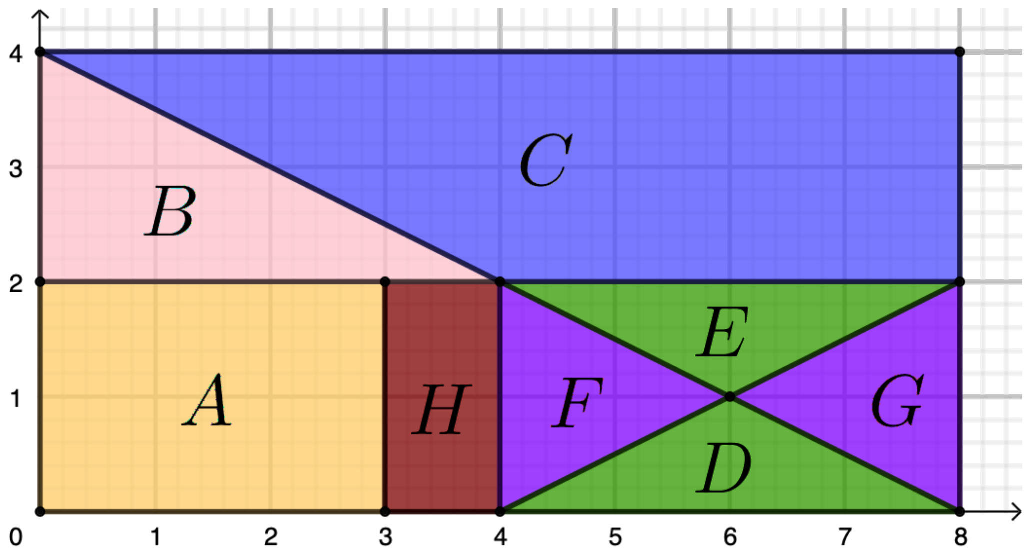
Problem of the Week
Problem B and Solution
Fraction Fun

Problem

A large rectangle is in the first quadrant of the Cartesian plane with its four vertices at $(0, 0)$, $(8, 0)$, $(8, 4)$, and $(0, 4)$. It is divided into eight regions labelled A, B, C, D, E, F, G, and H, as shown.



What fraction of the area of the large rectangle is the area of region A? the area of region B? the area of region C? the area of region D?

Solution

Solution 1

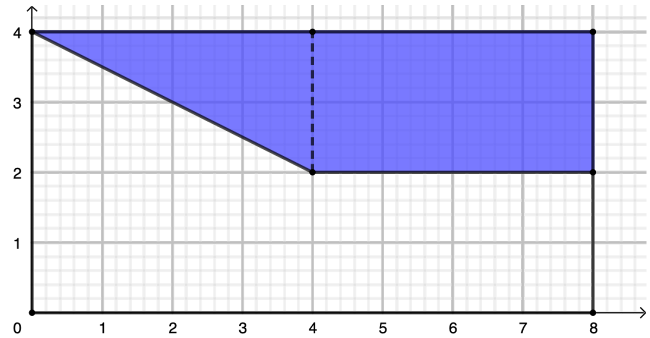
The large rectangle has a length of 8 units and a width of 4 units. Therefore, the area of the large rectangle is $8 \times 4 = 32$ square units.

Region A is a rectangle with a length of 3 units and a width of 2 units. Hence, its area is $3 \times 2 = 6$ square units. So, the area of region A is $\frac{6}{32} = \frac{3}{16}$ of the area of the large rectangle.

Region B is a triangle with a base of 4 units and height of 2 units. Hence, its area is $\frac{1}{2} \times 4 \times 2 = 4$ square units. So, the area of region B is $\frac{4}{32} = \frac{1}{8}$ of the area of the large rectangle.

Region D is a triangle with a base of 4 units and a height of 1 unit. Hence, its area is $\frac{1}{2} \times 1 \times 4 = 2$ square units. So the area of region D is $\frac{2}{32} = \frac{1}{16}$ of the area of the large rectangle.

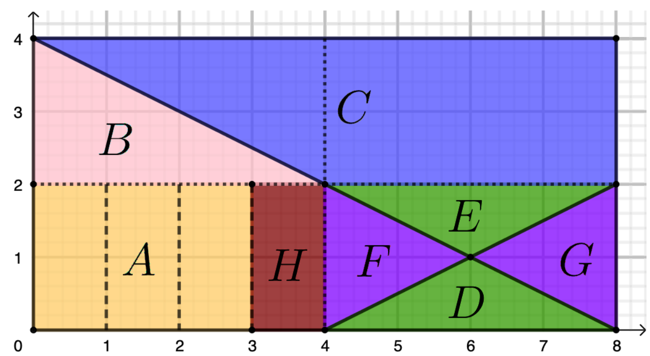
Region C is made up of a rectangle and a triangle as shown by the dashed line in the diagram below.



The rectangle has a length of 4 units and a width of 2 units. So, the area of the rectangle is $4 \times 2 = 8$ square units. The triangle has a base of 4 units and height of 2 units. So, the area of the triangle is $\frac{1}{2} \times 4 \times 2 = 4$ square units. Therefore, the area of region C is $8 + 4 = 12$ square units. Thus, the area of region C is $\frac{12}{32} = \frac{3}{8}$ of the area of the large rectangle.

Solution 2

We draw in dotted lines which divide the large rectangle into four equal parts, or quarters, and draw in dashed lines divide the lower left quarter further into quarters.

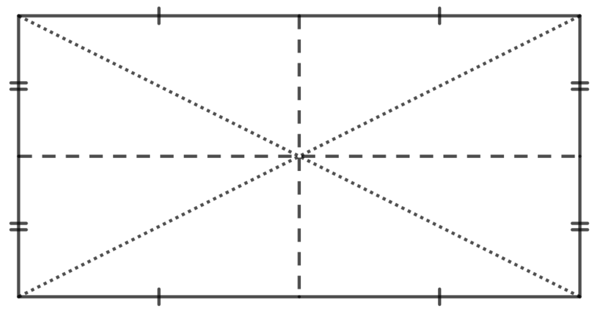


Since the dashed lines divide the lower left quarter of the rectangle further into quarters, the area of each of those four rectangles is $\frac{1}{4}$ of $\frac{1}{4}$ of the area the large rectangle, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle. Thus, the area of region A is $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$ of the area of the large rectangle.

The area of region B is half of the area of the top left quarter, and so is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ of the area of the large rectangle.

The area of region C is the area of the top half of the large rectangle, minus the area of region B, which is $\frac{1}{8}$ of the large rectangle. So in total, the area of region C is $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ of the area of the large rectangle.

Note that we can divide any rectangle into 4 smaller rectangles of equal area by joining the midpoints of opposite sides of the rectangles. When we construct the two diagonals of the large rectangle, we further divide each smaller rectangle into two triangles of equal areas. So, in the diagram below, the eight smaller triangles have equal area.



In our problem, the area of region D is equal to $\frac{2}{8}$ or $\frac{1}{4}$ of the area of the lower right rectangle. Therefore, the area of the region D is $\frac{1}{4}$ of $\frac{1}{4}$, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle.