be in position 5. Since Diana has two people ahead of her and two people behind her, she must be in position 3. That leaves us with positions 1, 2, and 4. Since Ji-Yeong has run farther than Ris but not as far as Manish, that tells us that Ris must be in position 1, Ji-Yeong must be in position 2, and Manish must be in position 4, as shown.

START Ris, Ji-Yeong, Diana, Manish, Isebel FINISH

(b) In order to determine which runner completed each fraction of the course, we must first write the fractions in order from smallest to largest. Then we can match the fractions with the runners in the order from part (a), since the runner who completed the smallest fraction of the course will be closest to the start, and the runner who completed the largest fraction of the course will be closest to the finish.

One way to compare the fractions is using diagrams, as shown.



Since each diagram is the same width, we can compare the shaded part of each diagram to place the fractions in order from smallest to largest. This gives us $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

Alternatively, we can use equivalent fractions. Using a common denominator of 12, our fractions can be written as follows.

$$\frac{2}{3} = \frac{8}{12}, \quad \frac{5}{6} = \frac{10}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{1}{4} = \frac{3}{12}$$

Now we can use the equivalent fractions to place the fractions in order from smallest to largest.

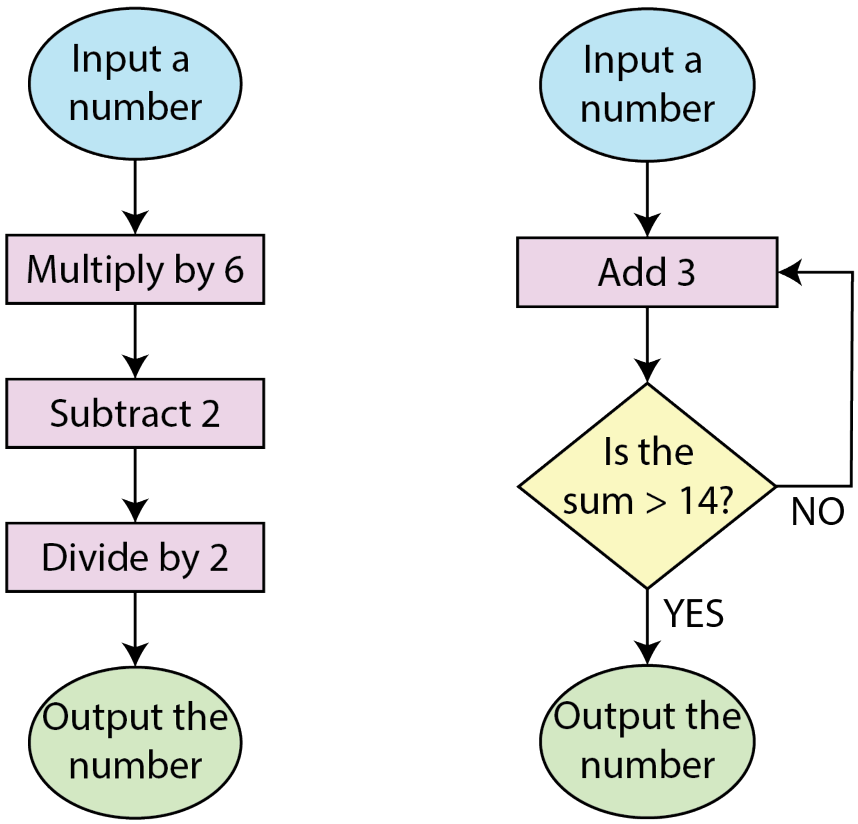
$$\frac{1}{4} = \frac{3}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{5}{6} = \frac{10}{12}$$

Once we have the fractions written in order from smallest to largest, we can match each runner to the fraction of the course they completed as shown.

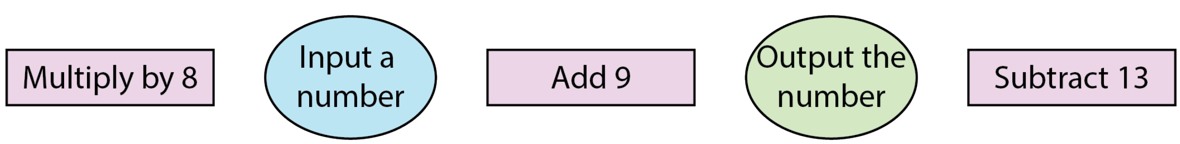
Runner	Ris	Ji-Yeong	Diana	Manish	Isebel
Fraction of Course Completed	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$

Problem of the Week
Problem B
Nice Flow

(a) For each flowchart below, determine the output value when the number 13 is the input number and when the number 10 is the input number.



(b) Using the symbols below, create all possible flowcharts.



(c) Which of the flowcharts in part (b) give an output of 248 for an input of 35?

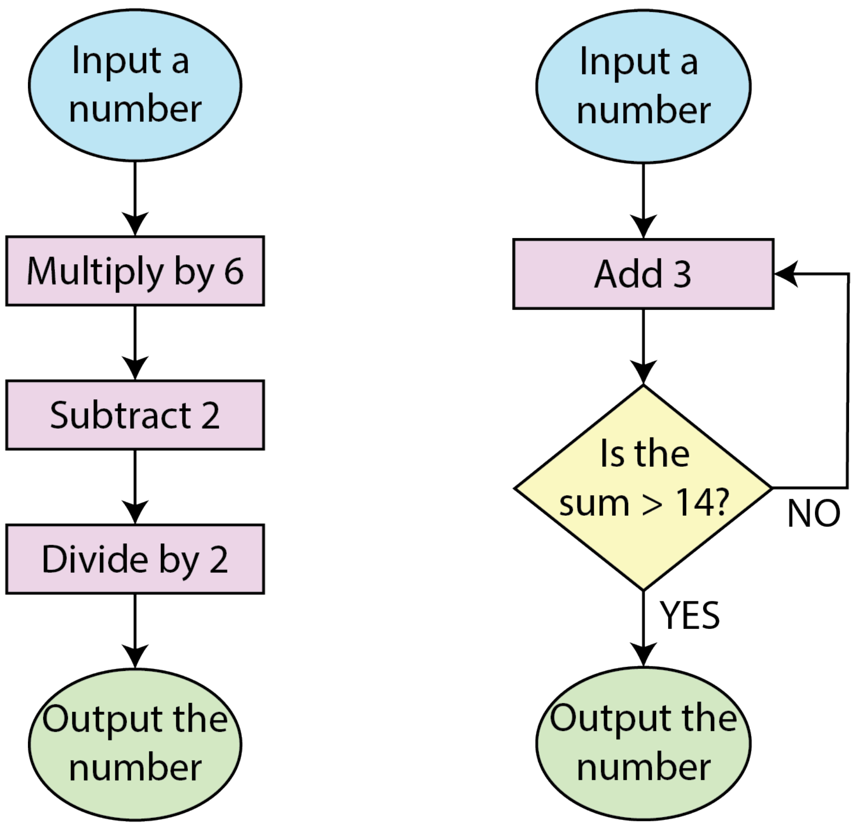
Problem of the Week

Problem B and Solution

Nice Flow

Problem

(a) For each flowchart below, determine the output value when the number 13 is the input number and when the number 10 is the input number.



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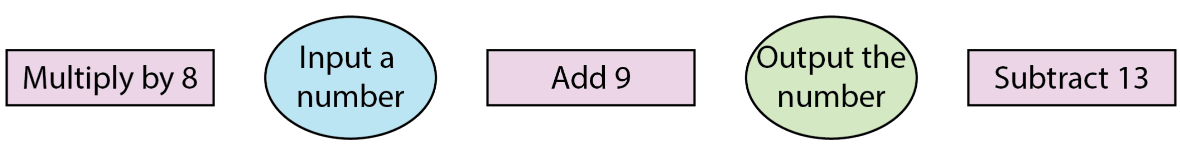


(c) Which of the flowcharts in part (b) give an output of 248 for an input of 35?

Solution

(a) For the flowchart on the left:

When we input 13, we first multiply by 6 to get 78.
Then, we subtract 2 to get 76.
Finally, we divide 76 by 2 to get 38.
Thus, the output is 38.

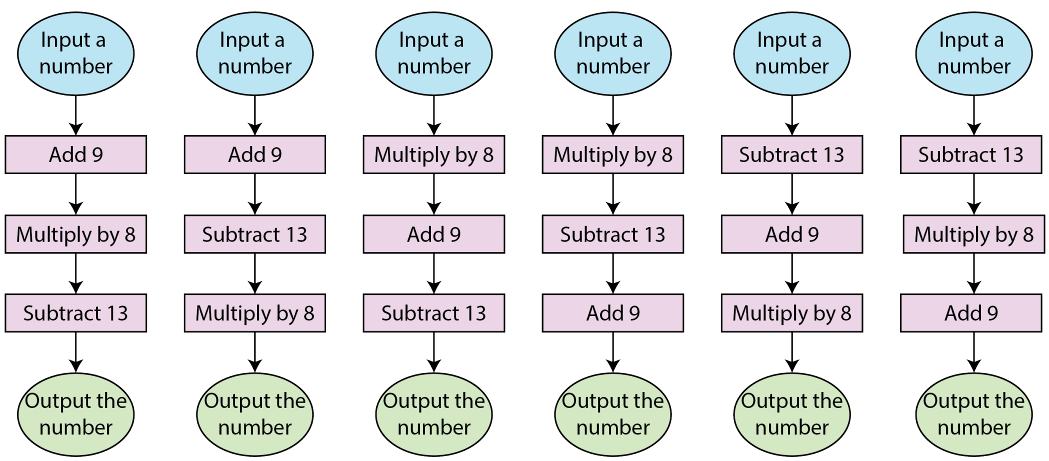
When we input 10, we first multiply by 6 to get 60.
Then, we subtract 2 to get 58.
Finally, we divide 58 by 2 to get 29.
Thus, the output is 29.

For the flowchart on the right:

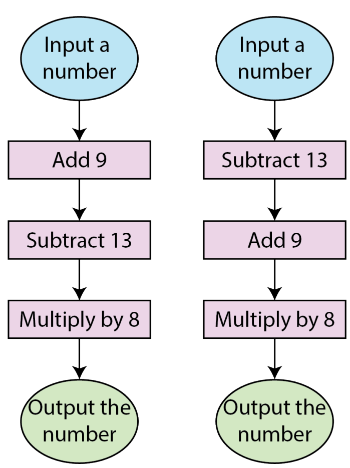
When we input 13, we first add 3 to get 16.
Since $16 > 14$, the output is 16.

When we input 10, we first add 3 to get 13.
Since 13 is not greater than 14, we add 3 again to get 16.
Since $16 > 14$, the output is 16.

(b) Here are the six possible flowcharts:



(c) Here are the two flowcharts that work:



Data Management (D)

Take me to the cover

Problem of the Week
Problem B
These Lakes Are Better Than Good

The table below shows data related to the five Great Lakes, which span the border between Canada and United States.

Lake	Area (miles^2)	Area (km^2)	Volume (miles^3)	Volume (km^3)
Superior	31 700	82 100	2900	12 070
Michigan	22 410	58 030	1180	4930
Huron	23 010	59 590	840	3520
Erie	9910	25 667	117	488
Ontario	7320	18 970	391	1631

(a) Find values for each of the following. Round your answers to two decimal places.

(i) How many times bigger is Lake Superior's area than Lake Ontario's?

(ii) How many times bigger is Lake Superior's volume than Lake Erie's?

(iii) What percentage of the total volume of all five lakes does Lake Superior contain?

(b) For the comparisons in part (a), does it matter whether you use the data based in miles, or in kilometres?

(c) What are the mean and the median areas of the Great Lakes in square kilometres?

(d) DISCOVERY: Lake Superior is the second largest lake in the world, by area, and Lake Huron is the fourth largest. Do some research to find the first and third largest lakes (by area). Try to discover some past data to see how their sizes have changed over time.

Problem of the Week
Problem B and Solution
These Lakes Are Better Than Good

Problem
The table below shows data related to the five Great Lakes, which span the border between Canada and United States.

Lake	Area (miles^2)	Area (km^2)	Volume (miles^3)	Volume (km^3)
Superior	31 700	82 100	2900	12 070
Michigan	22 410	58 030	1180	4930
Huron	23 010	59 590	840	3520
Erie	9910	25 667	117	488
Ontario	7320	18 970	391	1631

(a) Find values for each of the following. Round your answers to two decimal places.

(i) How many times bigger is Lake Superior's area than Lake Ontario's?

(ii) How many times bigger is Lake Superior's volume than Lake Erie's?

(iii) What percentage of the total volume of all five lakes does Lake Superior contain?

(b) For the comparisons in part (a), does it matter whether you use the data based in miles, or in kilometres?

(c) What are the mean and the median areas of the Great Lakes in square kilometres?

(d) DISCOVERY: Lake Superior is the second largest lake in the world, by area, and Lake Huron is the fourth largest. Do some research to find the first and third largest lakes (by area). Try to discover some past data to see how their sizes have changed over time.

Solution

(a) The approximate values of the comparisons using metric measures are as follows.

(i) Lake Superior's area is $\frac{82\,100}{18\,970} \approx 4.33$ times bigger than Lake Ontario's.

(ii) Lake Superior's volume is $\frac{12\,070}{488} \approx 24.73$ times bigger than Lake Erie's.

(iii) The total volume of all five lakes is $12\,070 + 4930 + 3520 + 488 + 1631 = 22\,639$ km^3. Thus, the percentage of the total volume of all five lakes contained by Lake Superior is $\frac{12\,070}{22\,639} \times 100\% \approx 53.32\%$.

(b) For these comparisons, it doesn't matter whether the data in miles, or in kilometres, is used, as long as the same unit is used for all lakes in the calculation. However, there may be slight variations in the values found in part (a), due to rounding and the precision of the values in the table.

(c) The mean area is $\frac{1}{5}(82\,100 + 58\,030 + 59\,590 + 25\,667 + 18\,970) = \frac{244\,357}{5} = 48\,871.4$ km^2. To find the median area we look for the third largest area, which is $58\,030$ km^2.

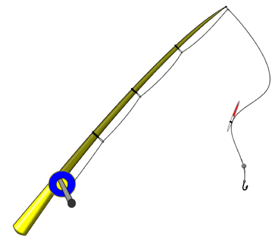
(d) The lake with the greatest area in the world is the Caspian Sea, which is surrounded by Kazakhstan, Russia, Turkmenistan, Azerbaijan, and Iran, and has an area of $389\,000$ km^2. The third largest lake is Victoria Lake in Africa, surrounded by Uganda, Kenya, and Tanzania, with an area of $59\,940$ km^2. Looking at variation in water level gives some idea of how the area and volume of a lake changes over time. For example, Lake Superior's mean water level varies by only one metre or so over the year, although climate change seems to be causing greater fluctuations. It is estimated that the Caspian Sea water level will drop as much as eight metres or more in this century.

Problem of the Week

Problem B

Something's Fishy Here

Nawal went to a cottage on Otter Lake and went fishing every day in their favourite spot. After seven days, their catch included 18 bass, 5 pike, 13 bluegill, 2 perch, and 1 trout.



(a) Based on that week's fishing, what is the *experimental* probability (as a fraction in lowest terms) that the next fish Nawal catches is a bluegill? What is the experimental probability that the next fish Nawal catches is a trout?

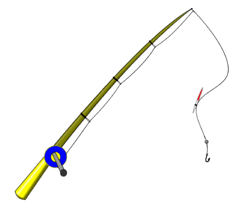
(b) Suppose Nawal went fishing for another seven days in the same spot. What are some things you could predict about their catch, based on their previous experience?

(c) If Nawal went fishing in a new spot recommended by a friend, what could you predict about their catch in this spot, based on their previous experience?

Problem of the Week
Problem B and Solution
Something's Fishy Here

Problem

Nawal went to a cottage on Otter Lake and went fishing every day in their favourite spot. After seven days, their catch included 18 bass, 5 pike, 13 bluegill, 2 perch, and 1 trout.



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(c) If Nawal went fishing in a new spot recommended by a friend, what could you predict about their catch in this spot, based on their previous experience?

Solution

(a) Nawal caught a total of $18 + 5 + 13 + 2 + 1 = 39$ fish.
Thus the experimental probability that the next fish Nawal catches is a bluegill is $\frac{13}{39} = \frac{1}{3}$. The experimental probability that the next fish Nawal catches is a trout is $\frac{1}{39}$.

(b) Since they are fishing in the same spot, we can predict that they will catch a total of about 39 fish, with a similar mix of bass, pike, bluegill, perch, and trout.
We can also predict that they will be more likely to catch bass or bluegill than pike, perch, or trout, given the proportions observed in their first week's catch.

(c) Since they are now fishing in a different spot, there wouldn't necessarily be the same types of fish, nor in the same proportions. On the bright side, since Nawal's friend recommended the spot, there may be more fish there than the previous spot.

Problem of the Week

Problem B

Equally Likely or A Sure Thing?

(a) Imagine you are drawing marbles one at a time from a bag which contains 1 red, 1 blue, 2 yellow, and 3 green marbles. You draw a marble without looking, record the colour, and return it to the bag. Suppose that each marble is equally likely to be drawn.

Which of the following events are equally likely to occur?

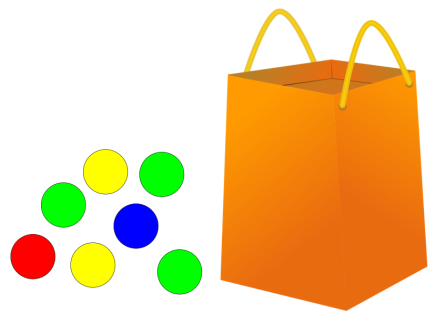
(i) You draw a red marble.

(ii) You draw a blue marble.

(iii) You draw a yellow marble.

(iv) You draw a green marble.

(v) You draw a red OR a blue marble.



Justify your answers by comparing the theoretical probabilities of the events.

(b) Suppose you have two unusual six-sided dice (number cubes), one with the even numbers 2, 4, 6, 8, 10, and 12 on its faces, and the other with the odd numbers 1, 3, 5, 7, 9, and 11 on its faces. When you roll the dice together, you find the sum of the two top faces. What is the probability of each of the following events?

(i) The sum is odd.

(ii) The sum is 7.

(iii) The sum is 25.

(c) Which of the events in part (b) can be called *certain*? Which of the events in part (b) can be called *impossible*?

Problem of the Week

Problem B and Solution

Equally Likely or A Sure Thing?

Problem

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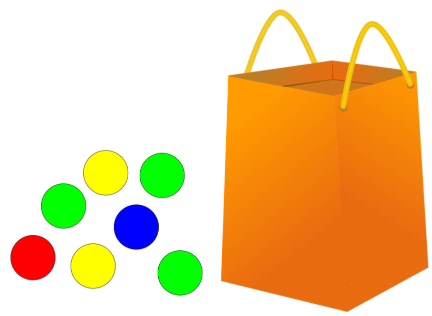

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(ii) The sum is 7.

(iii) The sum is 25.

(c) Which of the events in part (b) can be called *certain*? Which of the events in part (b) can be called *impossible*?

Solution

(a) The theoretical probability in each case is equal to

$$\frac{\text{the number of marbles of the desired colour(s)}}{\text{the total number of marbles}}$$

Using this and the fact that there are 7 marbles in total, we calculate the probability of each event.

(i) The probability of drawing a red marble is $\frac{1}{7}$, since there is only one red marble.

(ii) The probability of drawing a blue marble is $\frac{1}{7}$, since there is only one blue marble.

(iii) The probability of drawing a yellow marble is $\frac{2}{7}$, since there are two yellow marbles and either of the two could be drawn.

(iv) The probability of drawing a green marble is $\frac{3}{7}$, since there are three green marbles and any of the three could be drawn.

(v) The probability of drawing a red or a blue marble is $\frac{2}{7}$, since there are two marbles that are red or blue.

Therefore, events (i) and (ii) are equally likely to occur. Also, events (iii) and (v) are equally likely to occur.

(b) For each roll of the die with even numbers on its faces, there are 6 possible rolls for the die with odd numbers on its faces. Thus, since the the die with even numbers on its faces has 6 faces, there are $6 \times 6 = 36$ possible rolls. Using this, we can calculate the probability of each event.

(i) An odd number plus an even number is always odd, so every roll will produce an odd sum. Thus, the probability that the sum is odd is equal to $\frac{36}{36} = 1$.

(ii) A sum of 7 could be produced by rolling a 1 and a 6, rolling a 2 and a 5, or rolling a 3 and a 4. Thus, the probability that the sum is 7 is equal to $\frac{3}{36} = \frac{1}{12}$.

(iii) The maximum possible sum is $11 + 12 = 23$, so there is no way to roll a sum of 25. Thus, the probability that the sum is 25 is equal to $\frac{0}{36} = 0$.

(c) Since the probability of event (i) is 1, then event (i) can be called *certain*. Since the probability of event (iii) is 0, then event (iii) can be called *impossible*.

Problem of the Week

Problem B

Gamer!

Geoff plays a game using two standard six-sided dice: a black one and a white one. To win the game, Geoff must roll the dice and have the numbers on the two top faces sum to 11.

(a) What is the probability that he rolls a 7 with just the black die?

(b) What is the theoretical probability that he rolls a 1 on the black die and a 6 on the white die?

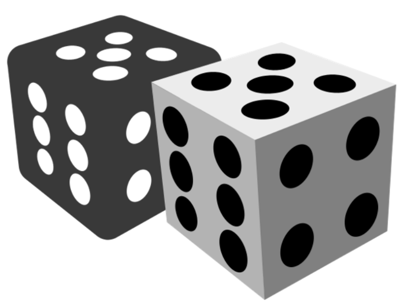
(c) If he rolls both dice and calculates the sum of the numbers on the two top faces, what sum(s) have the lowest theoretical probability of being rolled?

(d) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 7?

(e) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 11?

(f) Roll two dice thirty-six times and keep track of the number of times the numbers on the two top faces sum to 11. What was your empirical probability of rolling a sum of 11?

(g) Share your results in part (f) with your classmates. How many had their empirical probability of rolling a sum of 11 equal the theoretical probability of rolling a sum of 11?



Themes Data Management, Number Sense

Problem of the Week

Problem B and Solution

Gamer!

Problem

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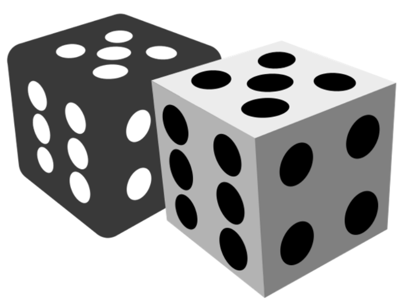

Solution

(a) Since the numbers on the faces of a standard six-sided die are 1, 2, 3, 4, 5, and 6, it is impossible to roll a 7. So the probability is 0.

(b) For each of the 6 possible numbers he could throw with the black die there are 6 possible numbers on the white die, so the total number of possible outcomes is $6 \times 6 = 36$. Thus, the theoretical probability that he throws a 1 on the black die and a 6 on the white die is 1 in 36, or $\frac{1}{36}$.

Alternatively, to solve this problem we can create a table where the columns show the possible numbers on the top face of the white die, the rows show the possible numbers on the top face of the black die, and each cell in the body of the table gives the sum of the corresponding pair of numbers.

		White Die					
		1	2	3	4	5	6
Black Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

From this table, we can conclude that there is 1 outcome out of 36 possible outcomes where the number on the top face of the black die is a 1 and the number on the top face of the white die is a 6. We will use this table in our answers to parts (c), (d), and (e).

(c) For each of the sums of 2 and 12, there is only one possible way to obtain that outcome (two ones or two sixes). Thus, each of these sums has the lowest theoretical probability, namely 1 in 36, or $\frac{1}{36}$.

(d) A sum of 7 can be obtained in 6 possible ways (as $1 + 6$ or $6 + 1$, $2 + 5$ or $5 + 2$, $3 + 4$ or $4 + 3$). So, there are six outcomes which give the desired sum. Thus, the theoretical probability that he rolls a 7 is 6 in 36, or $\frac{6}{36}$, which is equivalent to 1 in 6, or $\frac{1}{6}$.

(e) A sum of 11 can be obtained in 2 possible ways (as $5 + 6$ or $6 + 5$). Thus, the theoretical probability of rolling an 11 is 2 in 36, or $\frac{2}{36}$, which is equivalent to 1 in 18, or $\frac{1}{18}$.

(f) Answers will vary.

(g) Answers will vary.

Problem of the Week

Problem B

These Rates are Shocking

Most provinces take into consideration the time of day when they charge for electricity usage. The rates they charge are often referred to as Time-Of-Use (TOU) rates. Using the sample TOU rates in the table below, answer the questions that follow.

TOU Price Period	**November 1 - April 30 Time of Day**	**May 1 - October 31 Time of Day**	**TOU Rate (¢ per kWh)**
Off-Peak Hours	Weekdays 7 p.m. - 7 a.m., anytime on weekends	Weekdays 7 p.m. - 7 a.m., anytime on weekends	7.4
Mid-Peak Hours	Weekdays 11 a.m. - 5 p.m.	Weekdays 7 a.m. - 11 a.m. and 5 p.m. - 7 p.m.	10.2
On-Peak Hours	Weekdays 7 a.m. - 11 a.m. and 5 p.m. - 7 p.m	Weekdays 11 a.m. - 5 p.m.	15.1

(a) Garret's family used 50 kWh on a Saturday afternoon. What would be the charge for those 50 kWh?

(b) On November 10, when would be the best time of day to run your clothes dryer?

(c) When should you avoid using your clothes dryer in the summer?

(d) What might be a better way (environmentally and financially) to dry your clothes in the summer?

(e) Ramal's family used 1180 kWh hours of electricity in one month.

(i) What is the maximum amount of money (in dollars) they could have paid for electricity that month?

(ii) What is the minimum amount of money (in dollars) they could have paid for electricity that month?

Themes Data Management, Number Sense

Problem of the Week
Problem B and Solution
These Rates are Shocking

Problem

Most provinces take into consideration the time of day when they charge for electricity usage. The rates they charge are often referred to as Time-Of-Use (TOU) rates. Using the sample TOU rates in the table below, answer the questions that follow.

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(e) Ramal's family used 1180 kWh hours of electricity in one month.

  (i) What is the maximum amount of money (in dollars) they could have paid for electricity that month?

  (ii) What is the minimum amount of money (in dollars) they could have paid for electricity that month?

Solution

(a) The rate for any Saturday is 7.4¢ per kWh, which is \$0.074 per kWh. Therefore, the charge for 50 kWh would be $50 \times \$0.074 = \$3.70$.

(b) If November 10 falls on a weekday, the best time to run the dryer would be anytime before 7 a.m. or after 7 p.m. If November 10 falls on the weekend, you could run it anytime from Friday after 7 p.m. until Monday morning before 7 a.m.

(c) You should avoid running your dryer from 7 a.m. to 7 p.m. on weekdays, but it is most expensive to run your dryer between 11 a.m. and 5 p.m.

(d) You could hang your clothes out to dry in the summer which would have little or no cost, both environmentally and financially.

(e) (i) Ramal's family used 1180 kWh. The most they could have paid for electricity is \$0.151 per kWh. Therefore, the maximum amount they could have paid for electricity that month is $1180 \times \$0.151 = \$178.18$.

(ii) Ramal's family used 1180 kWh. The least they could have paid for electricity is \$0.074 per kWh. Therefore, the minimum amount they could have paid for electricity that month is $1180 \times \$0.074 = \$87.32$.

Problem of the Week

Problem B

The First Five Modern Olympics

In August 2024, Paris, France will host the Summer Olympic Games. The table below contains information about the sports that were at some of the first five modern Olympic Games.

Year	Location	Sports
1896	Athens, Greece	athletics, road cycling, track cycling, fencing, gymnastics, shooting, swimming, tennis, weightlifting, wrestling
1900	Paris, France	archery, athletics, Basque pelota, cricket, croquet, track cycling, equestrian, fencing, football, golf, gymnastics, polo, rowing, rugby, sailing, shooting, swimming, tennis, tug-of-war, water polo
1904	St. Louis, USA	archery, athletics, boxing, track cycling, diving, fencing, football, golf, gymnastics, lacrosse, roque, rowing, swimming, tennis, tug-of-war, water polo, weightlifting, wrestling
1908	London, England	archery, athletics, boxing, track cycling, diving, fencing, figure skating, football, field hockey, gymnastics, jeu de paume, lacrosse, polo, rackets, rowing, rugby, sailing, shooting, swimming, tennis, tug-of-war, water motorsports, water polo, wrestling
1912	Stockholm, Sweden	athletics, road cycling, diving, equestrian, fencing, football, gymnastics, modern pentathlon, rowing, sailing, shooting, swimming, tennis, tug-of-war, water polo, wrestling

(a) Create a graph that displays the number of sports in each of the first five modern Olympic Games.

(b) State five observations/conclusions from your graph.

Theme Data Management

Problem of the Week

Problem B and Solution

The First Five Modern Olympics

Problem

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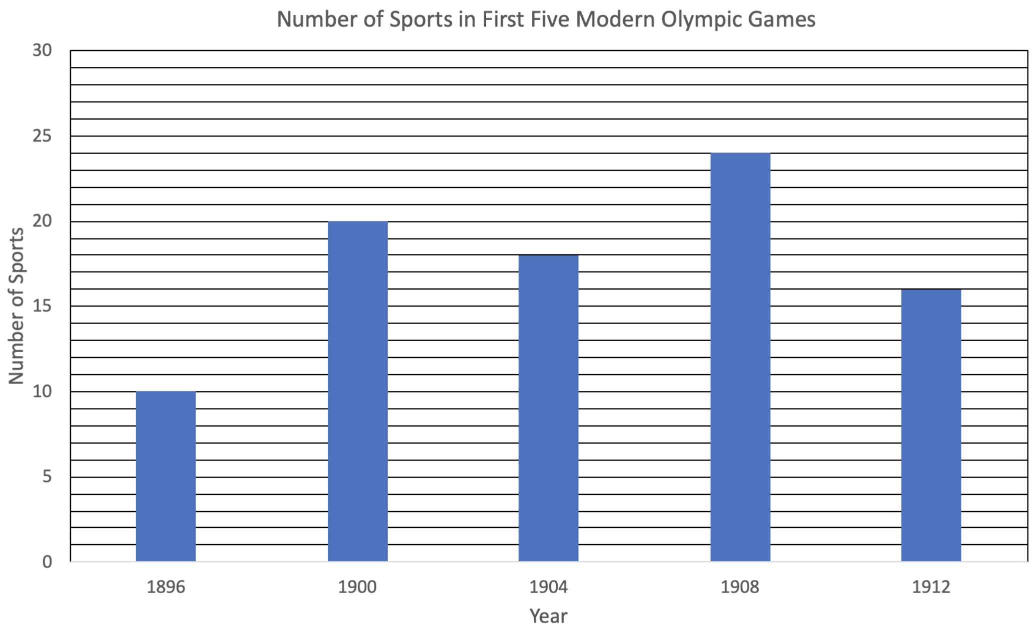
(a) Create a graph that displays the number of sports in each of the first five modern Olympic Games.

(b) State five observations/conclusions from your graph.

Solution

(a) Answers may vary. A bar graph displaying this information is shown below.



(b) Observations gleaned from the graphs will vary. Here are some possible observations.

- The fewest number of sports occurred in the year 1896.
- The most number of sports occurred in the year 1908.
- Other than 1896 and 1908, there were about the same number of sports (between 16 and 20).
- The number of sports doubled from 1896 to 1900.
- 18 is the median number of sports. (Since there are five bars, the third highest bar is the median.) This occurred in the year 1904.
- There doesn't seem to be a steady increase in the number of events, so it would be hard to predict how many events future games would hold.

Problem of the Week

Problem B

Introducing Olympic Sports

In August 2024, Paris, France will host the Summer Olympic Games. At these Olympics, breaking (also known as breakdancing) will be included for the first time. Over the years, the sports at the Summer Olympics have changed. The following tables show new sports at the Summer Olympics in the past 40 years, along with the year in which they were first introduced.

Sport	Year
Artistic Swimming	1984
Badminton	1992
Baseball	1992
Beach Volleyball	1996
BMX Freestyle	2020
BMX Racing	2008
Breaking	2024

Sport	Year
Karate	2020
Marathon Swimming	2008
Mountain Biking	1996
Rhythmic Gymnastics	1984
Rugby Sevens	2016
Skateboarding	2020
Softball	1996

Sport	Year
Sport Climbing	2020
Surfing	2020
Table Tennis	1988
Taekwondo	2000
Trampoline	2000
Triathlon	2000
3×3 Basketball	2020

(a) Organize and represent this data in a graph to show how many new sports were introduced each year.

(b) Create an infographic about new sports at the Summer Olympics using information from the table and/or your graph.

Theme Data Management

Problem of the Week
Problem B and Solution
Introducing Olympic Sports

Problem

In August 2024, Paris, France will host the Summer Olympic Games. At these Olympics, breaking (also known as breakdancing) will be included for the first time. Over the years, the sports at the Summer Olympics have changed. The following tables show new sports at the Summer Olympics in the past 40 years, along with the year in which they were first introduced.

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Surfing	2020
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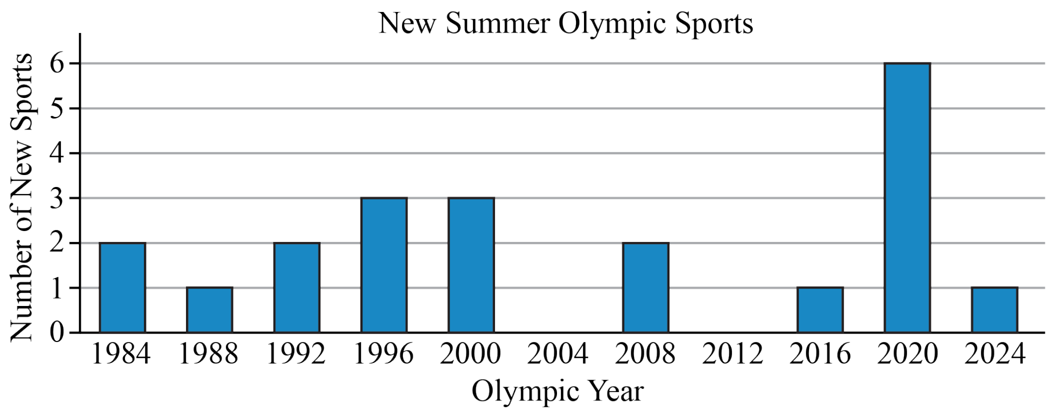
Taekwondo	2000
Trampoline	2000
Triathlon	2000
3×3 Basketball	2020

(a) Organize and represent this data in a graph to show how many new sports were introduced each year.

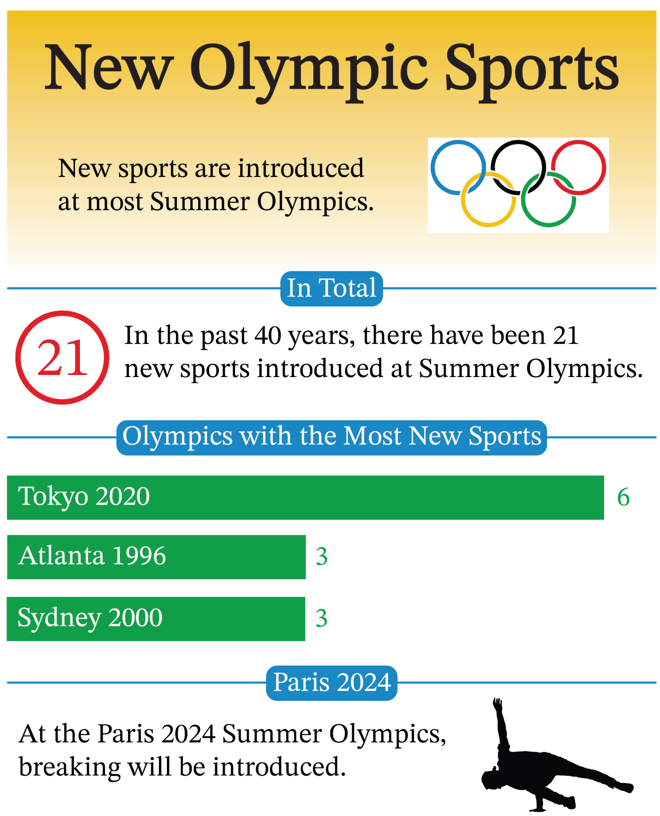
(b) Create an infographic about new sports at the Summer Olympics using information from the table and/or your graph.

Solution

(a) A bar graph can be used to show how many new sports were introduced each year. This is shown below.



(b) An example of an infographic is shown.



Geometry & Measurement (G)

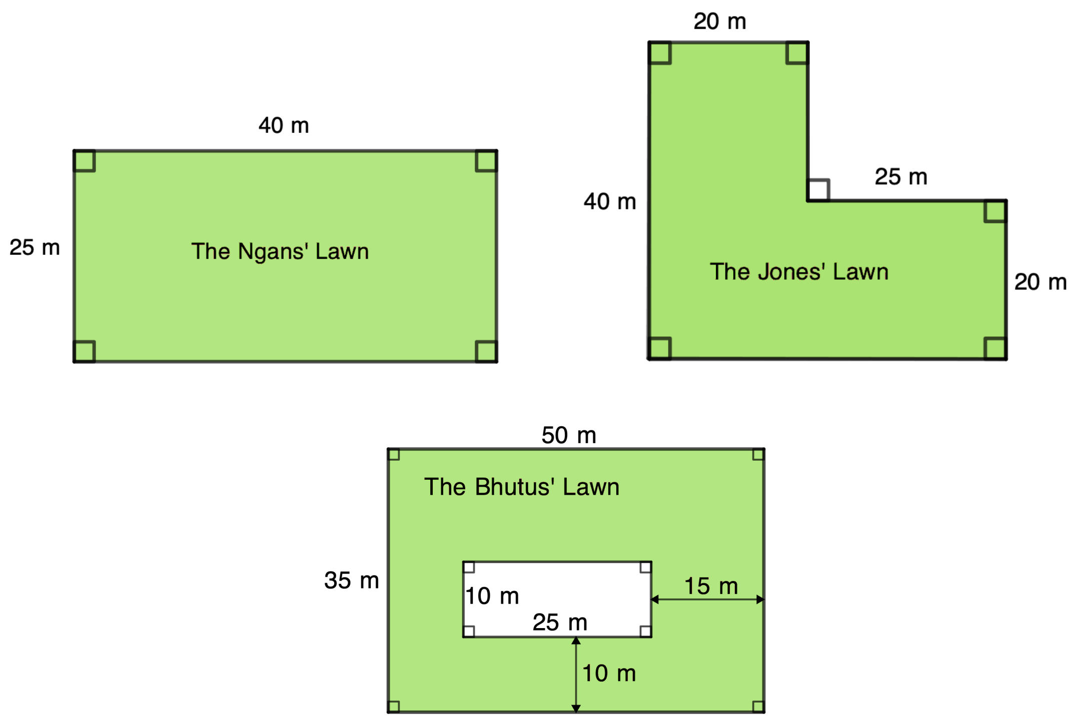
Take me to the cover

Problem of the Week

Problem B

Don't Get Mowed Over!

Jon has a grass cutting business. Three of the lawns that he cuts are shown. The lawns are shaded in green and all angles are right angles. His lawnmower can cut a swath of width 1 metre.



(a) Which lawn will take the longest to cut? Explain your reasoning.

(b) His lawnmower travels at 3 km per hour. What area of lawn, in m^2, can he cut in one hour?

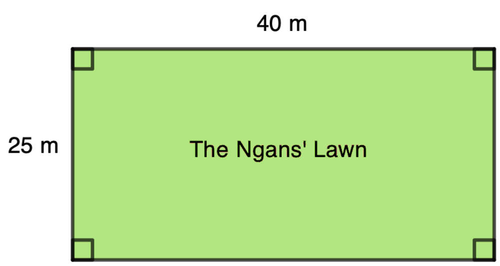
(c) How long, in minutes, will it take him to cut each lawn?

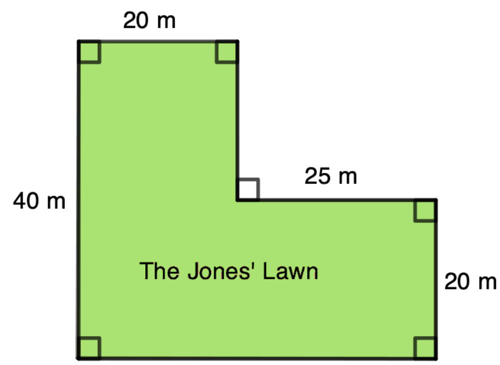
Themes Geometry & Measurement, Number Sense

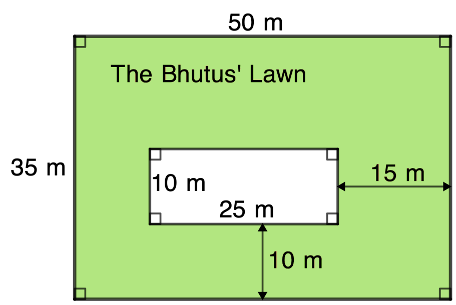
Problem of the Week
Problem B and Solution
Don't Get Mowed Over!

Problem

Jon has a grass cutting business. Three of the lawns that he cuts are shown. The lawns are shaded in green and all angles are right angles. His lawnmower can cut a swath of width 1 metre.







(a) Which lawn will take the longest to cut? Explain your reasoning.

(b) His lawnmower travels at 3 km per hour. What area of lawn, in m^2, can he cut in one hour?

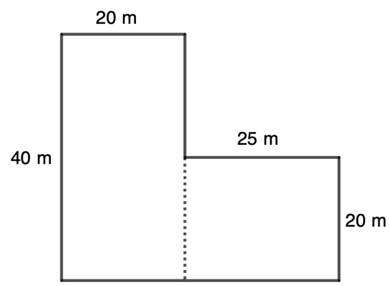
(c) How long, in minutes, will it take him to cut each lawn?

Solution

(a) Since each lawn is composed of regions that have integer lengths, in metres, and since the lawnmower can cut a swath of width 1 metre, we can compare the areas of the lawns to determine which will take the longest to cut.

The Ngans' lawn is a rectangle measuring 25 m $\times$ 40 m. Its total area is therefore 25 m $\times$ 40 m $= 1000\,\text{m}^2$.

The Jones' lawn can be divided into two smaller rectangles as shown.



The total area of the Jones' lawn is equal to the sum of the area of the rectangle on the left and the area of the rectangle on the right. The rectangle on the left has area equal to $40\text{ m} \times 20\text{ m} = 800\text{ m}^2$. The rectangle on the right has area equal to $20\text{ m} \times 25\text{ m} = 500\text{ m}^2$. Thus, the total area of the Jones' lawn is equal to $800\text{ m}^2 + 500\text{ m}^2 = 1300\text{ m}^2$.

The area of the Bhutus' lawn can be found by finding the area of the outer rectangle and subtracting the area of the inner rectangle. The area of the outer rectangle is equal to $50\text{ m} \times 35\text{ m} = 1750\text{ m}^2$. The area of the inner rectangle is equal to $10\text{ m} \times 25\text{ m} = 250\text{ m}^2$. Thus, the total area of the Bhutus' lawn is equal to $1750\text{ m}^2 - 250\text{ m}^2 = 1500\text{ m}^2$.

Since the Bhutus' lawn has the largest area, it will take the longest to cut.

NOTE: To determine the area of the Bhutus' lawn, we could have alternatively divided the lawn into smaller rectangles, and summed the areas of those rectangles.

(b) Jon's mower is 1 m wide and it travels at 3 km/h, or 3000 m/h. Therefore, he can cut $1\text{ m} \times 3000\text{ m} = 3000\text{ m}^2$ in one hour.

(c) The Ngans' lawn has area equal to 1000 m^2. Thus, it would take $1000 \div 3000 = 0.333$ (or $\frac{1}{3}$) of an hour, which is $\frac{1}{3} \times 60 = 20$ minutes to cut the lawn.

The Jones' lawn has area equal to 1300 m^2. It would take $1300 \div 3000 = 0.4333$ (or $\frac{13}{30}$) of an hour, which is $\frac{13}{30} \times 60 = 26$ minutes to cut the lawn.

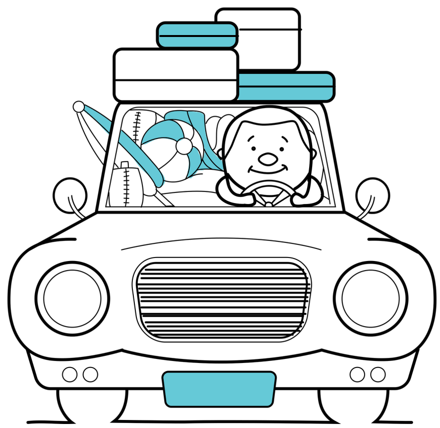
The Bhutus' lawn has area equal to 1500 m^2. It would take $1500 \div 3000 = 0.5$ (or $\frac{1}{2}$) of an hour, which is $\frac{1}{2} \times 60 = 30$ minutes to cut the lawn.

Problem of the Week
Problem B
Road Trip

Mr. Sand is going on a trip to the beach. The total distance to the beach is 263 km. His car has a 60 L gas tank and can travel 640 000 m on that tank of gas.

Suppose that there are two service stations available to Mr. Sand. Station A charges $40 for 25 L of gas, while Station B charges $51 for 30 L of gas.

Determine the cost of the gas for his trip if he fills up at Station A versus the cost if he fills up at Station B. Which is the more economical?



Themes Geometry & Measurement, Number Sense

Problem of the Week
Problem B and Solution
Road Trip

Problem

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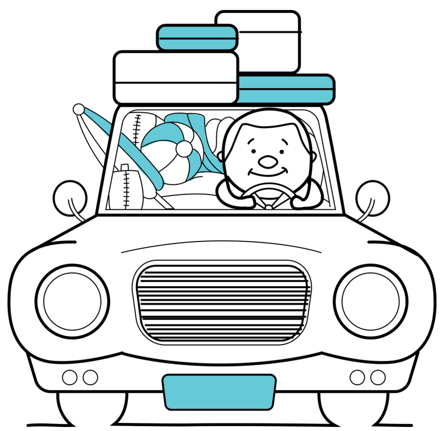


Solution

If his vehicle has a 60 L gas tank and will travel 640 000 m or 640 km on one full tank, then he is using $60 \div 640 = 0.09375$ L of gas per km.

Since the distance to the beach is 263 km, then this trip will take $263 \times 0.09375 \approx 24.656$ L of gas.

For Station A:

The cost is \$40 for 25 L. Therefore, the gas will cost $\frac{40}{25} = \$1.60$ per L.

Thus, the cost of the trip for Station A is $24.656 \times \$1.60 = \$39.45$.

For Station B:

The cost is \$51 for 30 L. Therefore, the gas will cost $\frac{51}{30} = \$1.70$ per L.

Thus, the cost of the trip for Station B is $24.656 \times \$1.70 = \$41.92$.

Therefore, Station A is more economical than Station B.

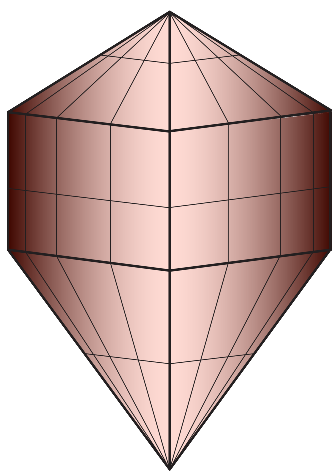
NOTE: Since the gas at Station A costs less per L than at Station B, then using gas from Station A will always cost less than using gas from Station B.

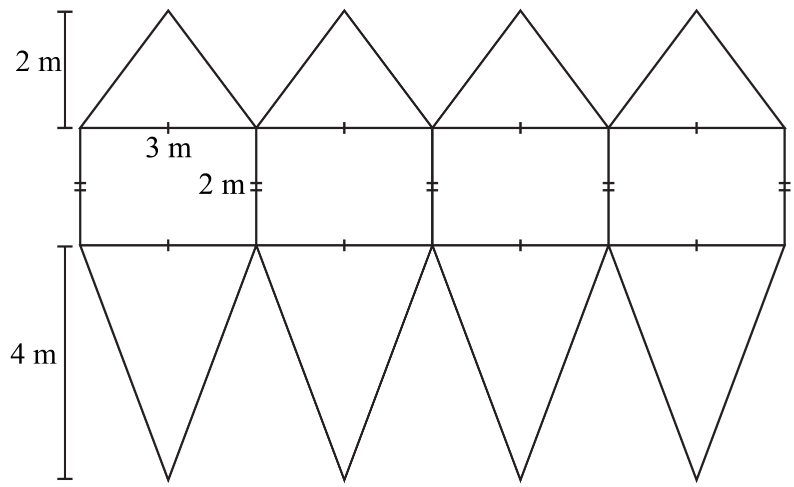
Problem of the Week

Problem B

How to Net a Balloon

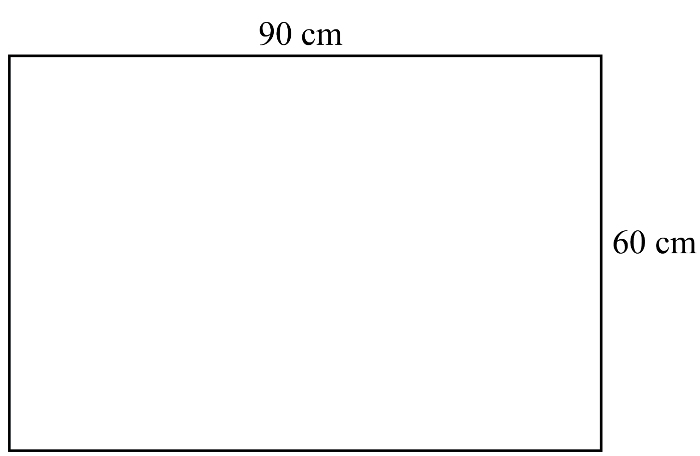
In 1783, the Montgolfier brothers launched the first hot air balloon flight in history, using a balloon that they created out of fabric and paper. The flight was short but successful. For an art exhibit, Vijay used cardboard to create a model of a balloon inspired by the Montgolfier brothers' balloon. Vijay's model, as well as its net, are shown below. Note that these diagrams are not drawn to scale.





(a) Calculate the total area of cardboard Vijay used in his model. This is also called the *surface area* of Vijay's model.

(b) Suppose you have a sheet of paper measuring 90 cm by 60 cm. Draw a net for a balloon that you could make using this sheet of paper. Write the dimensions for each shape on your net.



Theme Geometry & Measurement

Problem of the Week
Problem B and Solution
How to Net a Balloon

Problem

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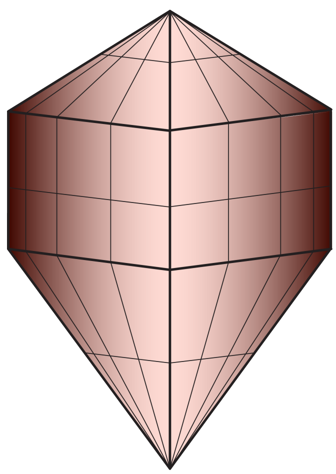

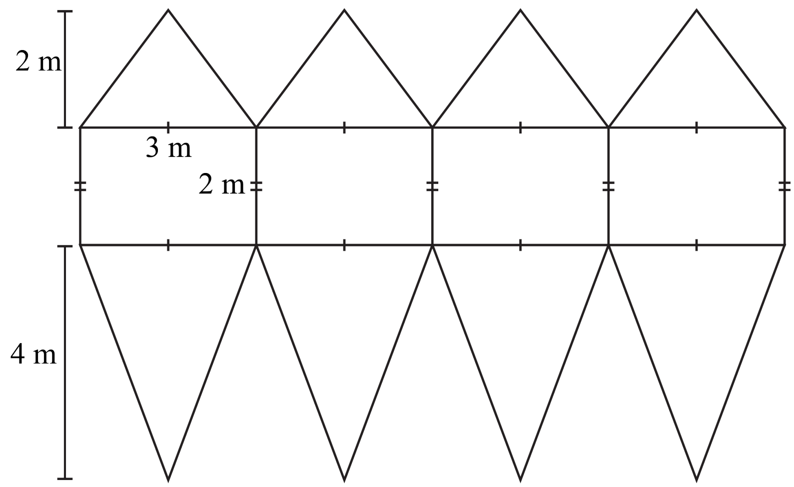

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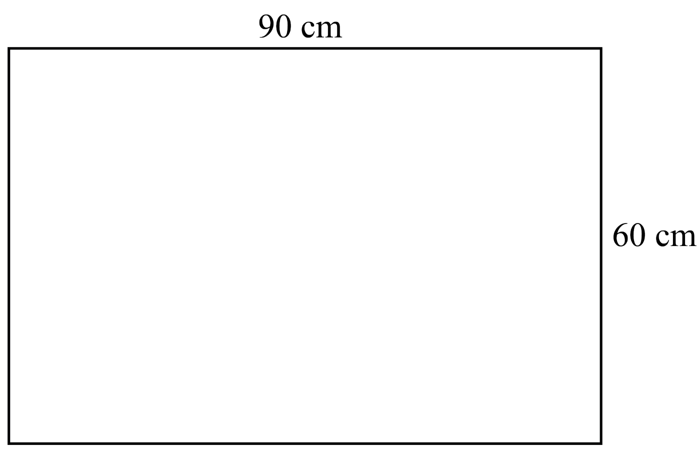


Solution

(a) To calculate the total area we notice that the net has four triangles on top, four rectangles, and four triangles on the bottom. We will calculate the area of each of these shapes separately and then add them together.

$$\begin{aligned}\text{Area of one top triangle} &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 3 \times 2\\ &= \frac{1}{2} \times 6\\ &= 3 \text{ m}^2\end{aligned}$$

Since there are four top triangles, the total area is $4 \times 3 = 12$ m^2.

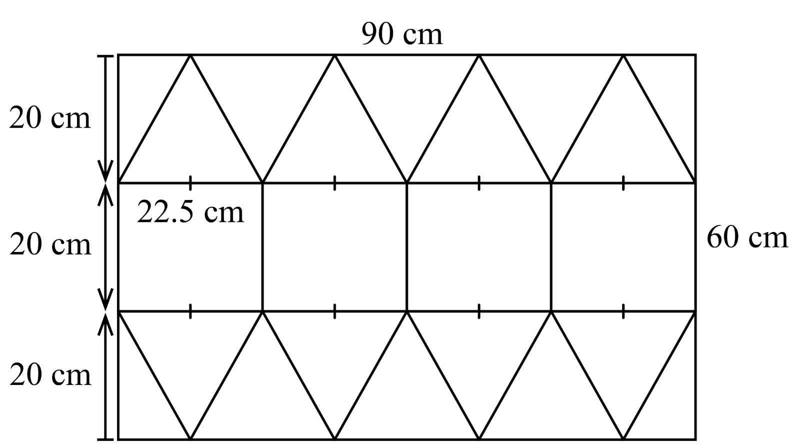
$$\begin{aligned}\text{Area of one rectangle} &= \text{length} \times \text{width}\\ &= 3 \times 2\\ &= 6 \text{ m}^2\end{aligned}$$

Since there are four rectangles, the total area is $4 \times 6 = 24$ m^2.

$$\begin{aligned}\text{Area of one bottom triangle} &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 3 \times 4\\ &= \frac{1}{2} \times 12\\ &= 6 \text{ m}^2\end{aligned}$$

Since there are four bottom triangles, the total area is $4 \times 6 = 24$ m^2.
Therefore, the total area of cardboard used is $12 + 24 + 24 = 60$ m^2.

(b) There are many possible nets. Here is one.



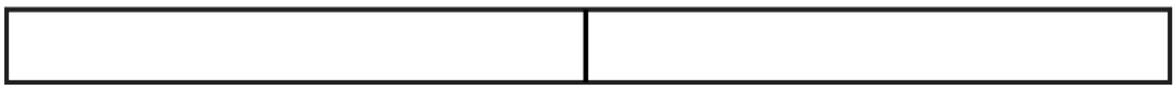
Problem of the Week

Problem B

Wrecked Tangles

Gaby drew a rectangle and called it *Diagram 1*.

She then drew a rectangle divided into two equal parts, and called *Diagram 2*.



She then counted the total number of rectangles in *Diagram 2*. There is 1 rectangle on the left, 1 rectangle on the right, and the original whole rectangle, which makes 3 rectangles in total.

Gaby then drew a rectangle divided into three equal parts, called *Diagram 3*.

Gaby counted a total of 6 rectangles in *Diagram 3*. Can you confirm this?

(a) Gaby continued drawing diagrams by dividing a rectangle into equal parts. *Diagram 4* is divided into four equal parts, *Diagram 5* is divided into five equal parts, and so on. Complete the table by determining the total number of rectangles in each diagram. Draw the diagrams to help you, and then look for a pattern in the total number of rectangles.

Diagram Number	Total Number of Rectangles
1	1
2	3
3	6
4	
5	
6	

(b) Use the pattern you found in part (a) to predict the total number of rectangles in *Diagram 12*.

Themes Algebra, Geometry & Measurement

Problem of the Week

Problem B and Solution

Wrecked Tangles

Problem

Gaby drew a rectangle and called it *Diagram 1*.

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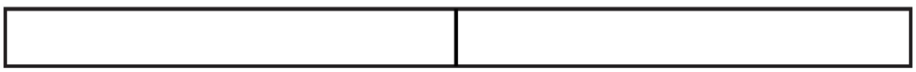

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Diagram Number	Total Number of Rectangles
1	1
2	3
3	6
4	
5	
6	

(b) Use the pattern you found in part (a) to predict the total number of rectangles in *Diagram 12*.

Solution

(a) For each rectangle, we will assign the smallest rectangle a length of one unit.

Diagram 4 is a rectangle divided into 4 equal parts. In this diagram, there are 4 rectangles of length one unit, 3 of length two units, 2 of length three units, and 1 of length four units. This is a total of $4 + 3 + 2 + 1 = 10$ rectangles.

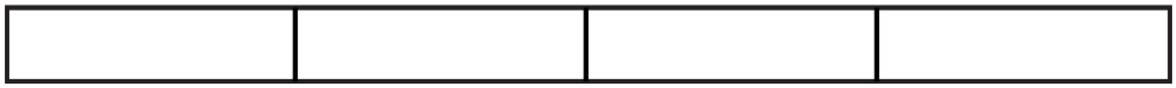
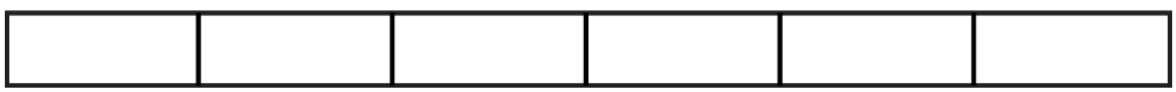


Diagram 5 is a rectangle divided into 5 equal parts. In this diagram, there are 5 rectangles of length one unit, 4 of length two units, 3 of length three units, 2 of length four units, and 1 of length five units. This is a total of $5 + 4 + 3 + 2 + 1 = 15$ rectangles.

Diagram 6 is a rectangle divided into 6 equal parts. In this diagram, there are 6 rectangles of length one unit, 5 of length two units, 4 of length three units, 3 of length four units, 2 of length five units, and 1 of length six units. This is a total of $6 + 5 + 4 + 3 + 2 + 1 = 21$ rectangles.



Now we see a pattern. The total number of rectangles for each diagram is equal to the sum of the diagram number and all the whole numbers smaller than it. Alternatively, the total number of rectangles for each diagram is equal to the diagram number plus the previous number of rectangles. So, the total number of rectangles in *Diagram 7* is equal to $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$, or $21 + 7 = 28$.

(b) Using the pattern from part (a), the total number of rectangles in *Diagram 12* is equal to $12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 78$, or $28 + 8 + 9 + 10 + 11 + 12 = 78$.

Problem of the Week
Problem B
Temperature Conversions

Two common units to measure temperature are degrees Celsius and degrees Fahrenheit. From time to time, we need to convert temperatures from degrees Celsius to degrees Fahrenheit.

(a) The exact conversion from degrees Celsius to degrees Fahrenheit is as follows:

Step 1: Take the temperature in degrees Celsius and multiply by 1.8.

Step 2: Take the result from Step 1 and add 32.

Using this exact conversion, convert the following temperatures in degrees Celsius to degrees Fahrenheit. The first has been done for you.

Temperature in degrees Celsius	Temperature in degrees Fahrenheit
100	212
30	
20	
10	
0	

(b) Sometimes when we want to convert between degrees Celsius and degrees Fahrenheit, we don't have a pencil and paper or calculator nearby. In that case, using an approximation and mental math can be helpful. One way to approximate the conversion from degrees Celsius to degrees Fahrenheit is as follows:

Step 1: Take the temperature in degrees Celsius and multiply by 2.

Step 2: Take the result from Step 1 and add 30.

Using this approximate conversion, convert the following temperatures in degrees Celsius to degrees Fahrenheit. The first has been done for you.

Temperature in degrees Celsius	Approximate temperature in degrees Fahrenheit
100	230
30	
20	
10	
0	

(c) Did any of the approximate conversions in part (b) give the same temperature as the exact conversion in part (a)?

Extension:
If you let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit, can you write formulas for the conversions in parts (a) and (b)?

Themes Geometry & Measurement, Number Sense

Problem of the Week

Problem B and Solution

Temperature Conversions

Problem

Two common units to measure temperature are degrees Celsius and degrees Fahrenheit. From time to time, we need to convert temperatures from degrees Celsius to degrees Fahrenheit.

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30	
20	
10	
0	

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Temperature in degrees Celsius	Approximate temperature in degrees Fahrenheit
100	230
30	
20	
10	
0	

(c) Did any of the approximate conversions in part (b) give the same temperature as the exact conversion in part (a)?

Extension:
If you let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit, can you write formulas for the conversions in parts (a) and (b)?

Solution

(a) The completed table is below.

Temperature in degrees Celsius	Temperature in degrees Fahrenheit
100	212
30	86
20	68
10	50
0	32

(b) The completed table is below.

Temperature in degrees Celsius	Approximate temperature in degrees Fahrenheit
100	230
30	90
20	70
10	50
0	30

(c) The conversion of 10°C to 50°F gave the same temperature when using both the approximate and exact conversions.

Extension:
In part (a), we have $F = 1.8 \times C + 32$.
In part (b), we have $F = 2 \times C + 30$.

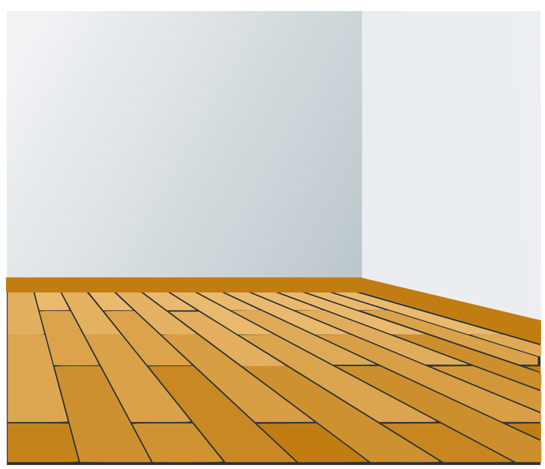
Problem of the Week
Problem B
Jafar's New Floor

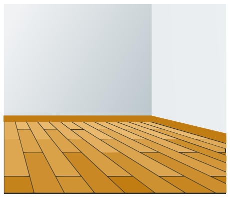
Jafar is laying new hardwood flooring in his rectangular living room, which has an area of 66 m^2. Each box of flooring has 8 identical wooden planks, and each plank has an area of 0.2 m^2.

(a) Assuming that there is no waste, how many planks will he need to cover the floor of his living room?

(b) If Jafar wants to buy an extra 10% for waste, how many boxes of flooring does he need to buy?

(c) If each box costs $74.50 and sales tax is 15%, what will be the total cost of the flooring in part (b)?





Problem of the Week
Problem B and Solution
Jafar's New Floor

Problem

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(a) Assuming that there is no waste, how many planks will he need to cover the floor of his living room?

(b) If Jafar wants to buy an extra 10% for waste, how many boxes of flooring does he need to buy?

(c) If each box costs \$74.50 and sales tax is 15%, what will be the total cost of the flooring in part (b)?

Solution

(a) The area of the living room floor is 66 m^2 and each plank has an area of 0.2 m^2. So the total number of planks needed is $66 \div 0.2 = 330$.

(b) The extra amount is 10% of 330, which is $0.10 \times 330 = 33$ planks.

So in total Jafar wants to buy $330 + 33 = 363$ planks. Since there are 8 planks in each box, the number of boxes required is $363 \div 8 = 45.375$. Therefore, he should buy 46 boxes.

(c) Jafar wants to buy 46 boxes, and each box costs \$74.50. The total cost before tax is $46 \times \$74.50 = \$3427$.

The amount of tax is 15% of \$3427, which is $0.15 \times \$3427 = \$514.05$.

Therefore, the total cost of the flooring is $\$3427 + \$514.05 = \$3941.05$.

Problem of the Week

Problem B

Not a Tetris™ Game

On June 6, 2024, the puzzle game Tetris™ will be 40 years old! The game of Tetris™ uses pieces called "tetrominoes", which are shapes composed of four identical squares, like the ones given at the bottom of this page. This problem is inspired by Tetris™.

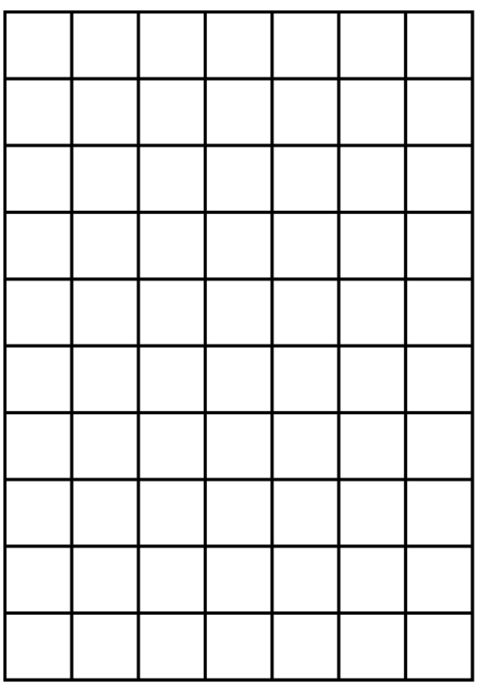
In this problem, tetromino pieces are to be placed in a grid according to the following rules:

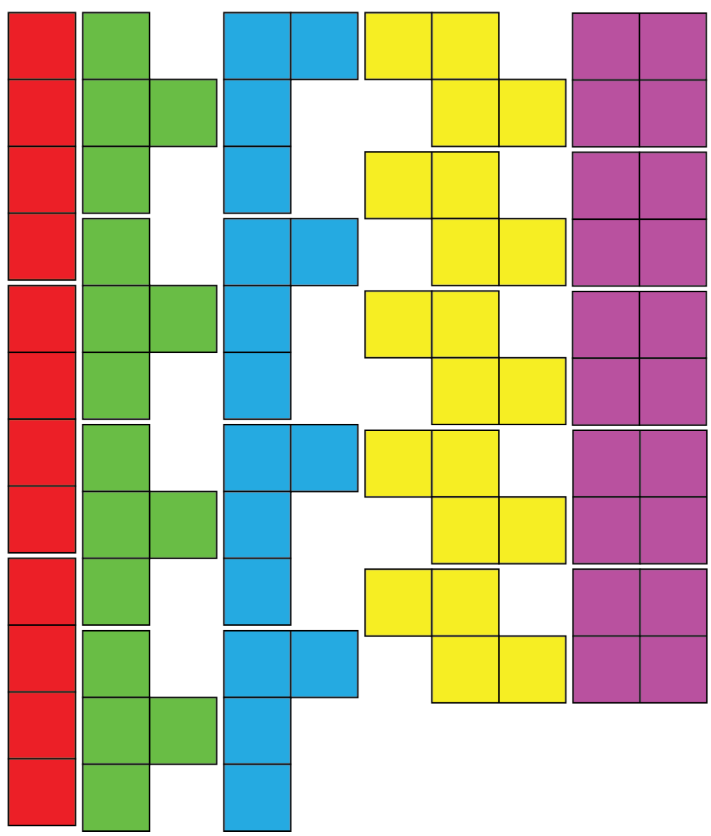
1. Pieces may be rotated or reflected (flipped over).
2. Pieces may **not** overlap each other and each square in a piece must be placed directly on top of a square in the grid.
3. Only the given pieces may be used, but you do not need to use all of them.

The goal is to cover as many squares in the grid as possible with the pieces. Is it possible to cover all the squares in the given grid? Explain why or why not.

When answering this question, you may find it helpful to cut out the given tetrominoes and place them on the grid.

Not printing this page? You can use our interactive worksheet.





Problem of the Week

Problem B and Solution

Not a Tetris$^{\text{TM}}$ Game

Problem

On June 6, 2024, the puzzle game Tetris$^{\text{TM}}$ will be 40 years old! The game of Tetris$^{\text{TM}}$ uses pieces called "tetrominoes", which are shapes composed of four identical squares, like the ones given at the bottom of this page. This problem is inspired by Tetris$^{\text{TM}}$.

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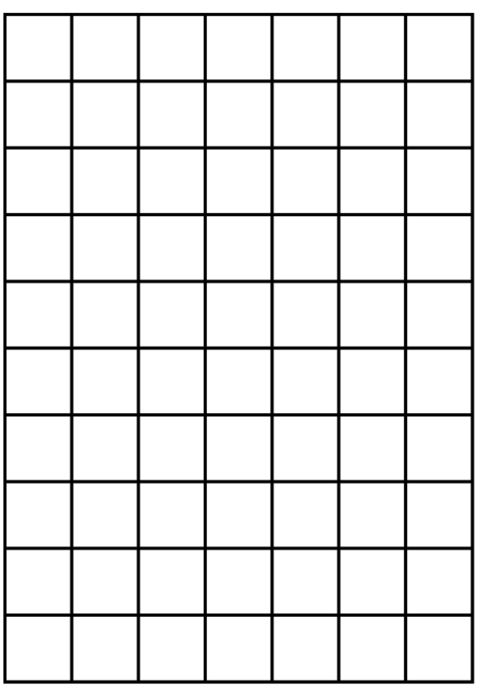
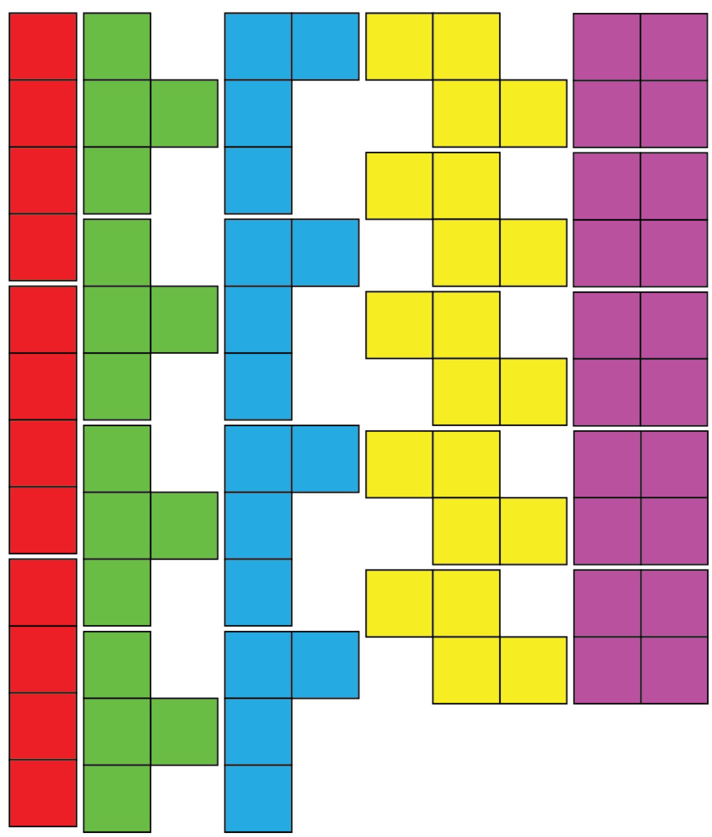
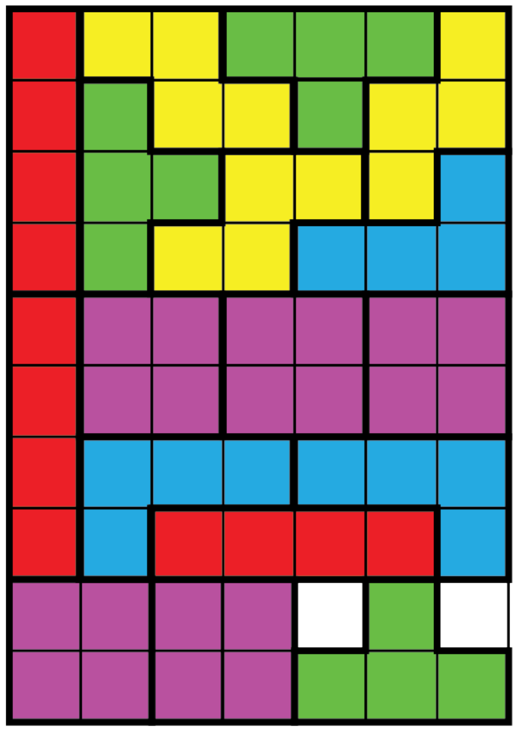




Solution

The grid has a total of $7 \times 10 = 70$ squares and each piece has 4 squares. Since $70 \div 4 = 17.5$, which is not a whole number, that tells us that 70 is not a multiple of 4. So it is not possible to cover all the squares in the grid. At most, we would be able to cover $17 \times 4 = 68$ of the squares. One such possibility is shown.



Problem of the Week
Problem B
Shelby's Code

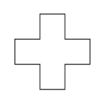
Shelby is using block coding to draw different shapes.

(a) Her first program is shown. What shape will be drawn after running this program?

```
on start
pen down
move forward 10 steps
turn right 90 degrees
move forward 10 steps
turn right 90 degrees
move forward 10 steps
turn right 90 degrees
move forward 10 steps
turn right 90 degrees
```

(b) Using the given blocks, write a program to draw the same shape as Shelby's program, using fewer blocks. Notice that some blocks contain an empty box to be filled with a number.

Blocks	Program
move forward ☐ steps turn right ☐ degrees repeat ☐ times	on start pen down

(c) Using the given blocks, write a program to draw the following shape: 

You may use a block more than once.

Blocks	Program
move forward ☐ steps turn right ☐ degrees turn left ☐ degrees repeat ☐ times	on start pen down

Themes Computational Thinking, Geometry & Measurement

Problem of the Week

Problem B and Solution

Shelby's Code

Problem

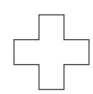
Shelby is using block coding to draw different shapes.

(a) Her first program is shown. What shape will be drawn after running this program?

```
on start
pen down
move forward 10 steps
turn right 90 degrees
move forward 10 steps
turn right 90 degrees
move forward 10 steps
turn right 90 degrees
move forward 10 steps
turn right 90 degrees
```

(b) Using the given blocks, write a program to draw the same shape as Shelby's program, using fewer blocks. Notice that some blocks contain an empty box to be filled with a number.

Blocks	Program
move forward ☐ steps turn right ☐ degrees repeat ☐ times	on start pen down

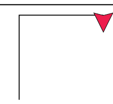
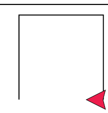
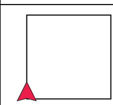
(c) Using the given blocks, write a program to draw the following shape: 

You may use a block more than once.

Blocks	Program
move forward ☐ steps turn right ☐ degrees turn left ☐ degrees repeat ☐ times	on start pen down

Solution

(a) This program will draw a square. The table below shows the drawing progress and position of the pen as we trace through the program.

Program Section	Drawing Progress
on start pen down	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	

(b) By using the repeat block, we can use fewer blocks in the program, as shown.

```
on start
pen down
repeat 4 times
    move forward 10 steps
    turn right 90 degrees
```

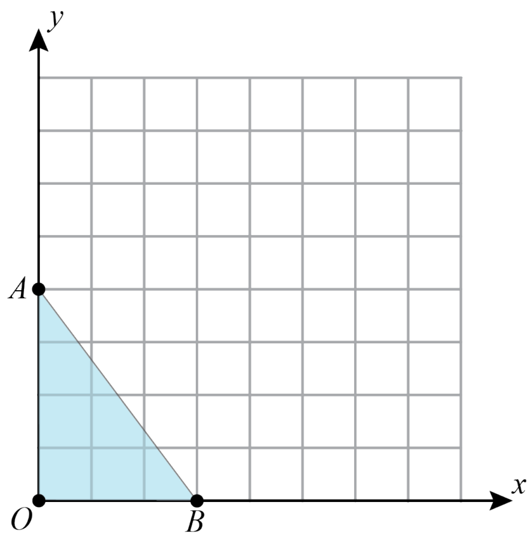
(c) There are several possible programs, depending on where the pen starts, and how many repeat blocks are used. Two programs are shown.

```
on start
pen down
repeat 4 times
    move forward 10 steps
    turn right 90 degrees
    move forward 10 steps
    turn right 90 degrees
    move forward 10 steps
    turn left 90 degrees
```

```
on start
pen down
repeat 4 times
    repeat 2 times
        move forward 10 steps
        turn right 90 degrees
    move forward 10 steps
    turn left 90 degrees
```

Problem of the Week
Problem B
Triangular Fun

Work through the parts that follow using the following coordinate plane, where grid lines are spaced 1 unit apart.



(a) Label the coordinates of the points A, O, and B.

(b) Plot point C on the y-axis so that OC is twice the length of OA. Then plot point D on the x-axis so that OD is twice the length of OB. Label the coordinates of points C and D.

(c) Show that the area of $\triangle COD$ is four times the area of $\triangle AOB$. To show this, you may use your diagram or an area formula.

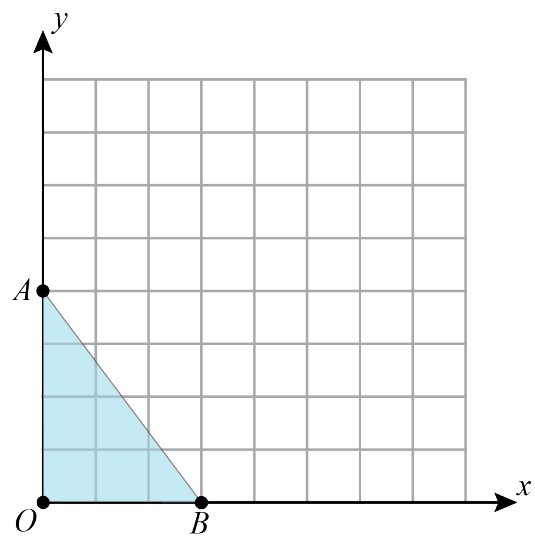
Extension: In general, if you double the lengths of the two perpendicular sides of any right-angled triangle, will the area of the new triangle be four times the area of the original triangle? Explain.

Not printing this page? You can use our interactive worksheet.

Problem of the Week
Problem B and Solution
Triangular Fun

Problem

Work through the parts that follow using the following coordinate plane, where grid lines are spaced 1 unit apart.



(a) Label the coordinates of the points A, O, and B.

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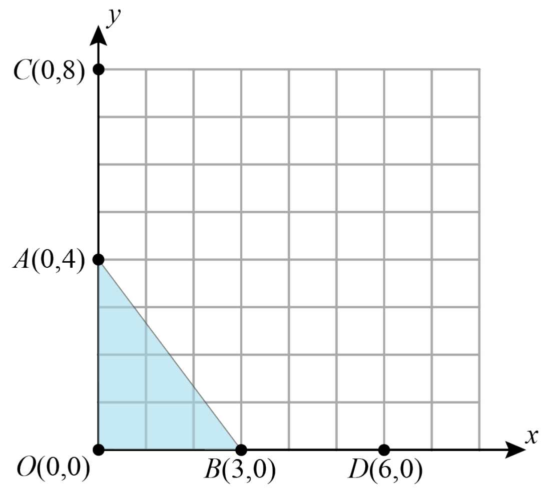
Extension: In general, if you double the lengths of the two perpendicular sides of any right-angled triangle, will the area of the new triangle be four times the area of the original triangle? Explain.

Not printing this page? You can use our interactive worksheet.

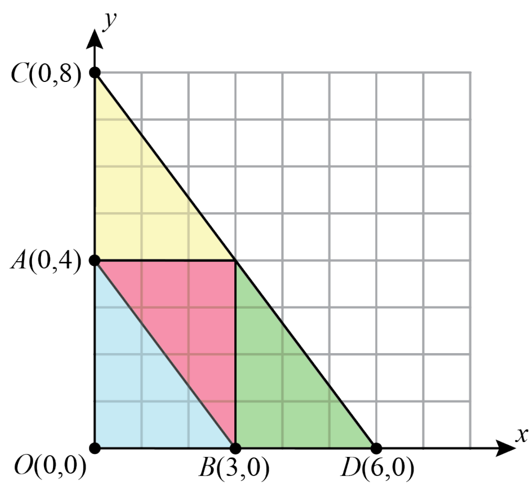
Solution

(a) The coordinates are $A(0,4)$, $O(0,0)$, and $B(3,0)$.

(b) Points C and D are plotted on the diagram, and their coordinates are $C(0,8)$ and $D(6,0)$, as shown.



(c) The diagram shows $\triangle COD$ divided into four smaller right-angled triangles, each congruent to $\triangle AOB$, with perpendicular sides of length 3 and 4. Therefore, the area of $\triangle COD$ is four times the area of $\triangle AOB$.



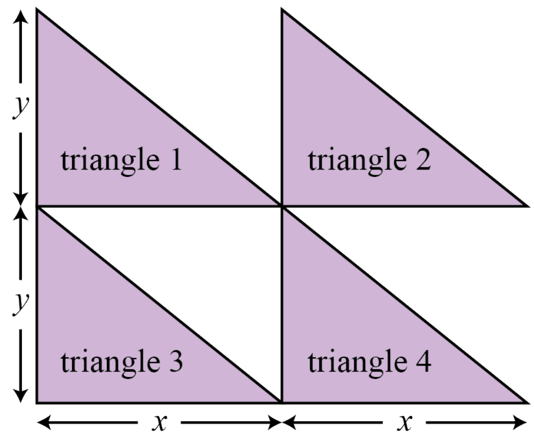
Alternatively, we can calculate the areas of $\triangle AOB$ and $\triangle COD$ using the area formula: Area = base $\times$ height $\div$ 2.

$$\begin{aligned} \text{Area of } \triangle AOB &= 3 \times 4 \div 2 \\ &= 12 \div 2 \\ &= 6 \end{aligned} \qquad \begin{aligned} \text{Area of } \triangle COD &= 6 \times 8 \div 2 \\ &= 48 \div 2 \\ &= 24 \end{aligned}$$

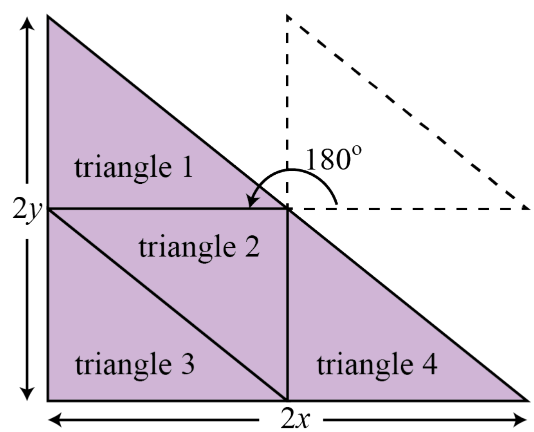
Since $6 \times 4 = 24$, the area of $\triangle COD$ is four times the area of $\triangle AOB$.

Extension Solution:

We will start with a right-angled triangle where the two perpendicular sides have lengths of x and y. We then create four copies of this triangle, numbered from 1 to 4, and arrange them as shown. The total area of the four triangles is four times the area of the original triangle.

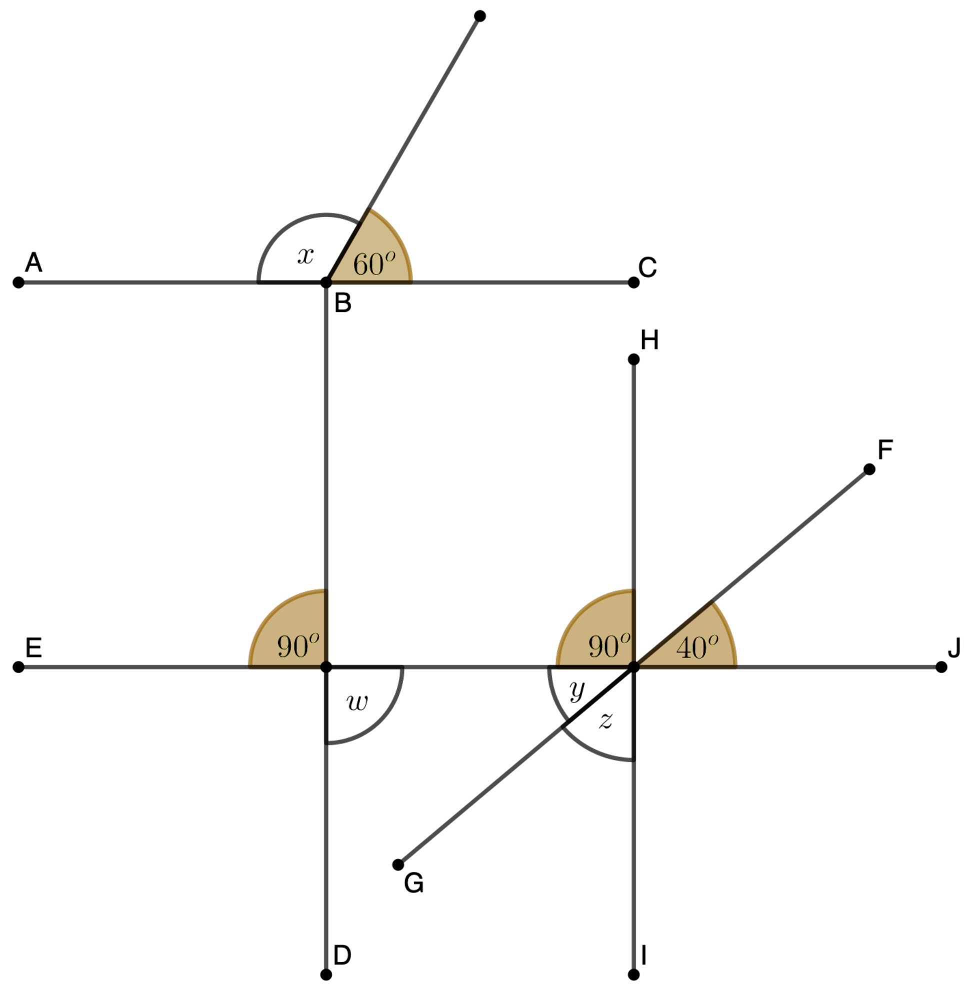


Now, if we rotate triangle 2 by 180°, the four triangles will be in the shape of a larger right-angled triangle where the lengths of the two perpendicular sides are $2x$ and $2y$. Thus, if you double the lengths of the two perpendicular sides of any right-angled triangle, the area of the new triangle will be four times the area of the original triangle.



Problem of the Week
Problem B
Angle Adventures

In the diagram below, AC, BD, EJ, HI, and FG are line segments. Determine the measure of each unknown angle w, x, y, and z.

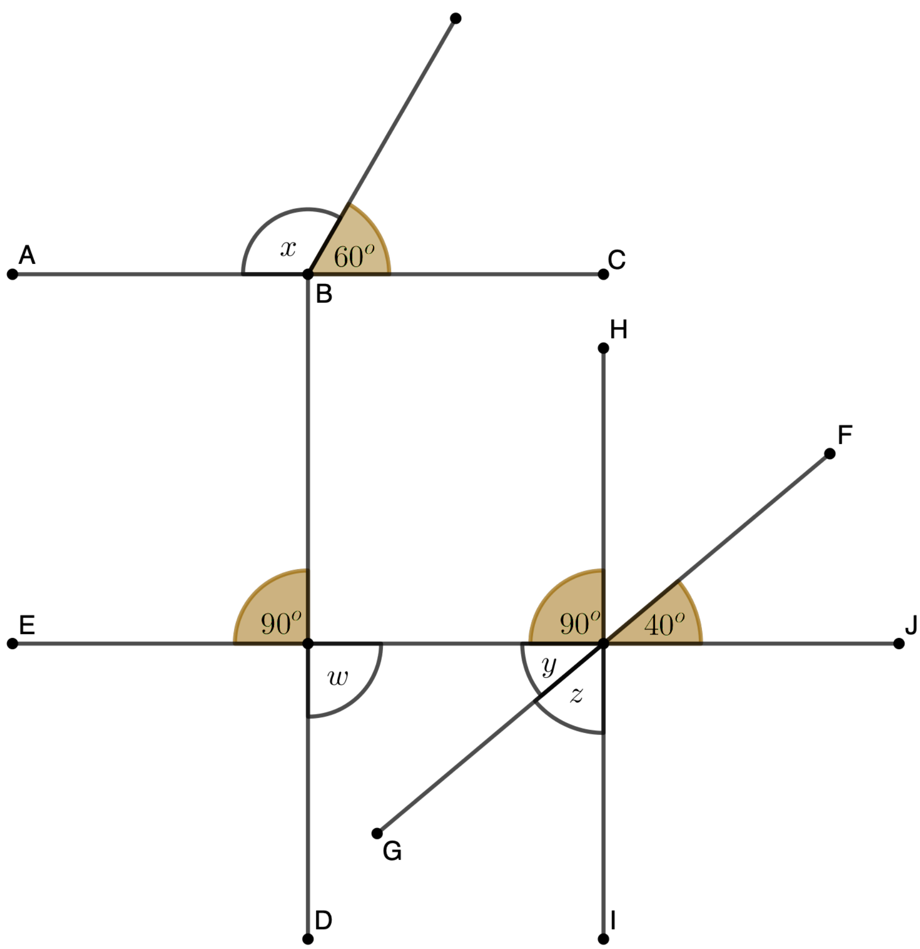


Theme Geometry & Measurement

Problem of the Week
Problem B and Solution
Angle Adventures

Problem

In the diagram below, AC, BD, EJ, HI, and FG are line segments. Determine the measure of each unknown angle w, x, y, and z.



Solution

Solution 1

Since $\angle w$ is opposite to $90°$, we know $\angle w = 90°$.

Since $\angle x$ supplementary to $60°$, we know that $\angle x = 180° - 60° = 120°$.

Since $\angle y$ is opposite to $40°$, we know $\angle y = 40°$.

We know that $90° + \angle y + \angle z = 180°$, so we must have $\angle y + \angle z = 90°$. Since $\angle y = 40°$, we have $\angle z = 50°$.

Solution 2

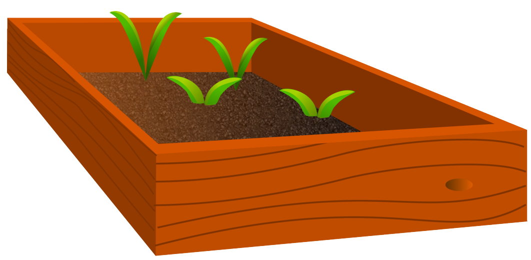
If we measure the given angles using a protractor, we will notice that the diagram is drawn to scale. Since the diagram is drawn to scale, you may use a protractor to find the angles.

Problem of the Week
Problem B
Jordyn's Garden

Jordyn's neighbourhood is building a community garden to grow some vegetables. They would like the garden bed to have an area of 48 square metres, and plan to put wooden fence boards around the edges of the garden bed. To reduce the cost of the project, they would like the garden bed to have the smallest possible perimeter.

(a) Determine the length and width of Jordyn's community garden bed. Assume the side lengths are whole numbers, in metres.

(b) The community decided to double the area of the garden bed, but would still like it to have the smallest possible perimeter. Again, assume the side lengths are whole numbers, in metres. Determine the length and width of the garden bed now.



Problem of the Week
Problem B and Solution
Jordyn's Garden

Problem
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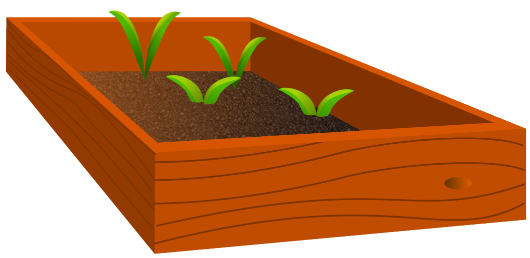


Solution

(a) Since the garden bed is in the shape of a rectangle and has an area of 48 square metres, it follows that length $\times$ width $= 48$. To determine the length and width, we need to find pairs of whole numbers that multiply to 48. These are called the factor pairs of 48, and are as follows: 1 and 48, 2 and 24, 3 and 16, 4 and 12, and 6 and 8. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

Width (metres)	Length (metres)	Perimeter (metres)
1	48	$2 \times (1 + 48) = 2 \times 49 = 98$
2	24	$2 \times (2 + 24) = 2 \times 26 = 52$
3	16	$2 \times (3 + 16) = 2 \times 19 = 38$
4	12	$2 \times (4 + 12) = 2 \times 16 = 32$
6	8	$2 \times (6 + 8) = 2 \times 14 = 28$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 8 metres and the width should be 6 metres.

(b) After they double the area of the garden bed it will have an area of $2 \times 48 = 96$ square metres. Using a similar approach to part (a), we need to find the factor pairs of 96, which are: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, and 8 and 12. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

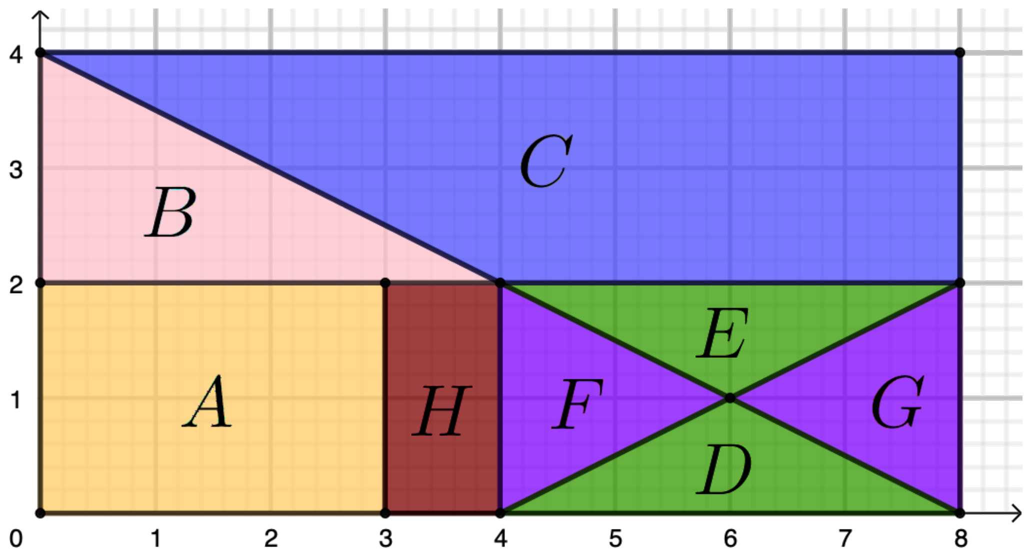
Width (metres)	Length (metres)	Perimeter (metres)
1	96	$2 \times (1 + 96) = 2 \times 97 = 194$
2	48	$2 \times (2 + 48) = 2 \times 50 = 100$
3	32	$2 \times (3 + 32) = 2 \times 35 = 70$
4	24	$2 \times (4 + 24) = 2 \times 28 = 56$
6	16	$2 \times (6 + 16) = 2 \times 22 = 44$
8	12	$2 \times (8 + 12) = 2 \times 20 = 40$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 12 metres and the width should be 8 metres.

Extension: Note that in each case, the minimum perimeter occurs for the factor pair whose positive difference is the smallest. Will this always happen? Why or why not?

Problem of the Week
Problem B
Fraction Fun

A large rectangle is in the first quadrant of the Cartesian plane with its four vertices at $(0, 0)$, $(8, 0)$, $(8, 4)$, and $(0, 4)$. It is divided into eight regions labelled A, B, C, D, E, F, G, and H, as shown.



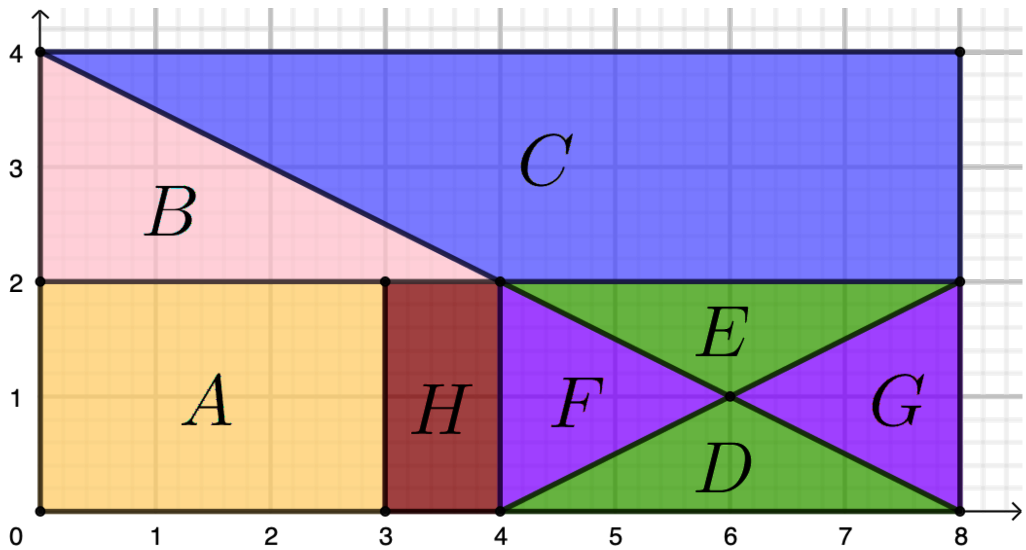
What fraction of the area of the large rectangle is the area of region A? the area of region B? the area of region C? the area of region D?

THEMES Geometry & Measurement, Number Sense

Problem of the Week
Problem B and Solution
Fraction Fun

Problem

A large rectangle is in the first quadrant of the Cartesian plane with its four vertices at $(0, 0)$, $(8, 0)$, $(8, 4)$, and $(0, 4)$. It is divided into eight regions labelled A, B, C, D, E, F, G, and H, as shown.



What fraction of the area of the large rectangle is the area of region A? the area of region B? the area of region C? the area of region D?

Solution

Solution 1

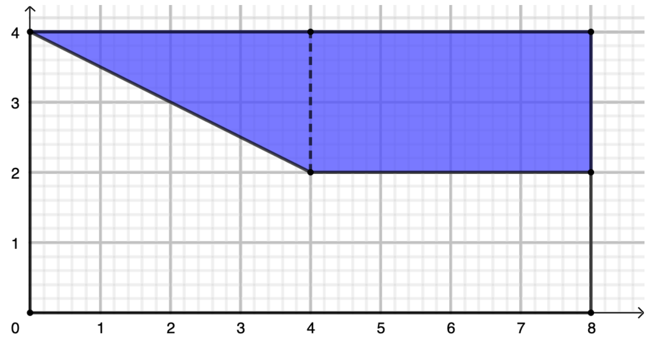
The large rectangle has a length of 8 units and a width of 4 units. Therefore, the area of the large rectangle is $8 \times 4 = 32$ square units.

Region A is a rectangle with a length of 3 units and a width of 2 units. Hence, its area is $3 \times 2 = 6$ square units. So, the area of region A is $\frac{6}{32} = \frac{3}{16}$ of the area of the large rectangle.

Region B is a triangle with a base of 4 units and height of 2 units. Hence, its area is $\frac{1}{2} \times 4 \times 2 = 4$ square units. So, the area of region B is $\frac{4}{32} = \frac{1}{8}$ of the area of the large rectangle.

Region D is a triangle with a base of 4 units and a height of 1 unit. Hence, its area is $\frac{1}{2} \times 1 \times 4 = 2$ square units. So the area of region D is $\frac{2}{32} = \frac{1}{16}$ of the area of the large rectangle.

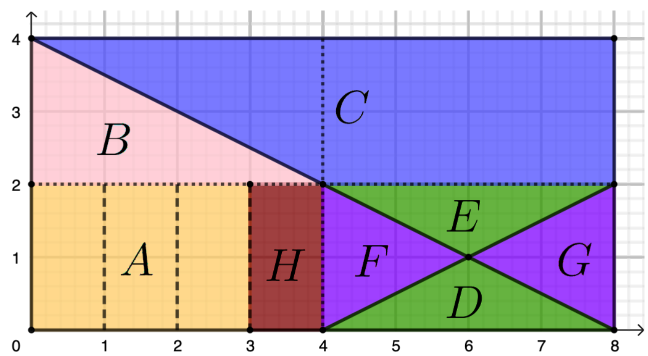
Region C is made up of a rectangle and a triangle as shown by the dashed line in the diagram below.



The rectangle has a length of 4 units and a width of 2 units. So, the area of the rectangle is $4 \times 2 = 8$ square units. The triangle has a base of 4 units and height of 2 units. So, the area of the triangle is $\frac{1}{2} \times 4 \times 2 = 4$ square units. Therefore, the area of region C is $8 + 4 = 12$ square units. Thus, the area of region C is $\frac{12}{32} = \frac{3}{8}$ of the area of the large rectangle.

Solution 2

We draw in dotted lines which divide the large rectangle into four equal parts, or quarters, and draw in dashed lines divide the lower left quarter further into quarters.

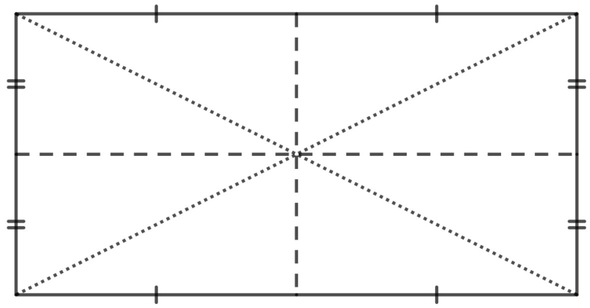


Since the dashed lines divide the lower left quarter of the rectangle further into quarters, the area of each of those four rectangles is $\frac{1}{4}$ of $\frac{1}{4}$ of the area the large rectangle, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle. Thus, the area of region A is $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$ of the area of the large rectangle.

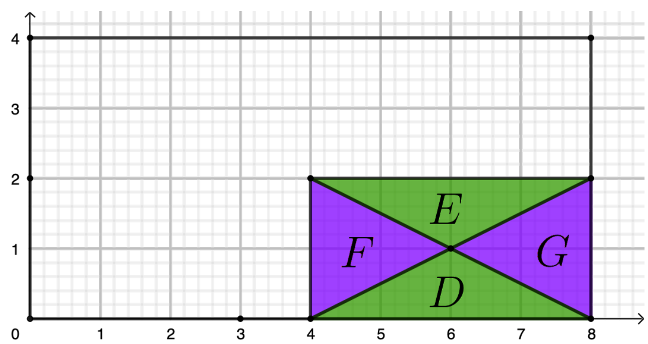
The area of region B is half of the area of the top left quarter, and so is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ of the area of the large rectangle.

The area of region C is the area of the top half of the large rectangle, minus the area of region B, which is $\frac{1}{8}$ of the large rectangle. So in total, the area of region C is $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ of the area of the large rectangle.

Note that we can divide any rectangle into 4 smaller rectangles of equal area by joining the midpoints of opposite sides of the rectangles. When we construct the two diagonals of the large rectangle, we further divide each smaller rectangle into two triangles of equal areas. So, in the diagram below, the eight smaller triangles have equal area.



In our problem, the area of region D is equal to $\frac{2}{8}$ or $\frac{1}{4}$ of the area of the lower right rectangle. Therefore, the area of the region D is $\frac{1}{4}$ of $\frac{1}{4}$, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle.

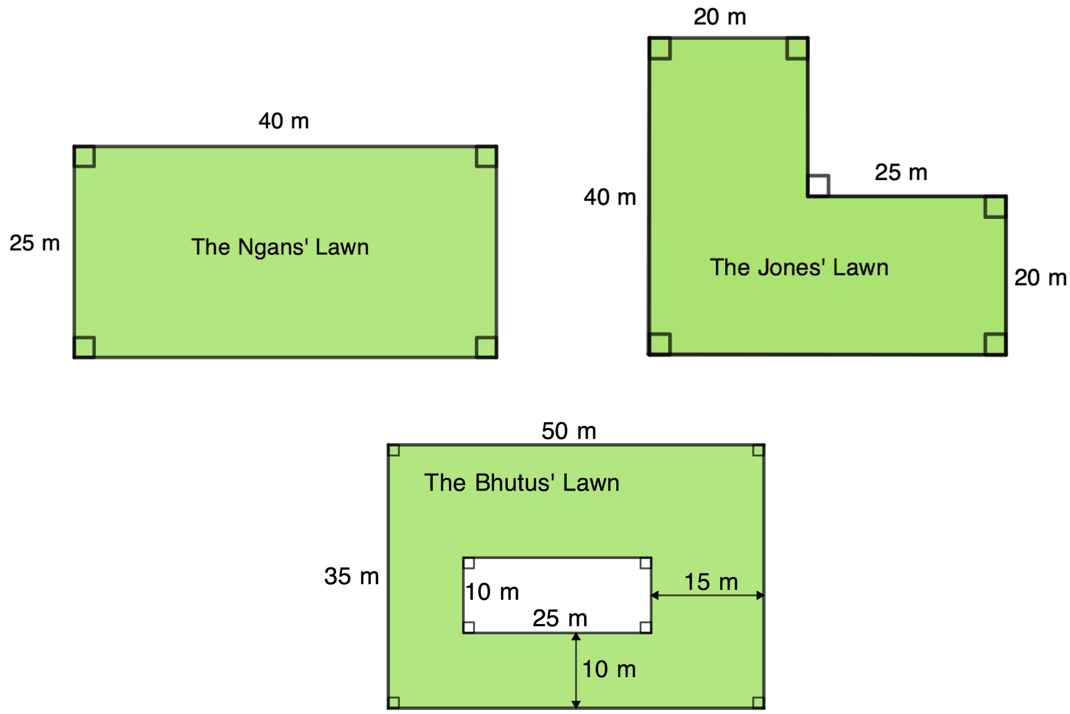


Number Sense (N)

Take me to the cover

Problem of the Week
Problem B
Don't Get Mowed Over!

Jon has a grass cutting business. Three of the lawns that he cuts are shown. The lawns are shaded in green and all angles are right angles. His lawnmower can cut a swath of width 1 metre.



(a) Which lawn will take the longest to cut? Explain your reasoning.

(b) His lawnmower travels at 3 km per hour. What area of lawn, in m^2, can he cut in one hour?

(c) How long, in minutes, will it take him to cut each lawn?

Problem of the Week

Problem B and Solution

Don't Get Mowed Over!

Problem

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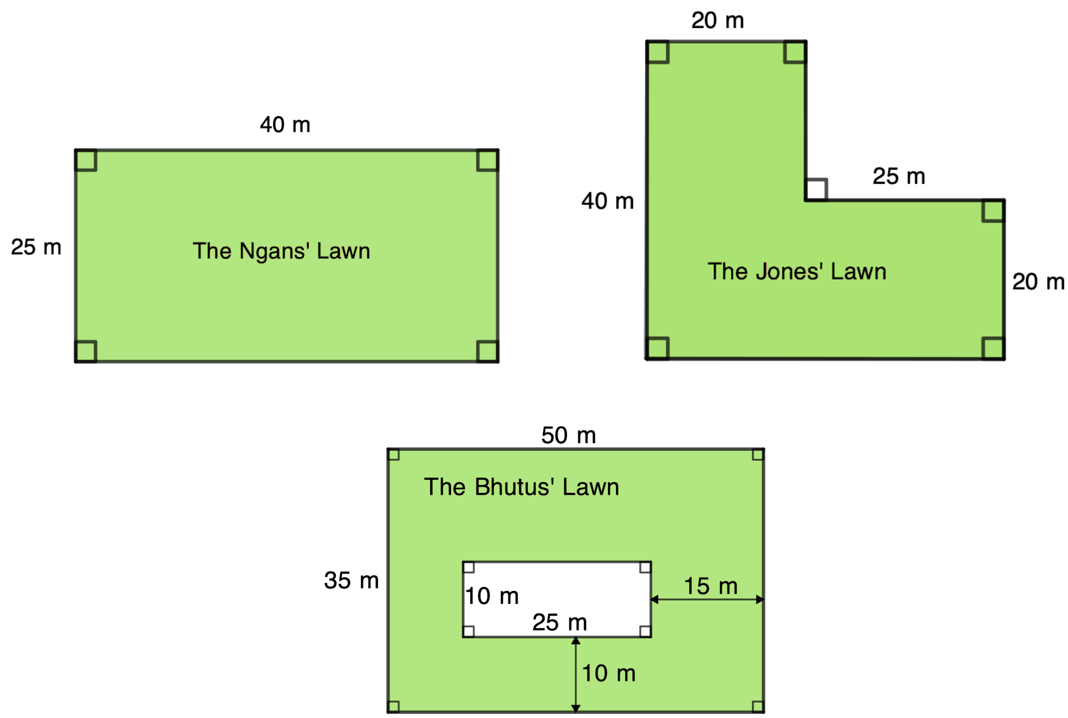


(a) Which lawn will take the longest to cut? Explain your reasoning.

(b) His lawnmower travels at 3 km per hour. What area of lawn, in m^2, can he cut in one hour?

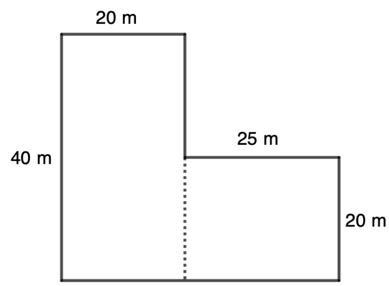
(c) How long, in minutes, will it take him to cut each lawn?

Solution

(a) Since each lawn is composed of regions that have integer lengths, in metres, and since the lawnmower can cut a swath of width 1 metre, we can compare the areas of the lawns to determine which will take the longest to cut.

The Ngans' lawn is a rectangle measuring 25 m $\times$ 40 m. Its total area is therefore 25 m $\times$ 40 m $= 1000\,\text{m}^2$.

The Jones' lawn can be divided into two smaller rectangles as shown.



The total area of the Jones' lawn is equal to the sum of the area of the rectangle on the left and the area of the rectangle on the right. The rectangle on the left has area equal to $40 \text{ m} \times 20 \text{ m} = 800 \text{ m}^2$. The rectangle on the right has area equal to $20 \text{ m} \times 25 \text{ m} = 500 \text{ m}^2$. Thus, the total area of the Jones' lawn is equal to $800 \text{ m}^2 + 500 \text{ m}^2 = 1300 \text{ m}^2$.

The area of the Bhutus' lawn can be found by finding the area of the outer rectangle and subtracting the area of the inner rectangle. The area of the outer rectangle is equal to $50 \text{ m} \times 35 \text{ m} = 1750 \text{ m}^2$. The area of the inner rectangle is equal to $10 \text{ m} \times 25 \text{ m} = 250 \text{ m}^2$. Thus, the total area of the Bhutus' lawn is equal to $1750 \text{ m}^2 - 250 \text{ m}^2 = 1500 \text{ m}^2$.

Since the Bhutus' lawn has the largest area, it will take the longest to cut.

NOTE: To determine the area of the Bhutus' lawn, we could have alternatively divided the lawn into smaller rectangles, and summed the areas of those rectangles.

(b) Jon's mower is 1 m wide and it travels at 3 km/h, or 3000 m/h. Therefore, he can cut $1 \text{ m} \times 3000 \text{ m} = 3000 \text{ m}^2$ in one hour.

(c) The Ngans' lawn has area equal to 1000 m^2. Thus, it would take $1000 \div 3000 = 0.333$ (or $\frac{1}{3}$) of an hour, which is $\frac{1}{3} \times 60 = 20$ minutes to cut the lawn.

The Jones' lawn has area equal to 1300 m^2. It would take $1300 \div 3000 = 0.4333$ (or $\frac{13}{30}$) of an hour, which is $\frac{13}{30} \times 60 = 26$ minutes to cut the lawn.

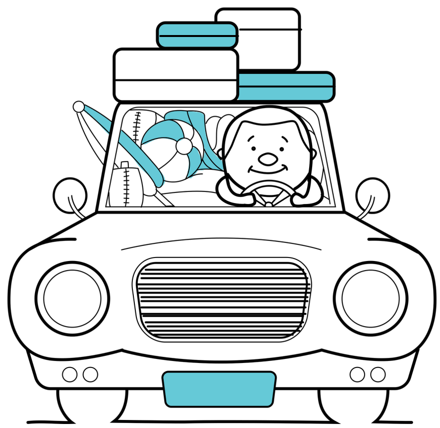
The Bhutus' lawn has area equal to 1500 m^2. It would take $1500 \div 3000 = 0.5$ (or $\frac{1}{2}$) of an hour, which is $\frac{1}{2} \times 60 = 30$ minutes to cut the lawn.

Problem of the Week
Problem B
Road Trip

Mr. Sand is going on a trip to the beach. The total distance to the beach is 263 km. His car has a 60 L gas tank and can travel 640 000 m on that tank of gas.

Suppose that there are two service stations available to Mr. Sand. Station A charges \$40 for 25 L of gas, while Station B charges \$51 for 30 L of gas.

Determine the cost of the gas for his trip if he fills up at Station A versus the cost if he fills up at Station B. Which is the more economical?



Themes Geometry & Measurement, Number Sense

Problem of the Week

Problem B and Solution

Road Trip

Problem

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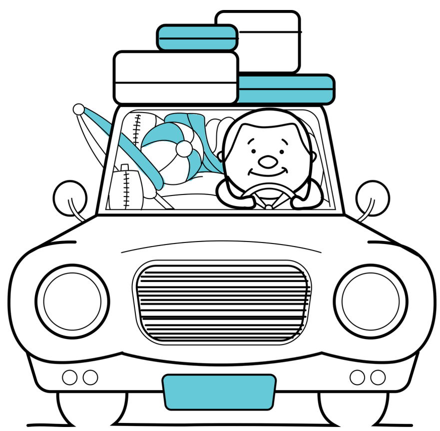


Solution

If his vehicle has a 60 L gas tank and will travel 640 000 m or 640 km on one full tank, then he is using $60 \div 640 = 0.09375$ L of gas per km.

Since the distance to the beach is 263 km, then this trip will take $263 \times 0.09375 \approx 24.656$ L of gas.

For Station A:

The cost is \$40 for 25 L. Therefore, the gas will cost $\frac{40}{25} = \$1.60$ per L.

Thus, the cost of the trip for Station A is $24.656 \times \$1.60 = \$39.45$.

For Station B:

The cost is \$51 for 30 L. Therefore, the gas will cost $\frac{51}{30} = \$1.70$ per L.

Thus, the cost of the trip for Station B is $24.656 \times \$1.70 = \$41.92$.

Therefore, Station A is more economical than Station B.

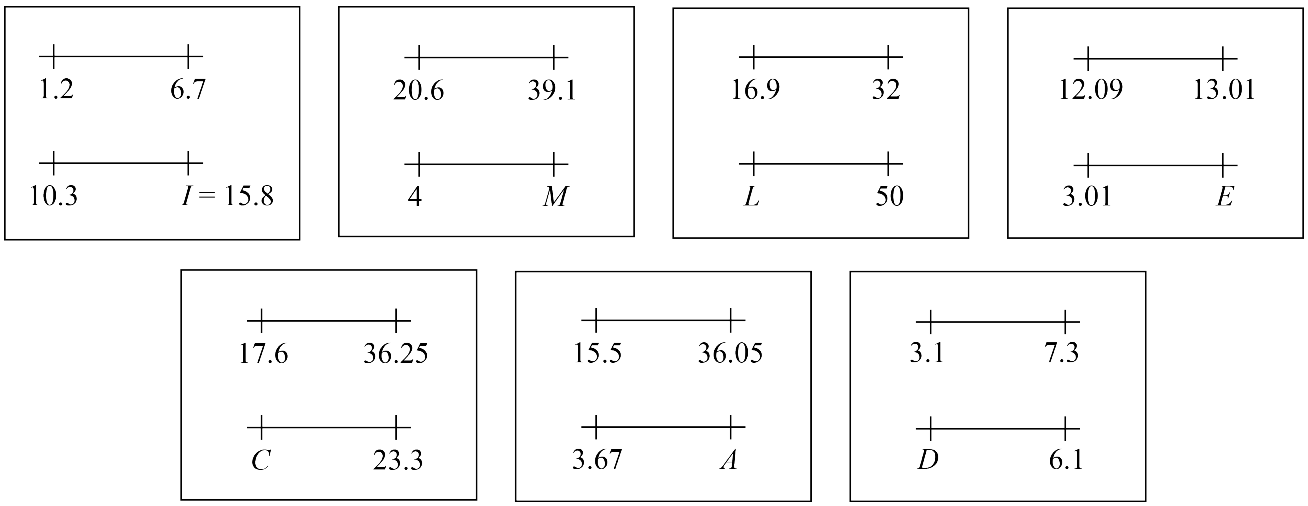
NOTE: Since the gas at Station A costs less per L than at Station B, then using gas from Station A will always cost less than using gas from Station B.

Problem of the Week

Problem B

Line Up These Letters!

Determine the number corresponding to each letter so that the two number lines in each box have the same range. The first letter has been done for you.



Then write the numbers in the table below, in order from least to greatest, along with their corresponding letters in the row below. The corresponding letters will spell a mathematical term!

Numbers				15.8			
Letters				*I*			

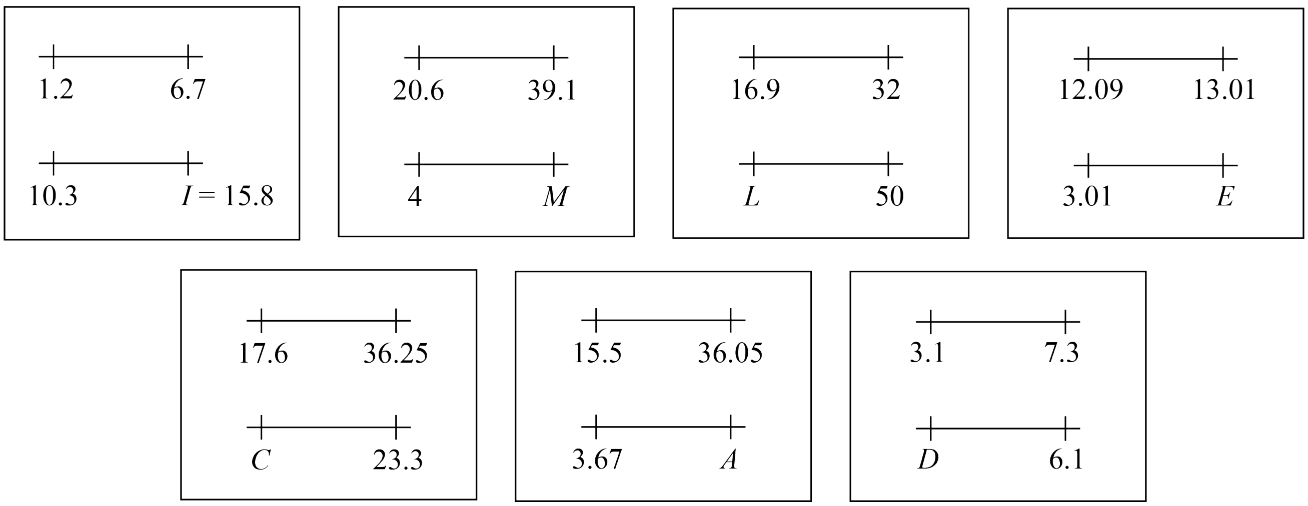
Problem of the Week

Problem B and Solution

Line Up These Letters!

Problem

Determine the number corresponding to each letter so that the two number lines in each box have the same range. The first letter has been done for you.



Then write the numbers in the table below, in order from least to greatest, along with their corresponding letters in the row below. The corresponding letters will spell a mathematical term!

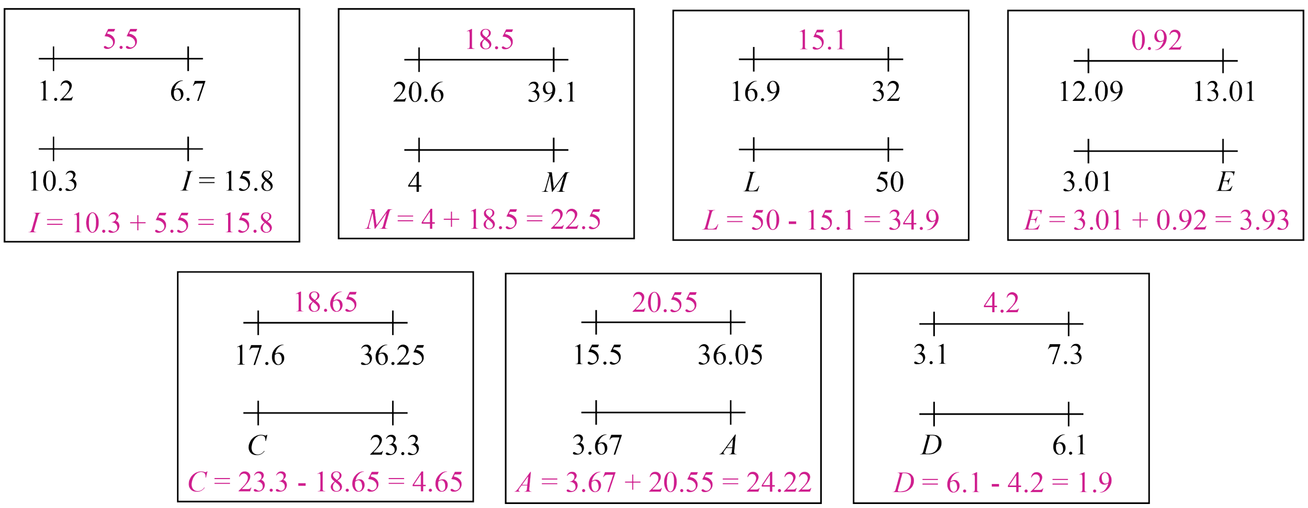
Numbers				15.8			
Letters				*I*			

Solution

In each case, the range of the number line with the given end points must first be determined. These are shown above the top number line in each box. For example, the range of the number line in the first box is $6.7 - 1.2 = 5.5$.

Then the unknown endpoint on the bottom number line can be determined by either addition or subtraction. For example, in the first box,
$I = 10.3 + 5.5 = 15.8$.

The calculations for each box are shown in the following diagram.



When the numbers are written in order from least to greatest, along with their corresponding letters, the letters spell the word DECIMAL, as shown.

Numbers	1.9	3.93	4.65	15.8	22.5	24.22	34.9
Letters	D	E	C	I	M	A	L

Problem of the Week
Problem B
Let the Leaves Fall Where They May

Masha lives in a house on a forested lot. The trees are lovely, but in the fall there is a lot of raking that needs to be done.

It took him 10 minutes to rake and fill his first bag of leaves, which had a mass of 11 kg. Over the course of the fall, he collected 35 bags of leaves.

(a) If he assumes that each bag has the same mass as the first bag, what is the expected total mass of all the leaves he collected?

(b) If he assumes that his time to rake and fill each bag was the same as for the first bag, what is his total expected time to collect all the leaves?

It actually took him 8 hours to do all his raking, and according to the weigh scale at the Environmental Transfer Station, he had 425 kg of leaves in total.

(c) What was the actual mean (average) mass of each bag of leaves? Round your answer to the nearest tenth of a kg.

(d) What was the actual mean (average) time that it took for him to rake the leaves for each bag? Round your answer to the nearest minute.

(e) TO THINK ABOUT: Was predicting his raking workload based on his first bag a good approach?

THEME NUMBER SENSE

Problem of the Week
Problem B and Solution
Let the Leaves Fall Where They May

Problem

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(e) To Think About: Was predicting his raking workload based on his first bag a good approach?

Solution

(a) If he collected 35 bags that each weighed 11 kg, the total mass of leaves he collected was 35×11 kg $= 385$ kg.

(b) If he collected 35 bags and took 10 minutes to collect the leaves for each one, his total time would have been 35 bags $\times$ 10 min/bag $= 350$ minutes or 5 hours and 50 minutes.

(c) The actual mean mass of each bag was $\dfrac{425}{35} \approx 12.1$ kg.

(d) It took him 8 hours or $8 \times 60 = 480$ minutes to rake the 35 bags. The mean time was $\dfrac{480}{35} \approx 14$ min/bag, rounded to the nearest minute.

(e) Answers will vary. Estimating based on what you know is usually a good way to make predictions. He might have gotten a better estimate if he had used the first few bags, rather than just the first.

Problem of the Week

Problem B

Take It In Stride

Donovan and his friend Usain each think they're faster than the other. Donovan has been practicing his running, and is hoping to beat Usain in a 0.75 km race. When Usain runs, his stride length is 120 cm.

(a) If the span of Donovan's running stride length is $\frac{2}{3}$ that of Usain's, how many more strides than Usain will he have to take in order to run the 0.75 km distance?

(b) It takes both Donovan and Usain exactly 255 seconds to run 0.75 km. Who takes more strides per second? Explain your reasoning.

(c) How many strides per second did Donovan take? Round your answer to one decimal place.

Theme Number Sense

Problem of the Week

Problem B and Solution

Take It In Stride

Problem

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(b) It takes both Donovan and Usain exactly 255 seconds to run 0.75 km. Who takes more strides per second? Explain your reasoning.

(c) How many strides per second did Donovan take? Round your answer to one decimal place.

Solution

(a) Since Usain's stride length is 120 cm, Donovan's stride length is

$$\frac{2}{3} \times 120 = \frac{2 \times 120}{3} = \frac{240}{3} = 80\,\text{cm}$$

So to run 0.75 km, or 75 000 cm, Donovan will take $75\,000 \div 80 = 937.5$ strides. Since he cannot take partial strides, this means he will take 938 strides.
For Usain, he will take $75\,000 \div 120 = 625$ strides.
Thus, Donovan will take $938 - 625 = 313$ more strides than Usain.

(b) Since Donovan has the smaller stride length, to run the same distance in the same time he must take more strides per second.

(c) The actual number of strides per second for Donovan was $938 \div 255 \approx 3.7$ strides per second.

Note: The actual number of strides per second for Usain was $625 \div 255 \approx 2.5$ strides per second.

Problem of the Week

Problem B

Rounding Equivalents

Sometimes the process of rounding numbers produces interesting results. For example, if you round the number 39.99 to the nearest ten, you get 40, if you round it to the nearest whole number, you get 40, and if you round it to the nearest tenth, you get 40.0. Notice that you get the same numerical value when rounding 39.99 to the nearest ten, whole number, and tenth.

(a) Find a number less than 100 with two decimal places such that when you round to the nearest tenth you get the same numerical value as when you round to the nearest whole number.

(b) Find a number less than 100 with two decimal places such that when you round to the nearest tenth you get the same numerical value as when you round to the nearest ten.

(c) Find the smallest number between 99 and 100 that has two decimal places that rounds to the same numerical value when you round to the nearest tenth, whole number, ten, and hundred.

Theme Number Sense

Problem of the Week
Problem B and Solution
Rounding Equivalents

Problem

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(a) Find a number less than 100 with two decimal places such that when you round to the nearest tenth you get the same numerical value as when you round to the nearest whole number.

(b) Find a number less than 100 with two decimal places such that when you round to the nearest tenth you get the same numerical value as when you round to the nearest ten.

(c) Find the smallest number between 99 and 100 that has two decimal places that rounds to the same numerical value when you round to the nearest tenth, whole number, ten, and hundred.

Solution

(a) Answers will vary. One possible answer is 18.96.
Rounding 18.96 to the nearest tenth yields 19.0.
Rounding 18.96 to the nearest whole number yields 19.

(b) Answers will vary. One possible answer is 20.03.
Rounding 20.03 to the nearest tenth yields 20.0.
Rounding 20.03 to the nearest ten yields 20.

(c) When the number is between 99 and 100, it must be 100 when rounded to the nearest hundred.
Therefore, the number rounded to the nearest tenth would be 100.0. The numbers less than 100 that have two decimal places that round to 100.0 when rounded to the nearest tenth are

$$99.99,\ 99.98,\ 99.97,\ 99.96 \text{ and } 99.95$$

(Note that 99.94 will round to 99.9 when rounded to the nearest tenth.) Therefore, the smallest of these numbers is 99.95.
Notice that 99.95 does indeed yield 100.0 or 100 when rounded to the nearest tenth, whole number, ten, or hundred.

Problem of the Week

Problem B

Birthday Presents

Paula and Quinn are twins. Their friends have saved money to buy them some gifts. Aleta has saved \$30, Benji has saved \$25, and Carolina has saved \$28. They have done some research on possible gifts and their costs:

- Both Paula and Quinn would like a pair of warm socks that are \$7 per pair.
- Paula wants some guitar picks for \$6 and a yo-yo that is \$7.
- Quinn would like some art pencils that are \$21.
- Both like chocolate mints that are \$4 per box.

(a) If they combine their savings, how much will be left after purchasing these gifts? You may ignore taxes.

(b) Other items they are considering are a hoodie for \$20, slippers for \$10, a diary for \$6, and a water bottle for \$16. They also have to buy some wrapping paper and ribbons, which is \$5 in total for both twins.

If they want to spend all their money and also spend about the same total amount on each person, which of the additional items could they buy for each of Paula and Quinn? You may ignore taxes.

Theme Number Sense

Problem of the Week
Problem B and Solution
Birthday Presents

Problem

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If they want to spend all their money and also spend about the same total amount on each person, which of the additional items could they buy for each of Paula and Quinn? You may ignore taxes.

Solution

(a) The three friends have saved a total of $\$30 + \$25 + \$28 = \$83$. The gifts cost $(2 \times \$7) + \$6 + \$7 + \$21 + (2 \times \$4) = \$56$ in total. Thus, there will be $\$83 - \$56 = \$27$ left over.

(b) First we will figure out approximately how much money they should spend on each person. Subtracting \$5 for wrapping paper and ribbons, the friends have a total of $\$83 - \$5 = \$78$ to spend on gifts. So they should spend about $\$78 \div 2 = \$39$ on each person.

- After buying the socks, art pencils, and chocolate mints for Quinn, they will have spent $\$7 + \$21 + \$4 = \$32$. Thus, they should spend approximately $\$39 - \$32 = \$7$ more on gifts for Quinn.
- After buying the socks, guitar picks, yo-yo, and chocolates for Paula, they will have spent $\$7 + \$6 + \$7 + \$4 = \$24$. Thus, they should spend approximately $\$39 - \$24 = \$15$ more on gifts for Paula.

After buying the wrapping paper and ribbons, they will have $\$27 - \$5 = \$22$ left, and want to spend it all on gifts. From the additional items, they can buy a diary and a water bottle for $\$6 + \$16 = \$22$, or two diaries and a pair of slippers for $\$6 + \$6 + \$10 = \$22$. No other combination of items equals \$22.

So they could buy a diary for \$6 for Quinn, and either a water bottle for \$16 or a diary and a pair of slippers for $\$6 + \$10$ for Paula. Either way they will have spent a total of $\$32 + \$6 = \$38$ on Quinn and $\$24 + \$16 = \$40$ on Paula.

Problem of the Week

Problem B

Temperature Conversions

Two common units to measure temperature are degrees Celsius and degrees Fahrenheit. From time to time, we need to convert temperatures from degrees Celsius to degrees Fahrenheit.

(a) The exact conversion from degrees Celsius to degrees Fahrenheit is as follows:

Step 1: Take the temperature in degrees Celsius and multiply by 1.8.

Step 2: Take the result from Step 1 and add 32.

Using this exact conversion, convert the following temperatures in degrees Celsius to degrees Fahrenheit. The first has been done for you.

Temperature in degrees Celsius	Temperature in degrees Fahrenheit
100	212
30	
20	
10	
0	

(b) Sometimes when we want to convert between degrees Celsius and degrees Fahrenheit, we don't have a pencil and paper or calculator nearby. In that case, using an approximation and mental math can be helpful. One way to approximate the conversion from degrees Celsius to degrees Fahrenheit is as follows:

Step 1: Take the temperature in degrees Celsius and multiply by 2.

Step 2: Take the result from Step 1 and add 30.

Using this approximate conversion, convert the following temperatures in degrees Celsius to degrees Fahrenheit. The first has been done for you.

Temperature in degrees Celsius	Approximate temperature in degrees Fahrenheit
100	230
30	
20	
10	
0	

(c) Did any of the approximate conversions in part (b) give the same temperature as the exact conversion in part (a)?

Extension:
If you let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit, can you write formulas for the conversions in parts (a) and (b)?

Themes Geometry & Measurement, Number Sense

Problem of the Week

Problem B and Solution

Temperature Conversions

Problem

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30	
20	
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0	

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Extension:

If you let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit, can you write formulas for the conversions in parts (a) and (b)?

Solution

(a) The completed table is below.

Temperature in degrees Celsius	Temperature in degrees Fahrenheit
100	212
30	86
20	68
10	50
0	32

(b) The completed table is below.

Temperature in degrees Celsius	Approximate temperature in degrees Fahrenheit
100	230
30	90
20	70
10	50
0	30

(c) The conversion of 10°C to 50°F gave the same temperature when using both the approximate and exact conversions.

EXTENSION:

In part (a), we have $F = 1.8 \times C + 32$.
In part (b), we have $F = 2 \times C + 30$.

Problem of the Week

Problem B

Grocery Shopping

A family of five is planning on having a special meal for lunch, while sticking to a \$30 budget. The three children (Sean, Siobhan, Saorise) are each to propose a menu for the lunch. Each menu is to consist of a main dish, along with a vegetable side and a dessert. When answering the questions that follow, use the tables of items and prices per item below.

(a) Sean's menu has wraps made up of chicken fingers, lettuce, tomatoes, and mayonnaise. He chooses celery for his vegetable side and apples for dessert. Does his menu cost less than \$30? If it does not, how can he change his menu so it costs less than \$30?

(b) Siobhan's menu has sandwiches made up of bread, cold cuts, cheese, mayonnaise, and mustard. She chooses carrots for her vegetable side and apple pie for dessert. Does her menu cost less than \$30? If it does not, how can she change her menu so that it costs less than \$30?

(c) Saorise would like to have two different options for the main part of the meal, along with one option for the vegetable side and one option for the dessert. Help her make a menu from the items below, without exceeding \$30.

Main Dish Items	
Item	**Price**
Bread	\$2.98
Chicken Fingers	\$7.99
Cold Cuts	\$6.99
Hot Dogs	\$3.49
Hot Dog Buns	\$2.99
Pasta	\$1.99
Pasta Sauce	\$2.98
Wraps	\$2.99
Cheese	\$4.98

Vegetables	
Item	**Price**
Carrots	\$2.99
Celery	\$2.99
Cucumbers	\$1.79
Lettuce	\$1.97
Tomatoes	\$1.49

Condiments	
Item	**Price**
Ketchup	\$3.99
Mayonnaise	\$3.59
Mustard	\$4.49
Relish	\$3.49

Dessert	
Item	**Price**
Apples	\$3.99
Apple Pie	\$5.99
Bananas	\$1.45
Cookies	\$2.99
Ice Cream	\$2.98

Theme Number Sense

Problem of the Week

Problem B and Solution

Grocery Shopping

Problem

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Dessert	
Item	**Price**
Apples	$3.99
Apple Pie	$5.99
Bananas	$1.45
Cookies	$2.99
Ice Cream	$2.98

Solution

(a) The total cost for Sean's menu is
$\$2.99 + \$7.99 + \$1.97 + \$1.49 + \$3.59 + \$2.99 + \$3.99 = \$25.01$. This menu does not exceed \$30.

(b) The total cost for for Siobhan's menu is
$\$2.98 + \$6.99 + \$4.98 + \$3.59 + \$4.49 + \$2.99 + \$5.99 = \$32.01$. This menu does exceed \$30 by \$2.01.

There are many options to change the menu so that cost does not exceed \$30. For example, removing either mayonnaise or mustard will make the cost less than \$30. Or she could have bananas for dessert instead of apple pie.

(c) Answers will vary. One possible main dish could be hot dogs with hot dog buns, ketchup, and mustard, which would cost
$\$3.49 + \$2.99 + \$3.99 + \$4.49 = \$14.96$. Another main dish could be pasta with pasta sauce, which would cost $\$1.99 + \$2.98 = \$4.97$. She could have cucumbers for her vegetable side and ice cream for dessert. The total cost for this menu is $\$14.96 + \$4.97 + \$1.79 + \$2.98 = \$24.70$.

Problem of the Week
Problem B
Gamer!

Geoff plays a game using two standard six-sided dice: a black one and a white one. To win the game, Geoff must roll the dice and have the numbers on the two top faces sum to 11.

(a) What is the probability that he rolls a 7 with just the black die?

(b) What is the theoretical probability that he rolls a 1 on the black die and a 6 on the white die?

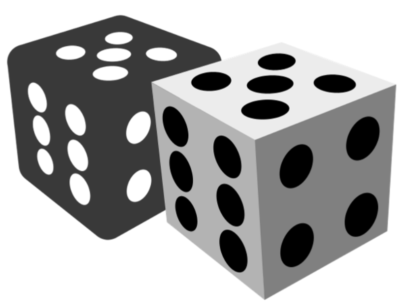
(c) If he rolls both dice and calculates the sum of the numbers on the two top faces, what sum(s) have the lowest theoretical probability of being rolled?

(d) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 7?

(e) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 11?

(f) Roll two dice thirty-six times and keep track of the number of times the numbers on the two top faces sum to 11. What was your empirical probability of rolling a sum of 11?

(g) Share your results in part (f) with your classmates. How many had their empirical probability of rolling a sum of 11 equal the theoretical probability of rolling a sum of 11?



Themes Data Management, Number Sense

Problem of the Week

Problem B and Solution

Gamer!

Problem

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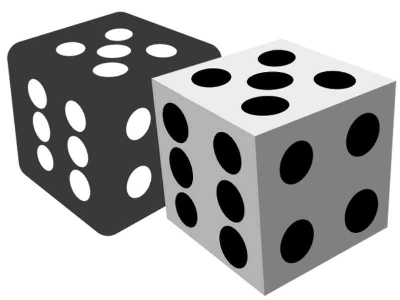


Solution

(a) Since the numbers on the faces of a standard six-sided die are 1, 2, 3, 4, 5, and 6, it is impossible to roll a 7. So the probability is 0.

(b) For each of the 6 possible numbers he could throw with the black die there are 6 possible numbers on the white die, so the total number of possible outcomes is $6 \times 6 = 36$. Thus, the theoretical probability that he throws a 1 on the black die and a 6 on the white die is 1 in 36, or $\frac{1}{36}$.

Alternatively, to solve this problem we can create a table where the columns show the possible numbers on the top face of the white die, the rows show the possible numbers on the top face of the black die, and each cell in the body of the table gives the sum of the corresponding pair of numbers.

		White Die					
		1	2	3	4	5	6
Black Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

From this table, we can conclude that there is 1 outcome out of 36 possible outcomes where the number on the top face of the black die is a 1 and the number on the top face of the white die is a 6. We will use this table in our answers to parts (c), (d), and (e).

(c) For each of the sums of 2 and 12, there is only one possible way to obtain that outcome (two ones or two sixes). Thus, each of these sums has the lowest theoretical probability, namely 1 in 36, or $\frac{1}{36}$.

(d) A sum of 7 can be obtained in 6 possible ways (as $1+6$ or $6+1$, $2+5$ or $5+2$, $3+4$ or $4+3$). So, there are six outcomes which give the desired sum. Thus, the theoretical probability that he rolls a 7 is 6 in 36, or $\frac{6}{36}$, which is equivalent to 1 in 6, or $\frac{1}{6}$.

(e) A sum of 11 can be obtained in 2 possible ways (as $5+6$ or $6+5$). Thus, the theoretical probability of rolling an 11 is 2 in 36, or $\frac{2}{36}$, which is equivalent to 1 in 18, or $\frac{1}{18}$.

(f) Answers will vary.

(g) Answers will vary.

Problem of the Week

Problem B

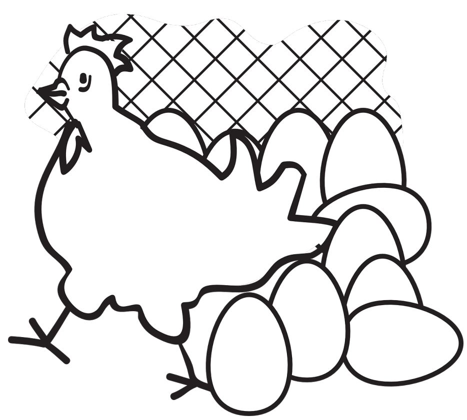
A Chicken and Egg Problem

Kamini raises chickens in her backyard, which lay an average total of 7 eggs per day. She has three customers who buy a dozen eggs from her each week, paying $5 per dozen. She keeps the remaining eggs for herself.

(a) How many eggs per week should Kamini expect to keep for herself?

(b) If the chickens continue to lay the same average number of eggs throughout the year and the customers continue to buy the same weekly amount, how much money can Kamini expect to make in one year?

(c) Chickens normally lay eggs for about four years. However, they do not lay eggs while moulting (losing then regrowing their feathers), which they do for 6 to 12 weeks per year. If Kamini's chickens maintain laying an average total of 7 eggs per day throughout the four years, except when they are moulting, and the customers continue to buy the same weekly amount, what is the maximum amount of money Kamini could make over those four years?



Problem of the Week
Problem B and Solution
A Chicken and Egg Problem

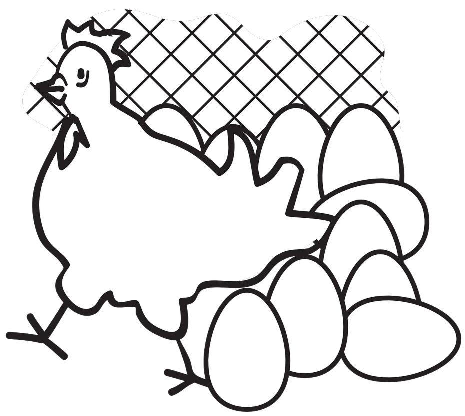
Problem

Kamini raises chickens in her backyard, which lay an average total of 7 eggs per day. She has three customers who buy a dozen eggs from her each week, paying \$5 per dozen. She keeps the remaining eggs for herself.

(a) How many eggs per week should Kamini expect to keep for herself?

(b) If the chickens continue to lay the same average number of eggs throughout the year and the customers continue to buy the same weekly amount, how much money can Kamini expect to make in one year?

(c) Chickens normally lay eggs for about four years. However, they do not lay eggs while moulting (losing then regrowing their feathers), which they do for 6 to 12 weeks per year. If Kamini's chickens maintain laying an average total of 7 eggs per day throughout the four years, except when they are moulting, and the customers continue to buy the same weekly amount, what is the maximum amount of money Kamini could make over those four years?



Solution

(a) Since her chickens lay 7 eggs per day for 7 days a week, Kamini can expect $7 \times 7 = 49$ eggs per week.

Since she has 3 customers that each buy 12 eggs per week, she has $3 \times 12 = 36$ eggs going to customers each week.

Thus, there are $49 - 36 = 13$ eggs left for Kamini to keep for herself each week.

(b) Each week she sells a dozen eggs at \$5 per dozen to 3 customers, so earns $3 \times \$5 = \$15$ each week.

Since there are 52 weeks in a year, in one year Kamini can expect to make $52 \times \$15 = \$780$.

(c) Kamini would make the maximum amount of money if the chickens only moult for 6 weeks in a year. So the chickens will be producing eggs for $52 - 6 = 46$ weeks each year.

Thus, the amount she would make in one year is $46 \times \$15 = \$690$. And the amount she would make in 4 years would be $4 \times \$690 = \$2760$.

Thus, Kamini could make a maximum of \$2760 over those 4 years.

Problem of the Week
Problem B
It's a Race

Manish, Diana, Isebel, Ris, and Ji-Yeong are the five runners in a 400 m race. Their friend cheered them on and took a photo partway through the race. The photo shows the following:

- Isebel is in the lead.
- Ji-Yeong has run farther than Ris, but not as far as Manish.
- Diana has two people ahead of her and two people behind her.

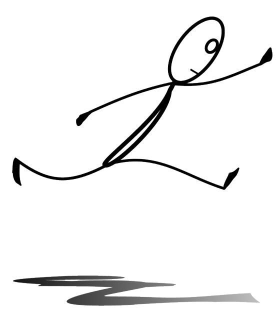
(a) Using the information given, determine the order of the runners in the photo. Fill in the blanks in the list shown below.

START ________, ________, ________, ________, ________ FINISH

(b) The following five fractions represent the fraction of the course that each runner had completed in the photo.

$$\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{3}, \frac{1}{4}$$

Which runner completed each fraction of the course? Show your work using diagrams or equivalent fractions.



THEMES COMPUTATIONAL THINKING, NUMBER SENSE

Problem of the Week

Problem B and Solution

It's a Race

Problem

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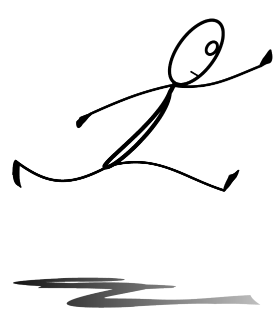


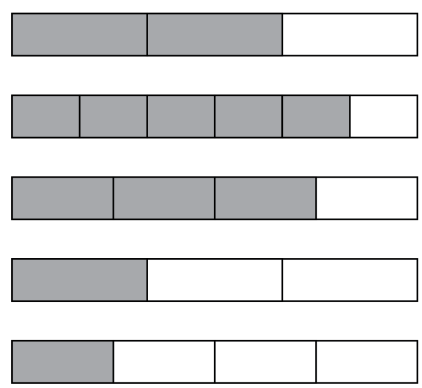
Solution

(a) We will number the positions from 1 to 5, starting on the left. Since Isebel is in the lead, she must be in position 5. Since Diana has two people ahead of her and two people behind her, she must be in position 3. That leaves us with positions 1, 2, and 4. Since Ji-Yeong has run farther than Ris but not as far as Manish, that tells us that Ris must be in position 1, Ji-Yeong must be in position 2, and Manish must be in position 4, as shown.

START Ris, Ji-Yeong, Diana, Manish, Isebel FINISH

(b) In order to determine which runner completed each fraction of the course, we must first write the fractions in order from smallest to largest. Then we can match the fractions with the runners in the order from part (a), since the runner who completed the smallest fraction of the course will be closest to the start, and the runner who completed the largest fraction of the course will be closest to the finish.

One way to compare the fractions is using diagrams, as shown.



$\frac{2}{3}$

$\frac{5}{6}$

$\frac{3}{4}$

$\frac{1}{3}$

$\frac{1}{4}$

Since each diagram is the same width, we can compare the shaded part of each diagram to place the fractions in order from smallest to largest. This gives us $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

Alternatively, we can use equivalent fractions. Using a common denominator of 12, our fractions can be written as follows.

$$\frac{2}{3} = \frac{8}{12}, \quad \frac{5}{6} = \frac{10}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{1}{4} = \frac{3}{12}$$

Now we can use the equivalent fractions to place the fractions in order from smallest to largest.

$$\frac{1}{4} = \frac{3}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{5}{6} = \frac{10}{12}$$

Once we have the fractions written in order from smallest to largest, we can match each runner to the fraction of the course they completed as shown.

Runner	Ris	Ji-Yeong	Diana	Manish	Isebel
Fraction of Course Completed	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$

Problem of the Week

Problem B

Server Satisfaction

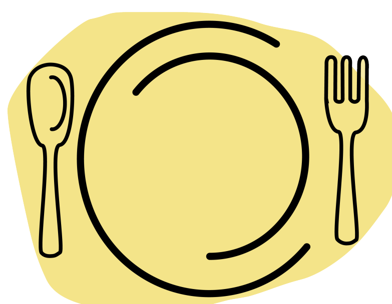
You've just enjoyed a delicious meal at a restaurant with your friend. The cost of the meal before tax and tip was \$35.10.

(a) Suppose the tax is 15% of the total cost. Estimate the dollar amount of tax using a mental math strategy. Then calculate the actual dollar amount of tax, and total cost including tax.

(b) If you want to tip the server an additional 20% after tax, how much would you pay in total?

(c) How much change would you receive if you paid with a \$100 bill?

Extension: Suppose you are a server. In general, when would you rather receive a \$20 tip instead of 20% of your bill? Justify your thinking.



Problem of the Week
Problem B and Solution
Server Satisfaction

Problem

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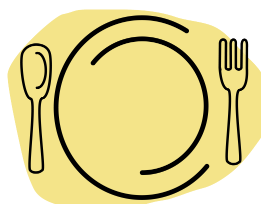

Solution

(a) To estimate the dollar amount of tax, we could first round the total cost to \$35. Then we could think of 15% as $10\% + 5\%$. Since 10% of \$35 is \$3.50, and half of that is \$1.75, we can estimate that the dollar amount of tax is $\$3.50 + \$1.75 = \$5.25$.

The actual dollar amount of tax is $\$35.10 \times 0.15 = \$5.265$, which rounds to \$5.27. Thus, the total cost including tax is $\$35.10 + \$5.27 = \$40.37$.

(b) If you want to tip the server an additional 20%, then we need to calculate 20% of \$40.37. We know that 10% of \$40.37 is \$4.037. We then double that to get 20%, which is \$8.074. Rounded to the nearest cent, this is \$8.07. Finally, adding that to the total cost including tax gives $\$40.37 + \$8.07 = \$48.44$.

(c) If you paid with a \$100 bill, your change would be $\$100 - \$48.44 = \$51.56$.

Extension: When 20% of the total cost including tax is less than \$20, then a \$20 tip would likely be preferred. When 20% of the total cost including tax is greater than \$20, then a tip of 20% would likely be preferred.

Problem of the Week

Problem B

These Rates are Shocking

Most provinces take into consideration the time of day when they charge for electricity usage. The rates they charge are often referred to as Time-Of-Use (TOU) rates. Using the sample TOU rates in the table below, answer the questions that follow.

TOU Price Period	**November 1 - April 30 Time of Day**	**May 1 - October 31 Time of Day**	**TOU Rate (¢ per kWh)**
Off-Peak Hours	Weekdays 7 p.m. - 7 a.m., anytime on weekends	Weekdays 7 p.m. - 7 a.m., anytime on weekends	7.4
Mid-Peak Hours	Weekdays 11 a.m. - 5 p.m.	Weekdays 7 a.m. - 11 a.m. and 5 p.m. - 7 p.m.	10.2
On-Peak Hours	Weekdays 7 a.m. - 11 a.m. and 5 p.m. - 7 p.m	Weekdays 11 a.m. - 5 p.m.	15.1

(a) Garret's family used 50 kWh on a Saturday afternoon. What would be the charge for those 50 kWh?

(b) On November 10, when would be the best time of day to run your clothes dryer?

(c) When should you avoid using your clothes dryer in the summer?

(d) What might be a better way (environmentally and financially) to dry your clothes in the summer?

(e) Ramal's family used 1180 kWh hours of electricity in one month.

  (i) What is the maximum amount of money (in dollars) they could have paid for electricity that month?

  (ii) What is the minimum amount of money (in dollars) they could have paid for electricity that month?

Themes Data Management, Number Sense

Problem of the Week
Problem B and Solution
These Rates are Shocking

Problem

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Solution

(a) The rate for any Saturday is 7.4¢ per kWh, which is \$0.074 per kWh. Therefore, the charge for 50 kWh would be $50 \times \$0.074 = \$3.70$.

(b) If November 10 falls on a weekday, the best time to run the dryer would be anytime before 7 a.m. or after 7 p.m. If November 10 falls on the weekend, you could run it anytime from Friday after 7 p.m. until Monday morning before 7 a.m.

(c) You should avoid running your dryer from 7 a.m. to 7 p.m. on weekdays, but it is most expensive to run your dryer between 11 a.m. and 5 p.m.

(d) You could hang your clothes out to dry in the summer which would have little or no cost, both environmentally and financially.

(e) (i) Ramal's family used 1180 kWh. The most they could have paid for electricity is \$0.151 per kWh. Therefore, the maximum amount they could have paid for electricity that month is $1180 \times \$0.151 = \$178.18$.

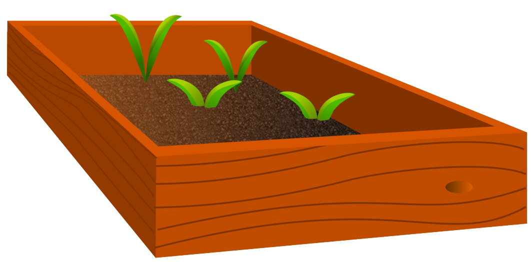
(ii) Ramal's family used 1180 kWh. The least they could have paid for electricity is \$0.074 per kWh. Therefore, the minimum amount they could have paid for electricity that month is $1180 \times \$0.074 = \$87.32$.

Problem of the Week
Problem B
Jordyn's Garden

Jordyn's neighbourhood is building a community garden to grow some vegetables. They would like the garden bed to have an area of 48 square metres, and plan to put wooden fence boards around the edges of the garden bed. To reduce the cost of the project, they would like the garden bed to have the smallest possible perimeter.

(a) Determine the length and width of Jordyn's community garden bed. Assume the side lengths are whole numbers, in metres.

(b) The community decided to double the area of the garden bed, but would still like it to have the smallest possible perimeter. Again, assume the side lengths are whole numbers, in metres. Determine the length and width of the garden bed now.



Themes Geometry & Measurement, Number Sense

Problem of the Week
Problem B and Solution
Jordyn's Garden

Problem

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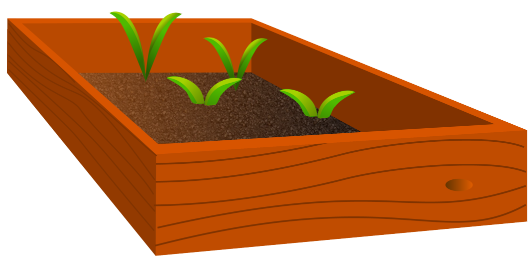


Solution

(a) Since the garden bed is in the shape of a rectangle and has an area of 48 square metres, it follows that length $\times$ width $= 48$. To determine the length and width, we need to find pairs of whole numbers that multiply to 48. These are called the factor pairs of 48, and are as follows: 1 and 48, 2 and 24, 3 and 16, 4 and 12, and 6 and 8. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

Width (metres)	Length (metres)	Perimeter (metres)
1	48	$2 \times (1 + 48) = 2 \times 49 = 98$
2	24	$2 \times (2 + 24) = 2 \times 26 = 52$
3	16	$2 \times (3 + 16) = 2 \times 19 = 38$
4	12	$2 \times (4 + 12) = 2 \times 16 = 32$
6	8	$2 \times (6 + 8) = 2 \times 14 = 28$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 8 metres and the width should be 6 metres.

(b) After they double the area of the garden bed it will have an area of $2 \times 48 = 96$ square metres. Using a similar approach to part (a), we need to find the factor pairs of 96, which are: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, and 8 and 12. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

Width (metres)	Length (metres)	Perimeter (metres)
1	96	$2 \times (1 + 96) = 2 \times 97 = 194$
2	48	$2 \times (2 + 48) = 2 \times 50 = 100$
3	32	$2 \times (3 + 32) = 2 \times 35 = 70$
4	24	$2 \times (4 + 24) = 2 \times 28 = 56$
6	16	$2 \times (6 + 16) = 2 \times 22 = 44$
8	12	$2 \times (8 + 12) = 2 \times 20 = 40$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 12 metres and the width should be 8 metres.

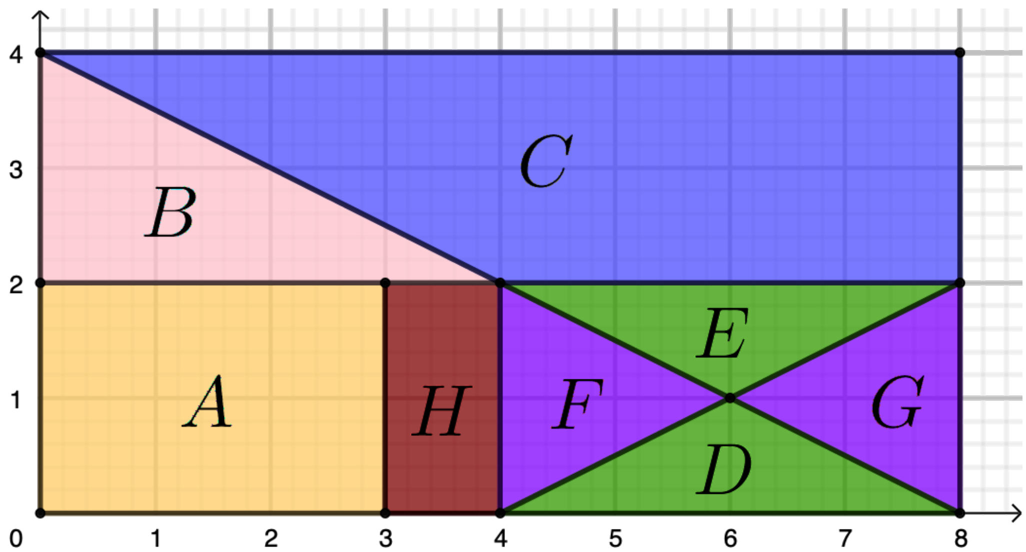
Extension: Note that in each case, the minimum perimeter occurs for the factor pair whose positive difference is the smallest. Will this always happen? Why or why not?

Problem of the Week

Problem B

Fraction Fun

A large rectangle is in the first quadrant of the Cartesian plane with its four vertices at $(0,0)$, $(8,0)$, $(8,4)$, and $(0,4)$. It is divided into eight regions labelled A, B, C, D, E, F, G, and H, as shown.



What fraction of the area of the large rectangle is the area of region A? the area of region B? the area of region C? the area of region D?

THEMES Geometry & Measurement, Number Sense

Problem of the Week
Problem B and Solution
Fraction Fun

Problem

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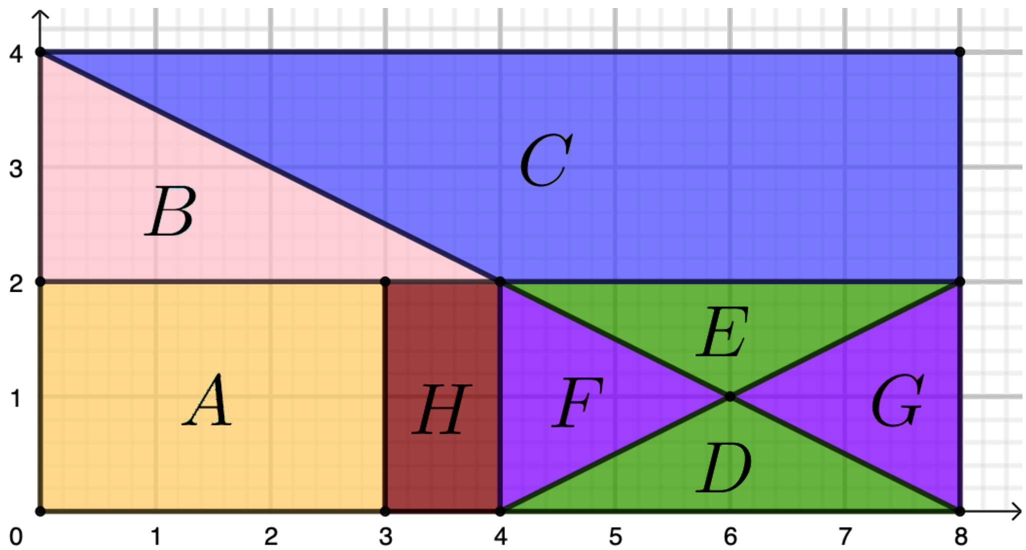


What fraction of the area of the large rectangle is the area of region A? the area of region B? the area of region C? the area of region D?

Solution

Solution 1

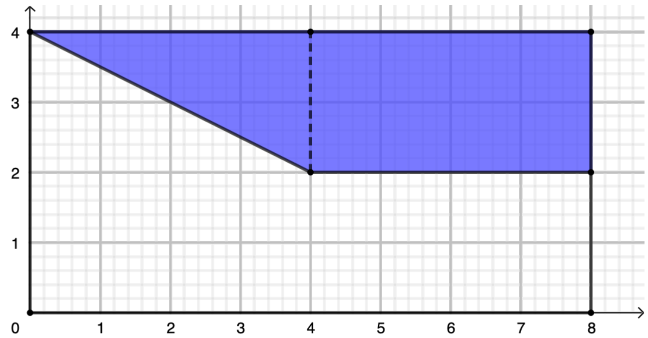
The large rectangle has a length of 8 units and a width of 4 units. Therefore, the area of the large rectangle is $8 \times 4 = 32$ square units.

Region A is a rectangle with a length of 3 units and a width of 2 units. Hence, its area is $3 \times 2 = 6$ square units. So, the area of region A is $\frac{6}{32} = \frac{3}{16}$ of the area of the large rectangle.

Region B is a triangle with a base of 4 units and height of 2 units. Hence, its area is $\frac{1}{2} \times 4 \times 2 = 4$ square units. So, the area of region B is $\frac{4}{32} = \frac{1}{8}$ of the area of the large rectangle.

Region D is a triangle with a base of 4 units and a height of 1 unit. Hence, its area is $\frac{1}{2} \times 1 \times 4 = 2$ square units. So the area of region D is $\frac{2}{32} = \frac{1}{16}$ of the area of the large rectangle.

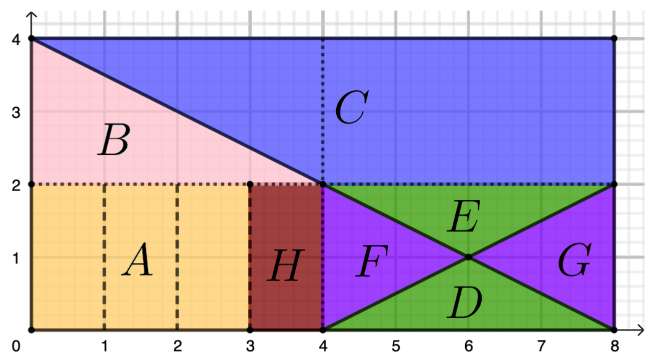
Region C is made up of a rectangle and a triangle as shown by the dashed line in the diagram below.



The rectangle has a length of 4 units and a width of 2 units. So, the area of the rectangle is $4 \times 2 = 8$ square units. The triangle has a base of 4 units and height of 2 units. So, the area of the triangle is $\frac{1}{2} \times 4 \times 2 = 4$ square units. Therefore, the area of region C is $8 + 4 = 12$ square units. Thus, the area of region C is $\frac{12}{32} = \frac{3}{8}$ of the area of the large rectangle.

Solution 2

We draw in dotted lines which divide the large rectangle into four equal parts, or quarters, and draw in dashed lines divide the lower left quarter further into quarters.

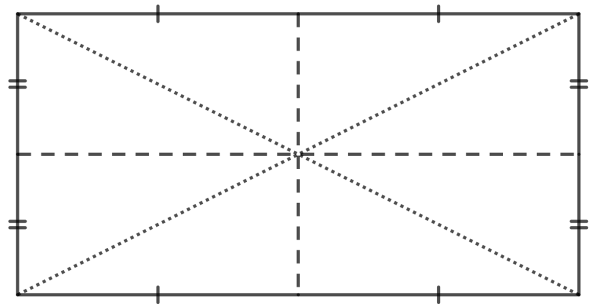


Since the dashed lines divide the lower left quarter of the rectangle further into quarters, the area of each of those four rectangles is $\frac{1}{4}$ of $\frac{1}{4}$ of the area the large rectangle, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle. Thus, the area of region A is $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$ of the area of the large rectangle.

The area of region B is half of the area of the top left quarter, and so is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ of the area of the large rectangle.

The area of region C is the area of the top half of the large rectangle, minus the area of region B, which is $\frac{1}{8}$ of the large rectangle. So in total, the area of region C is $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ of the area of the large rectangle.

Note that we can divide any rectangle into 4 smaller rectangles of equal area by joining the midpoints of opposite sides of the rectangles. When we construct the two diagonals of the large rectangle, we further divide each smaller rectangle into two triangles of equal areas. So, in the diagram below, the eight smaller triangles have equal area.



In our problem, the area of region D is equal to $\frac{2}{8}$ or $\frac{1}{4}$ of the area of the lower right rectangle. Therefore, the area of the region D is $\frac{1}{4}$ of $\frac{1}{4}$, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle.