Problème de la Semaine

Problèmes et Solutions 2023-2024

Problème B

5e/6e année

Le CENTRE d'ÉDUCATION
en MATHÉMATIQUES et en INFORMATIQUE
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Table des Matières

Les problèmes dans ce livret sont organisés par thème.
Un problème peut apparaître dans plusieurs thèmes.
Cliquer sur le nom du thème ci-dessous pour sauter à cette section.

Algèbre (A)

Amène-moi à la couverture

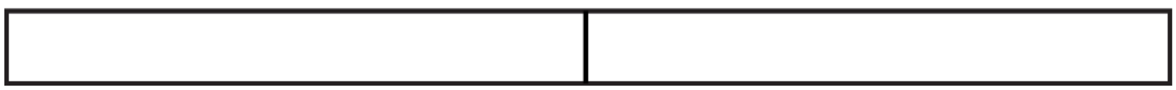
Problème de la semaine

Problème B

Évolution rectangulaire

Gaby a dessiné un rectangle et l'a nommé *Schéma 1*.

Ensuite, elle a dessiné un rectangle divisé en deux parties égales et l'a nommé *Schéma 2*.



Elle a ensuite compté le nombre total de rectangles dans *Schéma 2*. Il y a 1 rectangle à gauche, 1 rectangle à droite et le rectangle entier initial, soit un total de 3 rectangles.

Gaby a ensuite dessiné un rectangle divisé en trois parties égales, qu'elle a nommé *Schéma 3*.

Gaby a compté un total de 6 rectangles dans *Schéma 3*. Peux-tu le confirmer ?

(a) Gaby a continué à dessiner des schémas en divisant un rectangle en parties égales. *Schéma 4* est divisé en quatre parties égales, *Schéma 5* est divisé en cinq parties égales et ainsi de suite. Remplis le tableau en déterminant le nombre total de rectangles dans chaque schéma. Dessine les schémas pour t'aider, puis cherche une régularité dans le nombre total de rectangles.

Numéro du schéma	Nombre total de rectangles
1	1
2	3
3	6
4	
5	
6	

(b) Utilise la régularité que tu as trouvée dans la partie (a) pour prédire le nombre total de rectangles dans *Schéma 12*.

Thèmes Algèbre, Géométrie et mesure

Problem of the Week

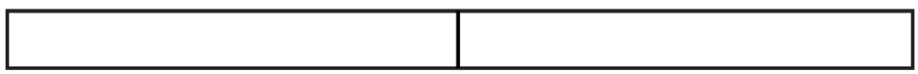
Problem B and Solution

Wrecked Tangles

Problem

Gaby drew a rectangle and called it *Diagram 1*.

She then drew a rectangle divided into two equal parts, and called *Diagram 2*.



She then counted the total number of rectangles in *Diagram 2*. There is 1 rectangle on the left, 1 rectangle on the right, and the original whole rectangle, which makes 3 rectangles in total.

Gaby then drew a rectangle divided into three equal parts, called *Diagram 3*.

Gaby counted a total of 6 rectangles in *Diagram 3*. Can you confirm this?

(a) Gaby continued drawing diagrams by dividing a rectangle into equal parts. *Diagram 4* is divided into four equal parts, *Diagram 5* is divided into five equal parts, and so on. Complete the table by determining the total number of rectangles in each diagram. Draw the diagrams to help you, and then look for a pattern in the total number of rectangles.

Diagram Number	Total Number of Rectangles
1	1
2	3
3	6
4	
5	
6	

(b) Use the pattern you found in part (a) to predict the total number of rectangles in *Diagram 12*.

Solution

(a) For each rectangle, we will assign the smallest rectangle a length of one unit.

Diagram 4 is a rectangle divided into 4 equal parts. In this diagram, there are 4 rectangles of length one unit, 3 of length two units, 2 of length three units, and 1 of length four units. This is a total of $4 + 3 + 2 + 1 = 10$ rectangles.

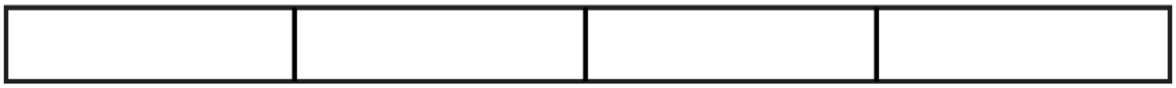
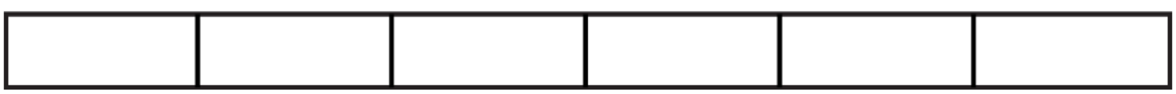


Diagram 5 is a rectangle divided into 5 equal parts. In this diagram, there are 5 rectangles of length one unit, 4 of length two units, 3 of length three units, 2 of length four units, and 1 of length five units. This is a total of $5 + 4 + 3 + 2 + 1 = 15$ rectangles.

Diagram 6 is a rectangle divided into 6 equal parts. In this diagram, there are 6 rectangles of length one unit, 5 of length two units, 4 of length three units, 3 of length four units, 2 of length five units, and 1 of length six units. This is a total of $6 + 5 + 4 + 3 + 2 + 1 = 21$ rectangles.



Now we see a pattern. The total number of rectangles for each diagram is equal to the sum of the diagram number and all the whole numbers smaller than it. Alternatively, the total number of rectangles for each diagram is equal to the diagram number plus the previous number of rectangles. So, the total number of rectangles in *Diagram 7* is equal to
$7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$, or $21 + 7 = 28$.

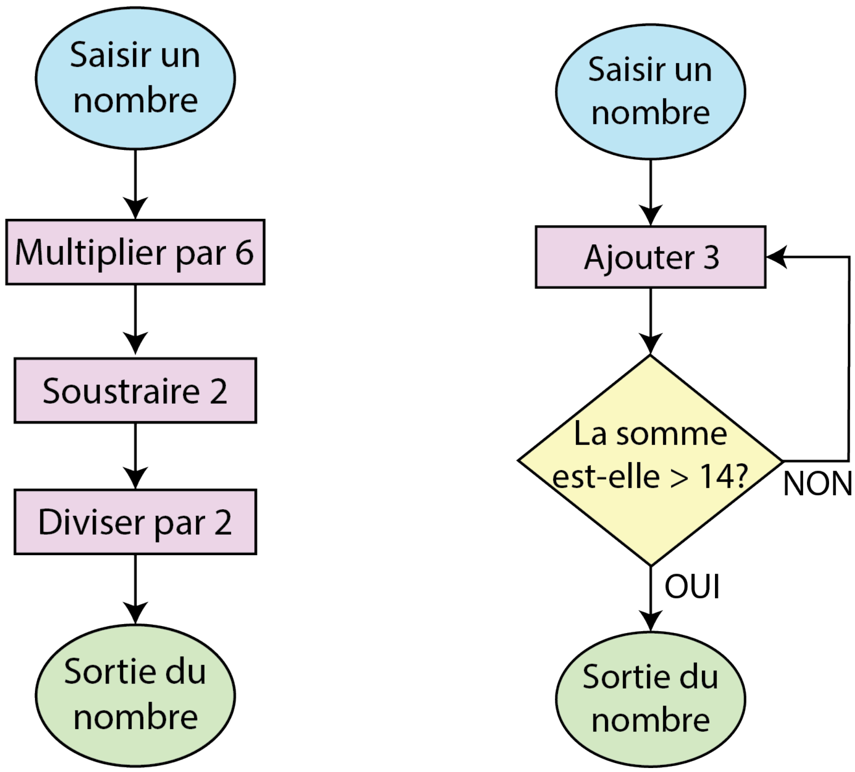
(b) Using the pattern from part (a), the total number of rectangles in *Diagram 12* is equal to $12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 78$, or $28 + 8 + 9 + 10 + 11 + 12 = 78$.

Problème de la semaine

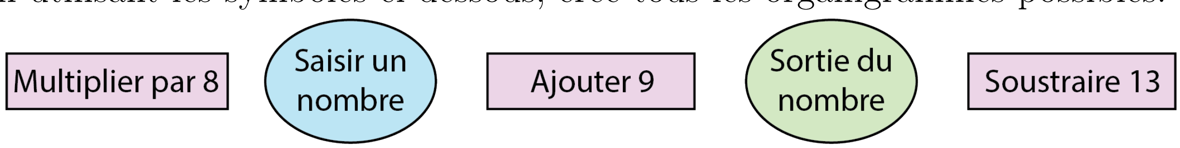
Problème B

Organigramme

(a) Pour chaque organigramme ci-dessous, détermine la valeur de sortie lorsque le nombre 13 est le nombre d'entrée et lorsque le nombre 10 est le nombre d'entrée.



(b) En utilisant les symboles ci-dessous, crée tous les organigrammes possibles.



(c) Parmi les organigrammes de la partie (b), lesquels donnent une sortie de 248 pour une entrée de 35 ?

Thèmes Algèbre, Pensée computationnelle

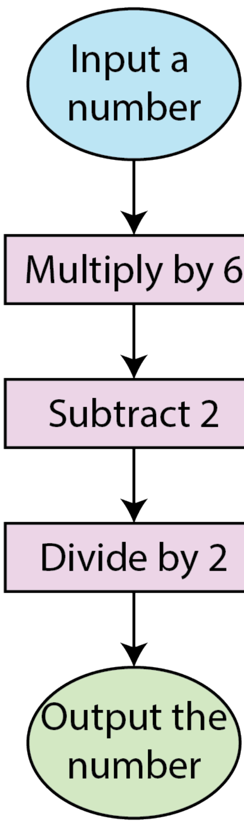
Problem of the Week

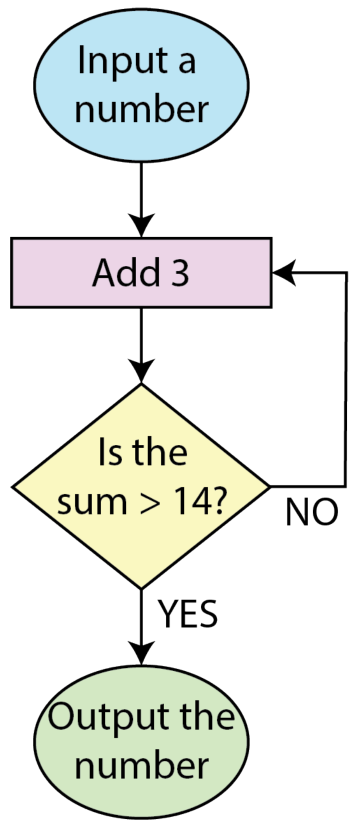
Problem B and Solution

Nice Flow

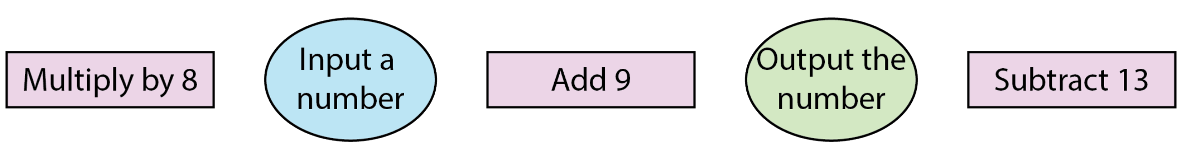
Problem

(a) For each flowchart below, determine the output value when the number 13 is the input number and when the number 10 is the input number.





(b) Using the symbols below, create all possible flowcharts.



(c) Which of the flowcharts in part (b) give an output of 248 for an input of 35?

Solution

(a) For the flowchart on the left:

When we input 13, we first multiply by 6 to get 78.
Then, we subtract 2 to get 76.
Finally, we divide 76 by 2 to get 38.
Thus, the output is 38.

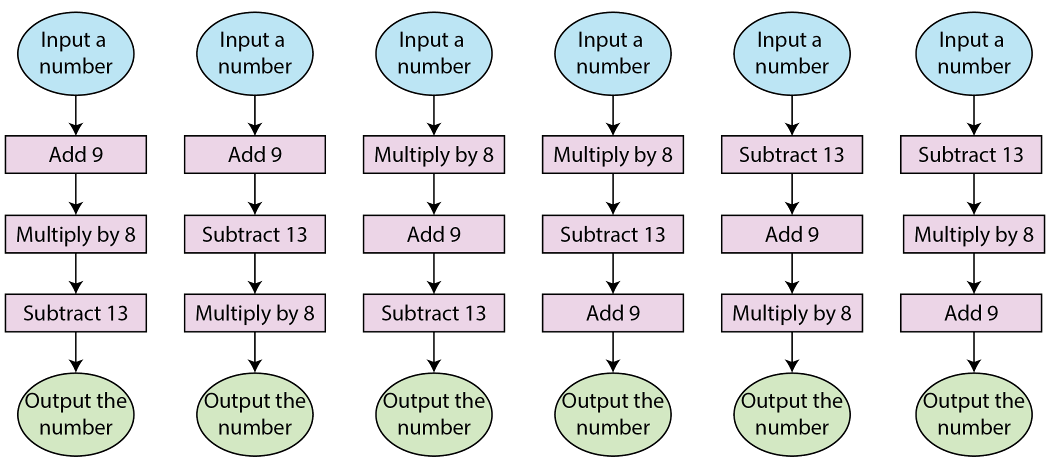
When we input 10, we first multiply by 6 to get 60.
Then, we subtract 2 to get 58.
Finally, we divide 58 by 2 to get 29.
Thus, the output is 29.

For the flowchart on the right:

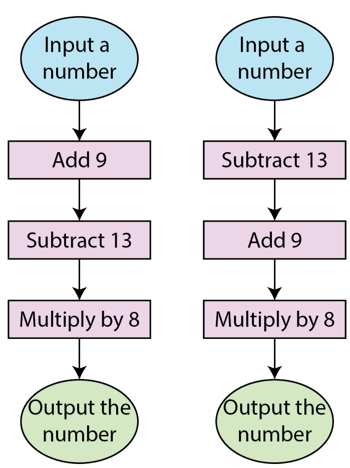
When we input 13, we first add 3 to get 16.
Since $16 > 14$, the output is 16.

When we input 10, we first add 3 to get 13.
Since 13 is not greater than 14, we add 3 again to get 16.
Since $16 > 14$, the output is 16.

(b) Here are the six possible flowcharts:



(c) Here are the two flowcharts that work:



Raisonnement informatiques (C)

Amène-moi à la couverture

Problème de la semaine

Problème B

Casse-tête de programmation

Viktoria écrit un programme que l'on peut utiliser pour deviner son nombre préféré. Ce dernier est 23.

(a) Utilise les blocs suivants pour créer un pseudo-code pour le programme de Viktoria. Note que tu n'auras peut-être pas besoin d'utiliser tous les blocs.

dire "Ce n'est pas correct."	dire "C'est correct."
sinon	dire "Devine mon nombre secret!"
si le nombre deviné $\neq 23$	si le nombre deviné $= 23$

(b) En utilisant les blocs supplémentaires ci-dessous, adapte ton pseudo-code de manière que, si le nombre deviné par l'utilisateur est incorrect, le programme lui indique si ce nombre est trop haut ou trop bas.

dire "C'est trop bas."	dire "C'est trop haut."
sinon si le nombre deviné est < 23	sinon si le nombre deviné est > 23

Tu n'arrives pas à imprimer cette page? Essaie notre feuille de travail interactive.

Problem of the Week

Problem B and Solution

Coding Conundrum

Problem

Viktoria is writing a program that people can use to guess her favourite number, which is 23.

(a) Use the following blocks to create pseudocode for Viktoria's program. Note that you may not need to use all of the blocks.

say "That's not correct."	say "That's correct."
else	say "Guess my secret number!"
if guess $\neq 23$	if guess $= 23$

(b) Using the additional blocks below, modify your pseudocode so that if the user's guess is incorrect, then the program will tell them whether their guess is too high or too low.

say "That's too low."	say "That's too high."
else if guess < 23	else if guess > 23

Not printing this page? You can use our interactive worksheet.

Solution

(a) Two possible solutions are shown below.

Solution 1

```
say "Guess my secret number!"
if guess = 23
   say "That's correct!"
else
   say "That's not correct."
```

Solution 2

```
say "Guess my secret number!"
if guess ≠ 23
   say "That's not correct!"
else
   say "That's correct."
```

(b) One possible solution is shown below.

```
say "Guess my secret number!"
if guess = 23
   say "That's correct!"
else if guess < 23
   say "That's too low."
else
   say "That's too high."
```

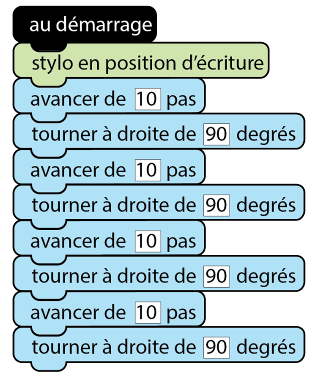
Problème de la semaine

Problème B

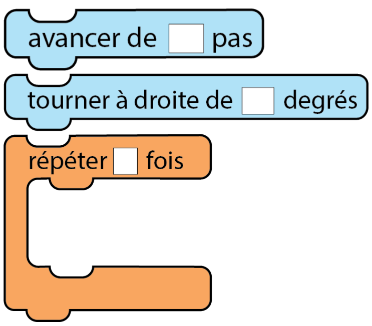
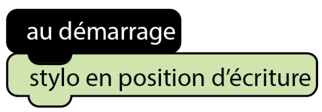
Le code de Shelby

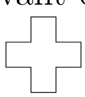
Shelby utilise le codage par blocs pour dessiner des formes.

(a) Le premier programme de Shelby est présenté dans la figure ci-contre. Quelle forme sera produite à l'issue de l'exécution de ce programme?

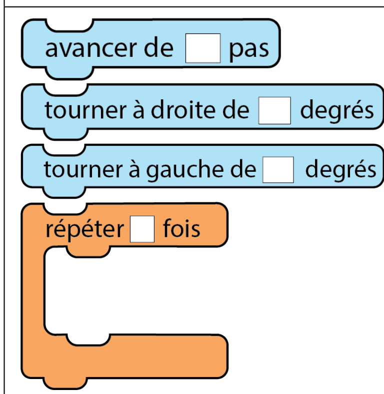
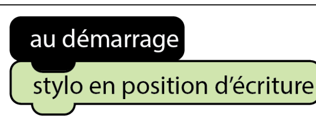


(b) En te servant des blocs fournis, conçois un programme qui dessine la même forme que celle de Shelby, mais en utilisant moins de blocs.

Blocs	Programme
	

(c) En te servant des blocs fournis, conçois un programme qui dessine la forme suivante: 

Un bloc peut être utilisé plus d'une fois.

Blocs	Programme
	

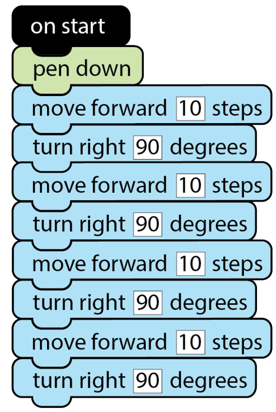
Thèmes Pensée computationnelle, Géométrie et mesure

Problem of the Week
Problem B and Solution
Shelby's Code

Problem

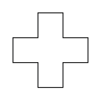
Shelby is using block coding to draw different shapes.

(a) Her first program is shown. What shape will be drawn after running this program?



(b) Using the given blocks, write a program to draw the same shape as Shelby's program, using fewer blocks. Notice that some blocks contain an empty box to be filled with a number.

Blocks	Program
move forward ☐ steps turn right ☐ degrees repeat ☐ times	on start pen down

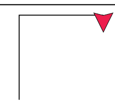
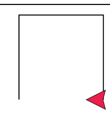
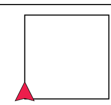
(c) Using the given blocks, write a program to draw the following shape: 

You may use a block more than once.

Blocks	Program
move forward ☐ steps turn right ☐ degrees turn left ☐ degrees repeat ☐ times	on start pen down

Solution

(a) This program will draw a square. The table below shows the drawing progress and position of the pen as we trace through the program.

Program Section	Drawing Progress
on start pen down	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	

(b) By using the repeat block, we can use fewer blocks in the program, as shown.

```
on start
pen down
repeat 4 times
    move forward 10 steps
    turn right 90 degrees
```

(c) There are several possible programs, depending on where the pen starts, and how many repeat blocks are used. Two programs are shown.

```
on start
pen down
repeat 4 times
    move forward 10 steps
    turn right 90 degrees
    move forward 10 steps
    turn right 90 degrees
    move forward 10 steps
    turn left 90 degrees
```

```
on start
pen down
repeat 4 times
    repeat 2 times
        move forward 10 steps
        turn right 90 degrees
    move forward 10 steps
    turn left 90 degrees
```

Problème de la semaine

Problème B

À vos marques, prêts, partez !

Manish, Diana, Isebel, Ris et Ji-Yeong participent à une course de 400 m. Leur ami est venu les encourager et a pris une photo pendant la course. Dans la photo:

— Isebel mène la course.
— Ji-Yeong est devant Ris, mais derrière Manish.
— Diana a deux coureurs devant elle et deux derrière.

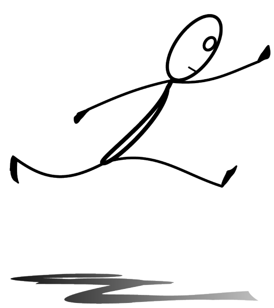
(a) À partir des indications fournies, détermine l'ordre des coureurs sur la photo. Remplis les blancs dans la liste ci-dessous.

DÉPART ________, ________, ________, ________, ________ ARRIVÉE

(b) Les cinq fractions suivantes représentent la fraction du parcours que chaque coureur avait complété sur la photo.

$$\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{3}, \frac{1}{4}$$

Associe chaque fraction au coureur correspondant. Montre ton travail à l'aide de diagrammes ou de fractions équivalentes.



Thèmes Pensée computationnelle, Sens du nombre

Problem of the Week

Problem B and Solution

It's a Race

Problem

Manish, Diana, Isebel, Ris, and Ji-Yeong are the five runners in a 400 m race. Their friend cheered them on and took a photo partway through the race. The photo shows the following:

- Isebel is in the lead.
- Ji-Yeong has run farther than Ris, but not as far as Manish.
- Diana has two people ahead of her and two people behind her.

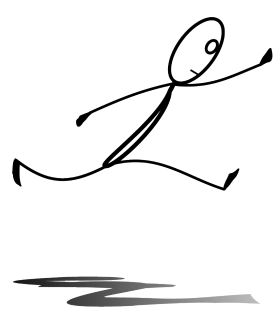
(a) Using the information given, determine the order of the runners in the photo. Fill in the blanks in the list shown below.

START ________, ________, ________, ________, ________ FINISH

(b) The following five fractions represent the fraction of the course that each runner had completed in the photo.

$$\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{3}, \frac{1}{4}$$

Which runner completed each fraction of the course? Show your work using diagrams or equivalent fractions.



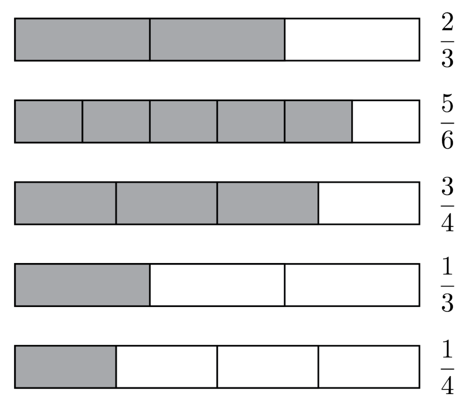
Solution

(a) We will number the positions from 1 to 5, starting on the left. Since Isebel is in the lead, she must be in position 5. Since Diana has two people ahead of her and two people behind her, she must be in position 3. That leaves us with positions 1, 2, and 4. Since Ji-Yeong has run farther than Ris but not as far as Manish, that tells us that Ris must be in position 1, Ji-Yeong must be in position 2, and Manish must be in position 4, as shown.

START Ris, Ji-Yeong, Diana, Manish, Isebel FINISH

(b) In order to determine which runner completed each fraction of the course, we must first write the fractions in order from smallest to largest. Then we can match the fractions with the runners in the order from part (a), since the runner who completed the smallest fraction of the course will be closest to the start, and the runner who completed the largest fraction of the course will be closest to the finish.

One way to compare the fractions is using diagrams, as shown.



Since each diagram is the same width, we can compare the shaded part of each diagram to place the fractions in order from smallest to largest. This gives us $\frac{1}{4}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$.

Alternatively, we can use equivalent fractions. Using a common denominator of 12, our fractions can be written as follows.

$$\frac{2}{3} = \frac{8}{12}, \quad \frac{5}{6} = \frac{10}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{1}{4} = \frac{3}{12}$$

Now we can use the equivalent fractions to place the fractions in order from smallest to largest.

$$\frac{1}{4} = \frac{3}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{5}{6} = \frac{10}{12}$$

Once we have the fractions written in order from smallest to largest, we can match each runner to the fraction of the course they completed as shown.

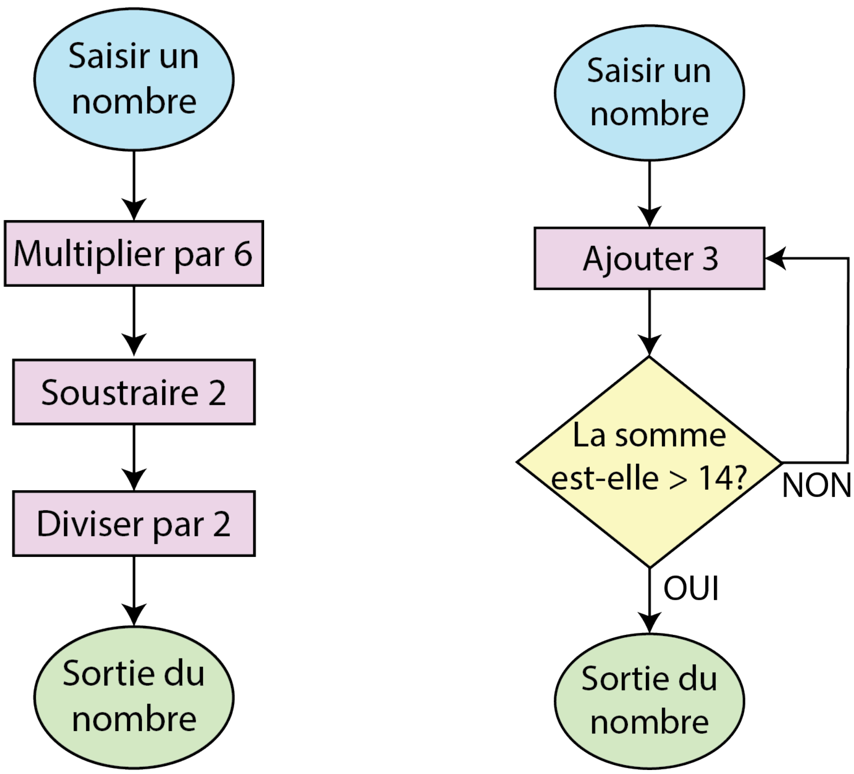
Runner	Ris	Ji-Yeong	Diana	Manish	Isebel
Fraction of Course Completed	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$

Problème de la semaine

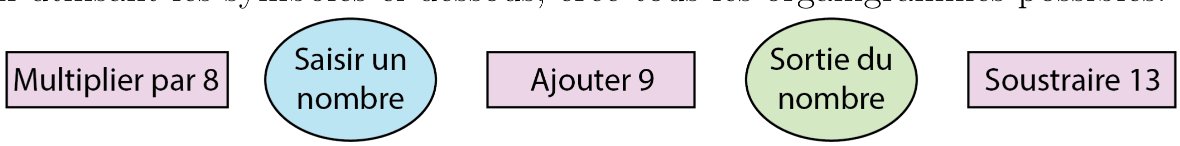
Problème B

Organigramme

(a) Pour chaque organigramme ci-dessous, détermine la valeur de sortie lorsque le nombre 13 est le nombre d'entrée et lorsque le nombre 10 est le nombre d'entrée.



(b) En utilisant les symboles ci-dessous, crée tous les organigrammes possibles.



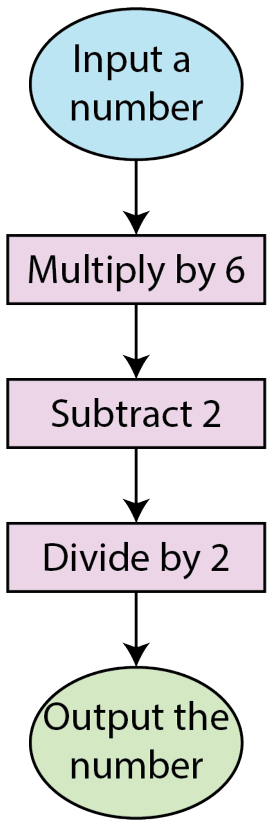
(c) Parmi les organigrammes de la partie (b), lesquels donnent une sortie de 248 pour une entrée de 35 ?

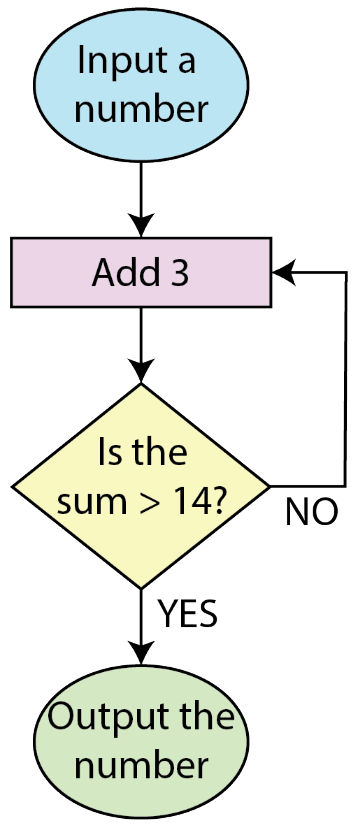
Thèmes Algèbre, Pensée computationnelle

Problem of the Week
Problem B and Solution
Nice Flow

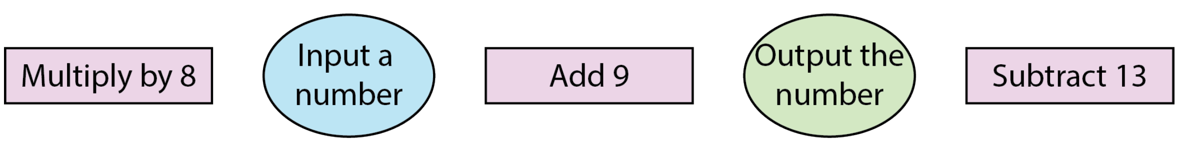
Problem

(a) For each flowchart below, determine the output value when the number 13 is the input number and when the number 10 is the input number.





(b) Using the symbols below, create all possible flowcharts.



(c) Which of the flowcharts in part (b) give an output of 248 for an input of 35?

Solution

(a) For the flowchart on the left:

When we input 13, we first multiply by 6 to get 78.
Then, we subtract 2 to get 76.
Finally, we divide 76 by 2 to get 38.
Thus, the output is 38.

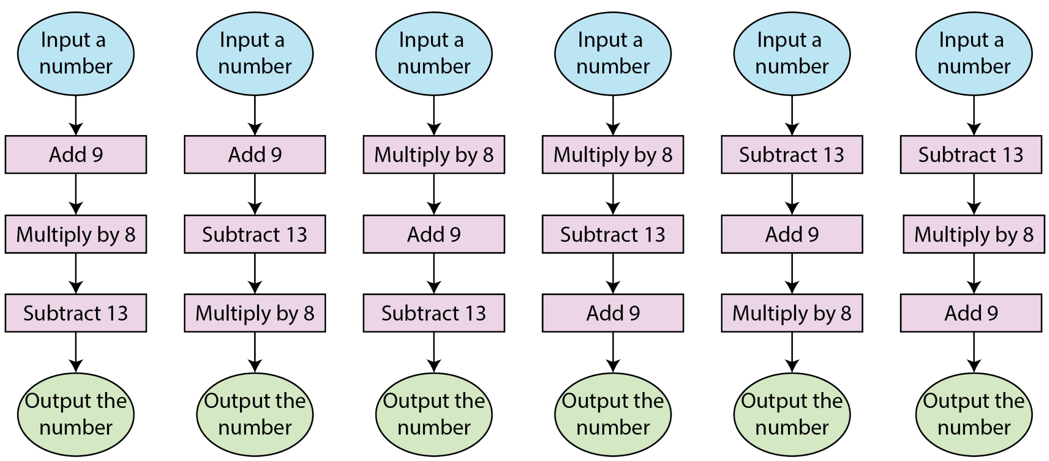
When we input 10, we first multiply by 6 to get 60.
Then, we subtract 2 to get 58.
Finally, we divide 58 by 2 to get 29.
Thus, the output is 29.

For the flowchart on the right:

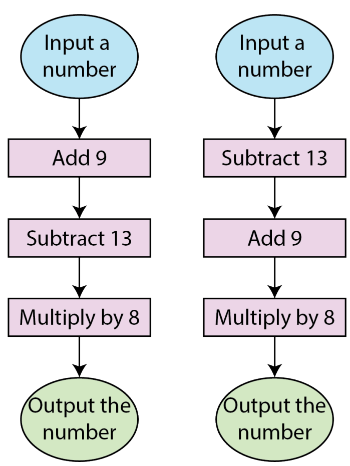
When we input 13, we first add 3 to get 16.
Since $16 > 14$, the output is 16.

When we input 10, we first add 3 to get 13.
Since 13 is not greater than 14, we add 3 again to get 16.
Since $16 > 14$, the output is 16.

(b) Here are the six possible flowcharts:



(c) Here are the two flowcharts that work:



Gestion des données (D)

Amène-moi à la couverture

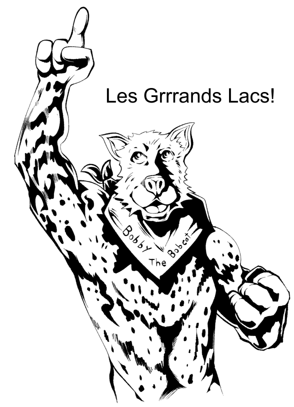
Problème de la semaine

Problème B

Que ces lacs sont grands !

Le tableau ci-dessous présente des données relatives aux cinq Grands Lacs. Ces lacs sont sur la frontière entre le Canada et les États-Unis.

Lac	Aire (milles^2)	Aire (km^2)	Volume (milles^3)	Volume (km^3)
Supérieur	31 700	82 100	2900	12 070
Michigan	22 410	58 030	1180	4930
Huron	23 010	59 590	840	3520
Érié	9910	25 667	117	488
Ontario	7320	18 970	391	1631



(a) Pour les questions suivantes, arrondis tes réponses au centième près.

  (i) Combien de fois la superficie du lac Supérieur est-elle supérieure à celle du lac Ontario ?

  (ii) Combien de fois le volume du lac Supérieur est-il supérieur à celui du lac Érié ?

  (iii) Le volume du lac Supérieur représente quel pourcentage du volume total des cinq lacs ?

(b) Pour les comparaisons dans la partie (a), est-ce que cela importe si tu utilises les données basées en milles ou en kilomètres ?

(c) Quelles sont les superficies moyenne et médiane des Grands Lacs en kilomètres carrés ?

(d) DÉCOUVERTE: En termes de superficie, le lac Supérieur est le deuxième plus grand lac du monde tandis que le lac Huron est le quatrième plus grand. Fais des recherches pour trouver les premier et troisième plus grands lacs au monde (en termes de superficie). Essaye de trouver des données passées pour voir comment leurs tailles ont évolué au fil du temps.

THÈME GESTION DES DONNÉES

Problem of the Week
Problem B and Solution
These Lakes Are Better Than Good

Problem

The table below shows data related to the five Great Lakes, which span the border between Canada and United States.

Lake	Area (miles^2)	Area (km^2)	Volume (miles^3)	Volume (km^3)
Superior	31 700	82 100	2900	12 070
Michigan	22 410	58 030	1180	4930
Huron	23 010	59 590	840	3520
Erie	9910	25 667	117	488
Ontario	7320	18 970	391	1631

(a) Find values for each of the following. Round your answers to two decimal places.

(i) How many times bigger is Lake Superior's area than Lake Ontario's?

(ii) How many times bigger is Lake Superior's volume than Lake Erie's?

(iii) What percentage of the total volume of all five lakes does Lake Superior contain?

(b) For the comparisons in part (a), does it matter whether you use the data based in miles, or in kilometres?

(c) What are the mean and the median areas of the Great Lakes in square kilometres?

(d) Discovery: Lake Superior is the second largest lake in the world, by area, and Lake Huron is the fourth largest. Do some research to find the first and third largest lakes (by area). Try to discover some past data to see how their sizes have changed over time.

Solution

(a) The approximate values of the comparisons using metric measures are as follows.

(i) Lake Superior's area is $\frac{82\,100}{18\,970} \approx 4.33$ times bigger than Lake Ontario's.

(ii) Lake Superior's volume is $\frac{12\,070}{488} \approx 24.73$ times bigger than Lake Erie's.

(iii) The total volume of all five lakes is $12\,070 + 4930 + 3520 + 488 + 1631 = 22\,639$ km^3. Thus, the percentage of the total volume of all five lakes contained by Lake Superior is $\frac{12\,070}{22\,639} \times 100\% \approx 53.32\%$.

(b) For these comparisons, it doesn't matter whether the data in miles, or in kilometres, is used, as long as the same unit is used for all lakes in the calculation. However, there may be slight variations in the values found in part (a), due to rounding and the precision of the values in the table.

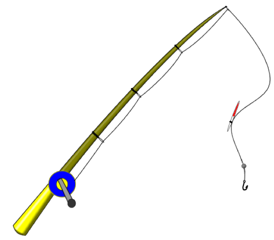
(c) The mean area is $\frac{1}{5}(82\,100 + 58\,030 + 59\,590 + 25\,667 + 18\,970) = \frac{244\,357}{5} = 48\,871.4$ km^2. To find the median area we look for the third largest area, which is $58\,030$ km^2.

(d) The lake with the greatest area in the world is the Caspian Sea, which is surrounded by Kazakhstan, Russia, Turkmenistan, Azerbaijan, and Iran, and has an area of $389\,000$ km^2. The third largest lake is Victoria Lake in Africa, surrounded by Uganda, Kenya, and Tanzania, with an area of $59\,940$ km^2. Looking at variation in water level gives some idea of how the area and volume of a lake changes over time. For example, Lake Superior's mean water level varies by only one metre or so over the year, although climate change seems to be causing greater fluctuations. It is estimated that the Caspian Sea water level will drop as much as eight metres or more in this century.

Problème de la semaine

Problème B

Il y a anguille sous roche...



Nawal est allée pêcher tous les jours à son endroit préféré au bord du lac Otter. Après sept jours, elle avait attrapé 18 bars, 5 brochets, 13 crapets arlequins, 2 perches et 1 truite.

(a) En se basant sur la pêche de la semaine dernière, quelle est la probabilité *expérimentale* (exprimée sous forme de fraction irréductible) que le prochain poisson que Nawal attrapera soit un crapet arlequin ? Quelle est la probabilité expérimentale que le prochain poisson que Nawal attrapera soit une truite ?

(b) Supposons que Nawal soit retournée pêcher pendant sept jours supplémentaires au même endroit. En te basant sur son expérience précédente, quelles prédictions peux-tu faire par rapport à ses prises ?

(c) Si Nawal est allée pêcher dans un nouvel endroit recommandé par un ami, que peux-tu prédire concernant ses prises à cet endroit en te basant sur son expérience précédente ?

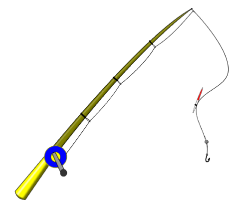
Problem of the Week

Problem B and Solution

Something's Fishy Here

Problem

Nawal went to a cottage on Otter Lake and went fishing every day in their favourite spot. After seven days, their catch included 18 bass, 5 pike, 13 bluegill, 2 perch, and 1 trout.



(a) Based on that week's fishing, what is the *experimental* probability (as a fraction in lowest terms) that the next fish Nawal catches is a bluegill? What is the experimental probability that the next fish Nawal catches is a trout?

(b) Suppose Nawal went fishing for another seven days in the same spot. What are some things you could predict about their catch, based on their previous experience?

(c) If Nawal went fishing in a new spot recommended by a friend, what could you predict about their catch in this spot, based on their previous experience?

Solution

(a) Nawal caught a total of $18 + 5 + 13 + 2 + 1 = 39$ fish.
Thus the experimental probability that the next fish Nawal catches is a bluegill is $\frac{13}{39} = \frac{1}{3}$. The experimental probability that the next fish Nawal catches is a trout is $\frac{1}{39}$.

(b) Since they are fishing in the same spot, we can predict that they will catch a total of about 39 fish, with a similar mix of bass, pike, bluegill, perch, and trout.
We can also predict that they will be more likely to catch bass or bluegill than pike, perch, or trout, given the proportions observed in their first week's catch.

(c) Since they are now fishing in a different spot, there wouldn't necessarily be the same types of fish, nor in the same proportions. On the bright side, since Nawal's friend recommended the spot, there may be more fish there than the previous spot.

Problème de la semaine

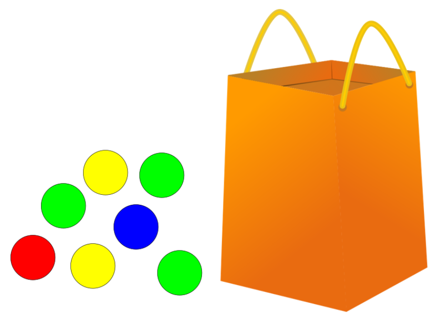
Problème B

C'est du cinquante-cinquante ou du sûr à cent pour cent ?

(a) Imagine que tu pioches des billes une à une dans un sac contenant 1 bille rouge, 1 bille bleue, 2 billes jaunes et 3 billes vertes. Lorsque tu pioches une bille du sac, tu la choisis au hasard, tu enregistres sa couleur, puis tu la replaces dans le sac. Supposons que chaque bille a la même probabilité d'être piochée.

Lesquels des événements suivants ont la même probabilité de se produire ?

- (i) Tu pioches une bille rouge.
- (ii) Tu pioches une bille bleue.
- (iii) Tu pioches une bille jaune.
- (iv) Tu pioches une bille verte.
- (v) Tu pioches une bille rouge OU une bille bleue.



Justifie tes réponses en comparant les probabilités théoriques des événements.

(b) Supposons que tu as deux dés à six faces. Sur l'un, figurent les nombres pairs 2, 4, 6, 8, 10 et 12, et sur l'autre, les nombres impairs 1, 3, 5, 7, 9 et 11. Tu jettes les deux dés simultanément puis tu détermines la somme des deux faces supérieures. Quelle est la probabilité de chacun des événements suivants ?

- (i) La somme est impaire.
- (ii) La somme est égale à 7.
- (iii) La somme est égale à 25.

(c) Parmi les événements de la partie (b), lesquels peuvent être qualifiés de *certains*? Lesquels des événements de la partie (b) peuvent être qualifiés d'*impossibles* ?

Problem of the Week

Problem B and Solution

Equally Likely or A Sure Thing?

Problem

(a) Imagine you are drawing marbles one at a time from a bag which contains 1 red, 1 blue, 2 yellow, and 3 green marbles. You draw a marble without looking, record the colour, and return it to the bag. Suppose that each marble is equally likely to be drawn.

Which of the following events are equally likely to occur?

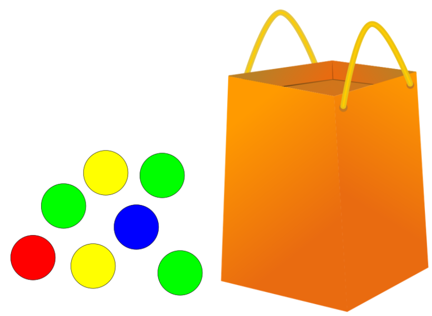
(i) You draw a red marble.

(ii) You draw a blue marble.

(iii) You draw a yellow marble.

(iv) You draw a green marble.

(v) You draw a red OR a blue marble.



Justify your answers by comparing the theoretical probabilities of the events.

(b) Suppose you have two unusual six-sided dice (number cubes), one with the even numbers 2, 4, 6, 8, 10, and 12 on its faces, and the other with the odd numbers 1, 3, 5, 7, 9, and 11 on its faces. When you roll the dice together, you find the sum of the two top faces. What is the probability of each of the following events?

(i) The sum is odd.

(ii) The sum is 7.

(iii) The sum is 25.

(c) Which of the events in part (b) can be called *certain*? Which of the events in part (b) can be called *impossible*?

Solution

(a) The theoretical probability in each case is equal to

$$\frac{\text{the number of marbles of the desired colour(s)}}{\text{the total number of marbles}}$$

Using this and the fact that there are 7 marbles in total, we calculate the probability of each event.

(i) The probability of drawing a red marble is $\frac{1}{7}$, since there is only one red marble.

(ii) The probability of drawing a blue marble is $\frac{1}{7}$, since there is only one blue marble.

(iii) The probability of drawing a yellow marble is $\frac{2}{7}$, since there are two yellow marbles and either of the two could be drawn.

(iv) The probability of drawing a green marble is $\frac{3}{7}$, since there are three green marbles and any of the three could be drawn.

(v) The probability of drawing a red or a blue marble is $\frac{2}{7}$, since there are two marbles that are red or blue.

Therefore, events (i) and (ii) are equally likely to occur. Also, events (iii) and (v) are equally likely to occur.

(b) For each roll of the die with even numbers on its faces, there are 6 possible rolls for the die with odd numbers on its faces. Thus, since the the die with even numbers on its faces has 6 faces, there are $6 \times 6 = 36$ possible rolls. Using this, we can calculate the probability of each event.

(i) An odd number plus an even number is always odd, so every roll will produce an odd sum. Thus, the probability that the sum is odd is equal to $\frac{36}{36} = 1$.

(ii) A sum of 7 could be produced by rolling a 1 and a 6, rolling a 2 and a 5, or rolling a 3 and a 4. Thus, the probability that the sum is 7 is equal to $\frac{3}{36} = \frac{1}{12}$.

(iii) The maximum possible sum is $11 + 12 = 23$, so there is no way to roll a sum of 25. Thus, the probability that the sum is 25 is equal to $\frac{0}{36} = 0$.

(c) Since the probability of event (i) is 1, then event (i) can be called *certain*. Since the probability of event (iii) is 0, then event (iii) can be called *impossible*.

Problème de la semaine

Problème B

Joueur !

Geoff joue à un jeu dans lequel on lance deux dés réguliers à six faces, l'un noir et l'autre blanc. Pour gagner, il doit lancer les dés et obtenir deux nombres dont la somme est 11.

(a) Quelle est la probabilité qu'il obtienne un 7 en utilisant uniquement le dé noir?

(b) Quelle est la probabilité théorique qu'il obtienne un 1 avec le dé noir et un 6 avec le dé blanc ?

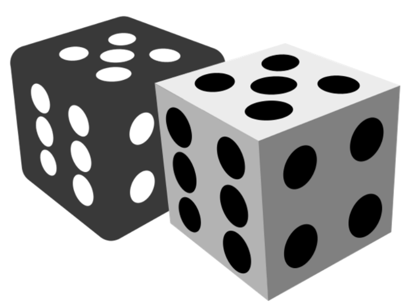
(c) S'il lance les deux dés et qu'il calcule la somme des nombres obtenus, quelle(s) somme(s) a (ont) la plus faible probabilité théorique d'être obtenue(s)?

(d) Quelle est la probabilité théorique d'obtenir une somme de 7 en lançant les deux dés ?

(e) Quelle est la probabilité théorique d'obtenir une somme de 11 en lançant les deux dés ?

(f) Lance les deux dés trente-six fois et compte le nombre de fois où tu obtiens une somme de 11. D'après tes observations, quelle était la probabilité empirique d'obtenir une somme de 11 ?

(g) Compare tes résultats de la partie (f) avec ceux de tes camarades. Pour combien d'entre eux la probabilité empirique d'obtenir une somme de 11 était-elle égale à la probabilité théorique d'obtenir une somme de 11 ?



Problem of the Week
Problem B and Solution
Gamer!

Problem

Geoff plays a game using two standard six-sided dice: a black one and a white one. To win the game, Geoff must roll the dice and have the numbers on the two top faces sum to 11.

(a) What is the probability that he rolls a 7 with just the black die?

(b) What is the theoretical probability that he rolls a 1 on the black die and a 6 on the white die?

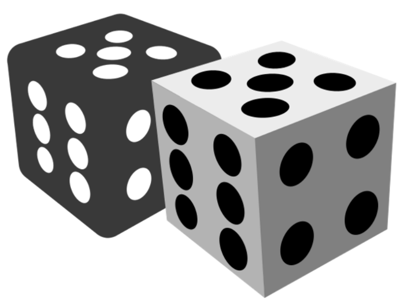
(c) If he rolls both dice and calculates the sum of the numbers on the two top faces, what sum(s) have the lowest theoretical probability of being rolled?

(d) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 7?

(e) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 11?

(f) Roll two dice thirty-six times and keep track of the number of times the numbers on the two top faces sum to 11. What was your empirical probability of rolling a sum of 11?

(g) Share your results in part (f) with your classmates. How many had their empirical probability of rolling a sum of 11 equal the theoretical probability of rolling a sum of 11?



Solution

(a) Since the numbers on the faces of a standard six-sided die are 1, 2, 3, 4, 5, and 6, it is impossible to roll a 7. So the probability is 0.

(b) For each of the 6 possible numbers he could throw with the black die there are 6 possible numbers on the white die, so the total number of possible outcomes is $6 \times 6 = 36$. Thus, the theoretical probability that he throws a 1 on the black die and a 6 on the white die is 1 in 36, or $\frac{1}{36}$.

Alternatively, to solve this problem we can create a table where the columns show the possible numbers on the top face of the white die, the rows show the possible numbers on the top face of the black die, and each cell in the body of the table gives the sum of the corresponding pair of numbers.

		White Die					
		1	2	3	4	5	6
Black Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

From this table, we can conclude that there is 1 outcome out of 36 possible outcomes where the number on the top face of the black die is a 1 and the number on the top face of the white die is a 6. We will use this table in our answers to parts (c), (d), and (e).

(c) For each of the sums of 2 and 12, there is only one possible way to obtain that outcome (two ones or two sixes). Thus, each of these sums has the lowest theoretical probability, namely 1 in 36, or $\frac{1}{36}$.

(d) A sum of 7 can be obtained in 6 possible ways (as $1 + 6$ or $6 + 1$, $2 + 5$ or $5 + 2$, $3 + 4$ or $4 + 3$). So, there are six outcomes which give the desired sum. Thus, the theoretical probability that he rolls a 7 is 6 in 36, or $\frac{6}{36}$, which is equivalent to 1 in 6, or $\frac{1}{6}$.

(e) A sum of 11 can be obtained in 2 possible ways (as $5 + 6$ or $6 + 5$). Thus, the theoretical probability of rolling an 11 is 2 in 36, or $\frac{2}{36}$, which is equivalent to 1 in 18, or $\frac{1}{18}$.

(f) Answers will vary.

(g) Answers will vary.

Problème de la semaine

Problème B

Ces taux sont choquants

La plupart des provinces considèrent l'heure de la journée lorsqu'elles établissent les tarifs de consommation d'électricité. Ces tarifs sont souvent appelés tarifs en fonction de l'heure de consommation (FHC). En utilisant les exemples de tarifs FHC dans le tableau ci-dessous, réponds aux questions suivantes.

Période de tarif FHC	**1^er^ novembre - 30 avril Heure du jour**	**1^er^ mai - 31 octobre Heure du jour**	**Tarif FHC (¢ par kWh)**
Période creuse	Les jours de semaine de 19 h à 7 h, à toute heure les fins de semaine	Les jours de semaine de 19 h à 7 h, à toute heure les fins de semaine	$7,4$
Période médiane	Les jours de semaine de 11 h à 17 h	Les jours de semaine de 7 h à 11 h et de 17 h à 19 h	$10,2$
Période de pointe	Les jours de semaine de 7 h à 11 h et de 17 h à 19 h	Les jours de semaine de 11 h à 17 h	$15,1$

(a) La famille de Garret a consommé 50 kWh un samedi après-midi. Combien ces 50 kWh ont-ils coûté ?

(b) Le 10 novembre, quel serait le meilleur moment de la journée pour utiliser ta sécheuse ?

(c) À quel moment devrais-tu éviter d'utiliser ta sécheuse en été ?

(d) Quelle serait une méthode plus avantageuse (écologiquement et financièrement) pour sécher tes vêtements en été ?

(e) La famille de Ramal a consommé 1180 kWh d'électricité en un mois.

   (i) Quel est le montant maximal (en dollars) qu'ils auraient pu payer pour l'électricité ce mois-là ?

   (ii) Quel est le montant minimal (en dollars) qu'ils auraient pu payer pour l'électricité ce mois-là ?

Thèmes Gestion des données, Sens du nombre

Problem of the Week

Problem B and Solution

These Rates are Shocking

Problem

Most provinces take into consideration the time of day when they charge for electricity usage. The rates they charge are often referred to as Time-Of-Use (TOU) rates. Using the sample TOU rates in the table below, answer the questions that follow.

TOU Price Period	November 1 - April 30 Time of Day	May 1 - October 31 Time of Day	TOU Rate (¢ per kWh)
Off-Peak Hours	Weekdays 7 p.m. - 7 a.m., anytime on weekends	Weekdays 7 p.m. - 7 a.m., anytime on weekends	7.4
Mid-Peak Hours	Weekdays 11 a.m. - 5 p.m.	Weekdays 7 a.m. - 11 a.m. and 5 p.m. - 7 p.m.	10.2
On-Peak Hours	Weekdays 7 a.m. - 11 a.m. and 5 p.m. - 7 p.m	Weekdays 11 a.m. - 5 p.m.	15.1

(a) Garret's family used 50 kWh on a Saturday afternoon. What would be the charge for those 50 kWh?

(b) On November 10, when would be the best time of day to run your clothes dryer?

(c) When should you avoid using your clothes dryer in the summer?

(d) What might be a better way (environmentally and financially) to dry your clothes in the summer?

(e) Ramal's family used 1180 kWh hours of electricity in one month.

  (i) What is the maximum amount of money (in dollars) they could have paid for electricity that month?

  (ii) What is the minimum amount of money (in dollars) they could have paid for electricity that month?

Solution

(a) The rate for any Saturday is 7.4¢ per kWh, which is \$0.074 per kWh. Therefore, the charge for 50 kWh would be $50 \times \$0.074 = \$3.70$.

(b) If November 10 falls on a weekday, the best time to run the dryer would be anytime before 7 a.m. or after 7 p.m. If November 10 falls on the weekend, you could run it anytime from Friday after 7 p.m. until Monday morning before 7 a.m.

(c) You should avoid running your dryer from 7 a.m. to 7 p.m. on weekdays, but it is most expensive to run your dryer between 11 a.m. and 5 p.m.

(d) You could hang your clothes out to dry in the summer which would have little or no cost, both environmentally and financially.

(e) (i) Ramal's family used 1180 kWh. The most they could have paid for electricity is \$0.151 per kWh. Therefore, the maximum amount they could have paid for electricity that month is $1180 \times \$0.151 = \$178.18$.

(ii) Ramal's family used 1180 kWh. The least they could have paid for electricity is \$0.074 per kWh. Therefore, the minimum amount they could have paid for electricity that month is $1180 \times \$0.074 = \$87.32$.

Problème de la semaine

Problème B

Les cinq premiers Jeux olympiques modernes

En août 2024, Paris, France, accueillera les Jeux olympiques d'été. Le tableau ci-dessous présente les sports inscrits aux programmes des cinq premiers Jeux olympiques modernes.

Année	Lieu	Sports
1896	Athènes, Grèce	l'athlétisme, le cyclisme sur route, le cyclisme sur piste, l'escrime, la gymnastique, le tir, la natation, le tennis, l'haltérophilie, la lutte
1900	Paris, France	le tir à l'arc, l'athlétisme, la pelote basque, le cricket, le croquet, le cyclisme sur piste, l'équitation, l'escrime, le football, le golf, la gymnastique, le polo, l'aviron, le rugby, la voile, le tir, la natation, le tennis, le tir à la corde, le water-polo
1904	Saint-Louis, États-Unis	le tir à l'arc, l'athlétisme, la boxe, le cyclisme sur piste, le plongeon, l'escrime, le football, le golf, la gymnastique, la crosse, le roque, l'aviron, la natation, le tennis, le tir à la corde, le water-polo, l'haltérophilie, la lutte
1908	Londres, Angleterre	le tir à l'arc, l'athlétisme, la boxe, le cyclisme sur piste, le plongeon, l'escrime, le patinage artistique, le football, le hockey sur gazon, la gymnastique, le jeu de paume, la crosse, le polo, le jeu de raquettes, l'aviron, le rugby, la voile, le tir, la natation, le tennis, le tir à la corde, le motonautisme, le water-polo, la lutte
1912	Stockholm, Suède	l'athlétisme, le cyclisme sur route, le plongeon, l'équitation, l'escrime, le football, la gymnastique, le pentathlon moderne, l'aviron, la voile, le tir, la natation, le tennis, le tir à la corde, le water-polo, la lutte

(a) Crée un graphique qui représente le nombre de sports inscrits aux programmes des cinq premiers jeux olympiques modernes.

(b) Formule cinq observations ou conclusions à partir de ton graphique.

Thème Gestion des données

Problem of the Week
Problem B and Solution
The First Five Modern Olympics

Problem

In August 2024, Paris, France will host the Summer Olympic Games. The table below contains information about the sports that were at some of the first five modern Olympic Games.

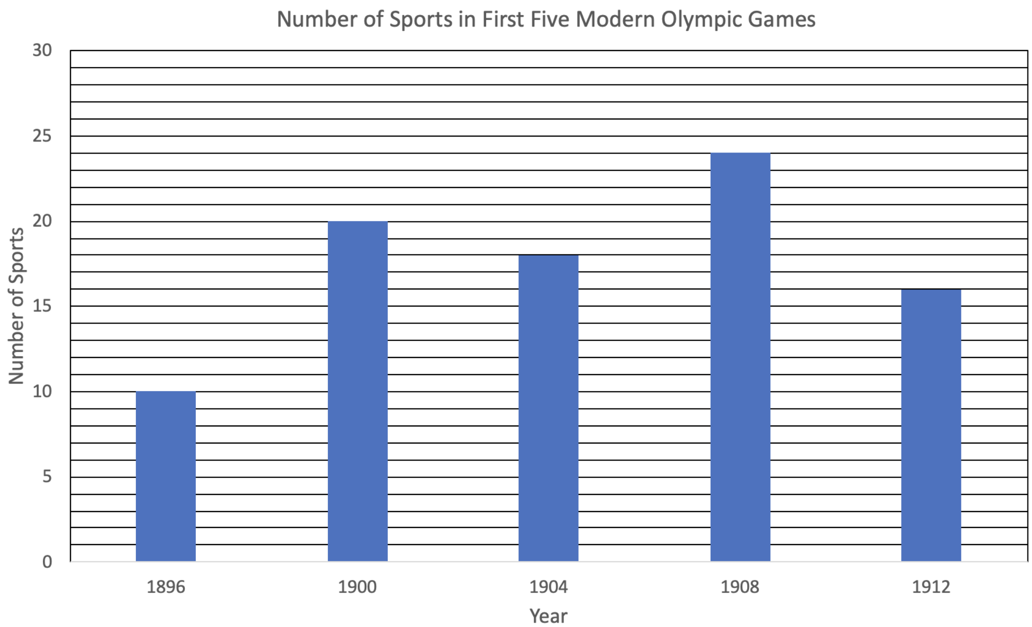
Year	Location	Sports
1896	Athens, Greece	athletics, road cycling, track cycling, fencing, gymnastics, shooting, swimming, tennis, weightlifting, wrestling
1900	Paris, France	archery, athletics, Basque pelota, cricket, croquet, track cycling, equestrian, fencing, football, golf, gymnastics, polo, rowing, rugby, sailing, shooting, swimming, tennis, tug-of-war, water polo
1904	St. Louis, USA	archery, athletics, boxing, track cycling, diving, fencing, football, golf, gymnastics, lacrosse, roque, rowing, swimming, tennis, tug-of-war, water polo, weightlifting, wrestling
1908	London, England	archery, athletics, boxing, track cycling, diving, fencing, figure skating, football, field hockey, gymnastics, jeu de paume, lacrosse, polo, rackets, rowing, rugby, sailing, shooting, swimming, tennis, tug-of-war, water motorsports, water polo, wrestling
1912	Stockholm, Sweden	athletics, road cycling, diving, equestrian, fencing, football, gymnastics, modern pentathlon, rowing, sailing, shooting, swimming, tennis, tug-of-war, water polo, wrestling

(a) Create a graph that displays the number of sports in each of the first five modern Olympic Games.

(b) State five observations/conclusions from your graph.

Solution

(a) Answers may vary. A bar graph displaying this information is shown below.



(b) Observations gleaned from the graphs will vary. Here are some possible observations.

- The fewest number of sports occurred in the year 1896.
- The most number of sports occurred in the year 1908.
- Other than 1896 and 1908, there were about the same number of sports (between 16 and 20).
- The number of sports doubled from 1896 to 1900.
- 18 is the median number of sports. (Since there are five bars, the third highest bar is the median.) This occurred in the year 1904.
- There doesn't seem to be a steady increase in the number of events, so it would be hard to predict how many events future games would hold.

Problème de la semaine

Problème B

Nouveaux sports olympiques

En août 2024, Paris, France accueillera les Jeux olympiques d'été. Lors de ces Jeux olympiques, le breakdance deviendra une discipline olympique pour la première fois. Au fil des années, les sports inscrits aux programmes des Jeux olympiques ont évolué. Les tableaux ci-dessous présentent les nouveaux sports introduits lors des Jeux olympiques d'été des 20 dernières années, ainsi que leur année d'introduction.

Sport	Année
La natation artistique	1984
Le badminton	1992
Le baseball	1992
Le volleyball de plage	1996
Le BMX Freestyle	2020
Le BMX Racing	2008
Le breakdance	2024

Sport	Année
Le karaté	2020
La natation marathon	2008
Le vélo tout terrain	1996
La gymnastique rythmique	1984
Le rugby à sept	2016
Le skateboard	2020
Le softball	1996

Sport	Année
L'escalade sportive	2020
Le surf	2020
Le tennis de table	1988
Le taekwondo	2000
Le trampoline	2000
Le triathlon	2000
Le basketball 3×3	2020

(a) Organise et représente ces données à l'aide d'un graphique pour illustrer le nombre de nouveaux sports qui ont été introduits chaque année.

(b) Crée une infographie sur les nouveaux sports aux Jeux olympiques d'été en utilisant les informations du tableau et/ou de ton graphique.

Thème Gestion des données

Problem of the Week
Problem B and Solution
Introducing Olympic Sports

Problem

In August 2024, Paris, France will host the Summer Olympic Games. At these Olympics, breaking (also known as breakdancing) will be included for the first time. Over the years, the sports at the Summer Olympics have changed. The following tables show new sports at the Summer Olympics in the past 40 years, along with the year in which they were first introduced.

Sport	Year
Artistic Swimming	1984
Badminton	1992
Baseball	1992
Beach Volleyball	1996
BMX Freestyle	2020
BMX Racing	2008
Breaking	2024

Sport	Year
Karate	2020
Marathon Swimming	2008
Mountain Biking	1996
Rhythmic Gymnastics	1984
Rugby Sevens	2016
Skateboarding	2020
Softball	1996

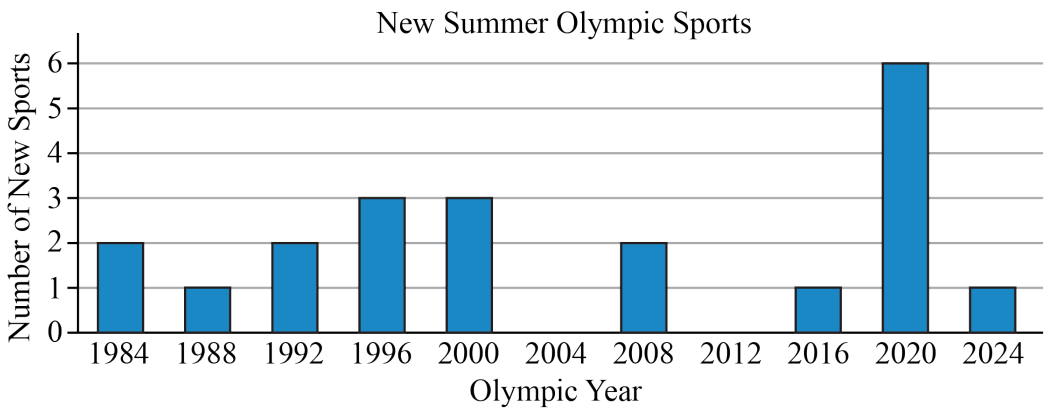
Sport	Year
Sport Climbing	2020
Surfing	2020
Table Tennis	1988
Taekwondo	2000
Trampoline	2000
Triathlon	2000
3×3 Basketball	2020

(a) Organize and represent this data in a graph to show how many new sports were introduced each year.

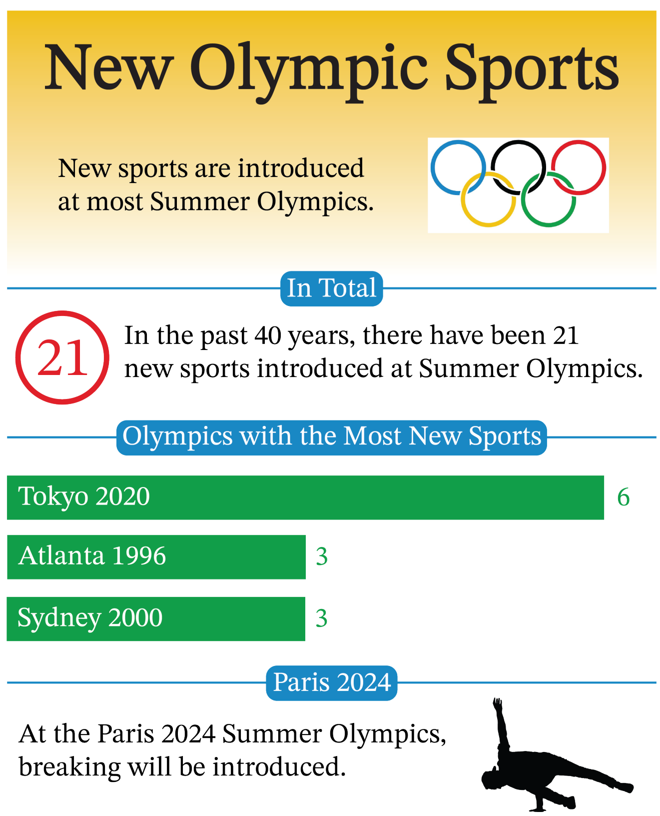
(b) Create an infographic about new sports at the Summer Olympics using information from the table and/or your graph.

Solution

(a) A bar graph can be used to show how many new sports were introduced each year. This is shown below.



(b) An example of an infographic is shown.



Géométrie et mesure (G)

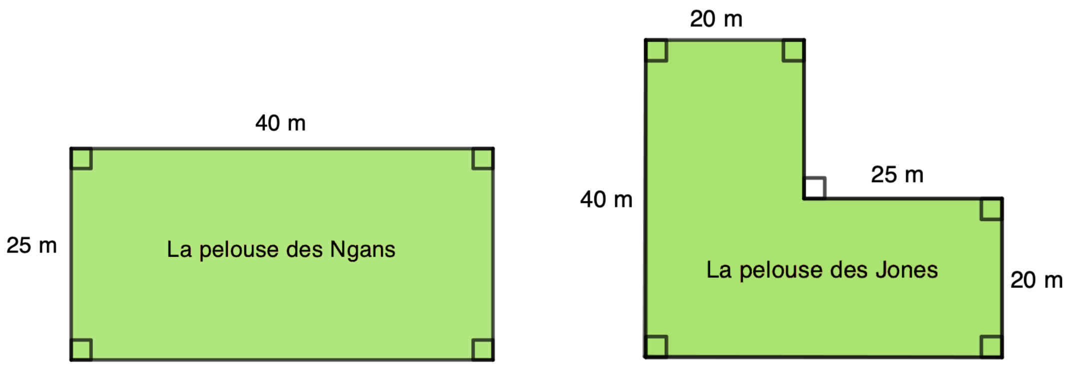
Amène-moi à la couverture

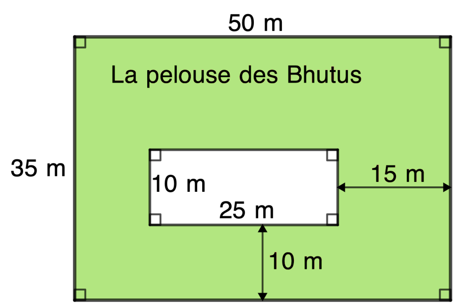
Problème de la semaine

Problème B

Tondre des pelouses

Jon a une entreprise de tonte de pelouse. Trois des pelouses qu'il tond sont représentées dans les figures ci-dessous. Les pelouses sont ombrées en vert et tous les angles sont des angles droits. Sa tondeuse peut couper une rangée d'herbe d'une largeur de 1 mètre.





(a) Laquelle des pelouses nécessitera le plus de temps à tondre? Explique ton raisonnement.

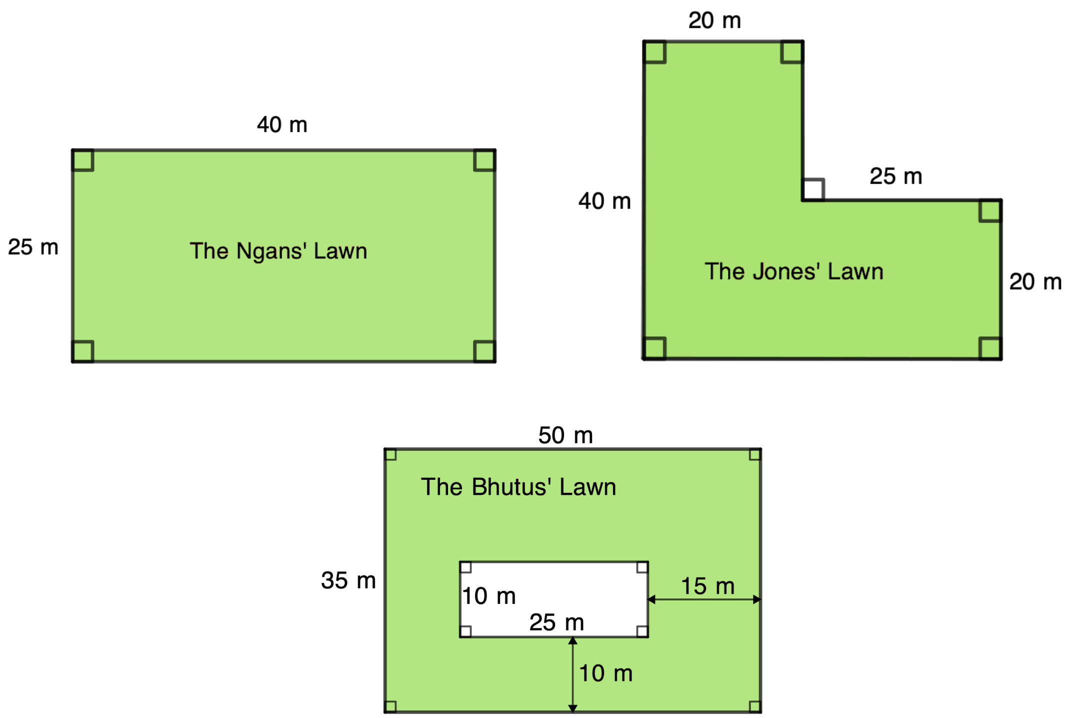
(b) Sachant que sa tondeuse se déplace à une vitesse de 3 km par heure, quelle est la superficie, en m^2, qu'il peut tondre en une heure?

(c) Combien de temps, en minutes, lui faudra-t-il pour tondre chaque pelouse?

Problem of the Week
Problem B and Solution
Don't Get Mowed Over!

Problem

Jon has a grass cutting business. Three of the lawns that he cuts are shown. The lawns are shaded in green and all angles are right angles. His lawnmower can cut a swath of width 1 metre.



(a) Which lawn will take the longest to cut? Explain your reasoning.

(b) His lawnmower travels at 3 km per hour. What area of lawn, in m^2, can he cut in one hour?

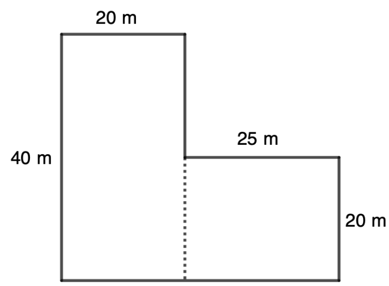
(c) How long, in minutes, will it take him to cut each lawn?

Solution

(a) Since each lawn is composed of regions that have integer lengths, in metres, and since the lawnmower can cut a swath of width 1 metre, we can compare the areas of the lawns to determine which will take the longest to cut.

The Ngans' lawn is a rectangle measuring 25 m $\times$ 40 m. Its total area is therefore 25 m $\times$ 40 m $= 1000\,\text{m}^2$.

The Jones' lawn can be divided into two smaller rectangles as shown.



The total area of the Jones' lawn is equal to the sum of the area of the rectangle on the left and the area of the rectangle on the right. The rectangle on the left has area equal to 40 m $\times$ 20 m $=$ 800 m^2. The rectangle on the right has area equal to 20 m $\times$ 25 m $=$ 500 m^2. Thus, the total area of the Jones' lawn is equal to 800 m^2 + 500 m^2 = 1300 m^2.

The area of the Bhutus' lawn can be found by finding the area of the outer rectangle and subtracting the area of the inner rectangle. The area of the outer rectangle is equal to 50 m $\times$ 35 m $=$ 1750 m^2. The area of the inner rectangle is equal to 10 m $\times$ 25 m $=$ 250 m^2. Thus, the total area of the Bhutus' lawn is equal to 1750 m^2 $-$ 250 m^2 $=$ 1500 m^2.

Since the Bhutus' lawn has the largest area, it will take the longest to cut.

NOTE: To determine the area of the Bhutus' lawn, we could have alternatively divided the lawn into smaller rectangles, and summed the areas of those rectangles.

(b) Jon's mower is 1 m wide and it travels at 3 km/h, or 3000 m/h. Therefore, he can cut 1 m $\times$ 3000 m $=$ 3000 m^2 in one hour.

(c) The Ngans' lawn has area equal to 1000 m^2. Thus, it would take $1000 \div 3000 = 0.333$ (or $\frac{1}{3}$) of an hour, which is $\frac{1}{3} \times 60 = 20$ minutes to cut the lawn.

The Jones' lawn has area equal to 1300 m^2. It would take $1300 \div 3000 = 0.4333$ (or $\frac{13}{30}$) of an hour, which is $\frac{13}{30} \times 60 = 26$ minutes to cut the lawn.

The Bhutus' lawn has area equal to 1500 m^2. It would take $1500 \div 3000 = 0.5$ (or $\frac{1}{2}$) of an hour, which is $\frac{1}{2} \times 60 = 30$ minutes to cut the lawn.

Problème de la semaine

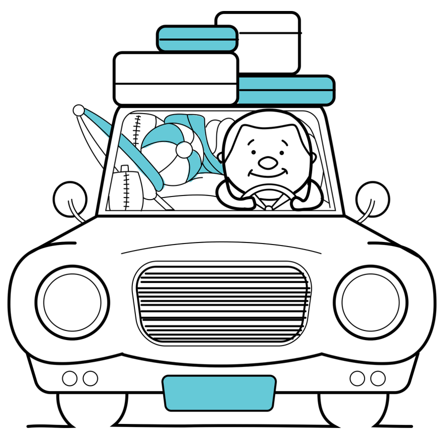
Problème B

Virée en voiture

M. Sand prévoit un voyage à la plage. La distance totale jusqu'à la plage est de 263 km. Sa voiture possède un réservoir de 60 L et peut parcourir 640 000 m avec ce plein.

Supposons que M. Sand a accès à deux stations-service. À la station A, 25 L d'essence coûtent 40 \$, tandis qu'à la station B, 30 L d'essence coûtent 51 \$.

Détermine le coût de l'essence pour son voyage s'il fait le plein à la station A par rapport au coût s'il fait le plein à la station B. Laquelle des stations est la plus économique ?



Thèmes Géométrie et mesure, Sens du nombre

Problem of the Week

Problem B and Solution

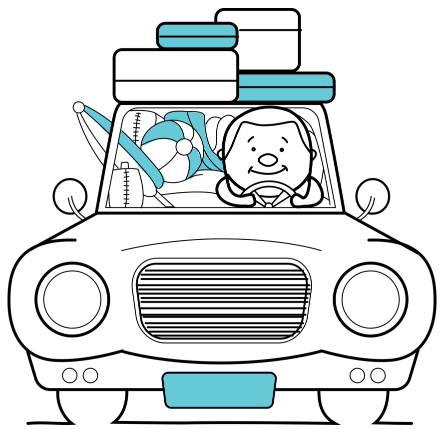
Road Trip

Problem

Mr. Sand is going on a trip to the beach. The total distance to the beach is 263 km. His car has a 60 L gas tank and can travel 640 000 m on that tank of gas.

Suppose that there are two service stations available to Mr. Sand. Station A charges \$40 for 25 L of gas, while Station B charges \$51 for 30 L of gas.

Determine the cost of the gas for his trip if he fills up at Station A versus the cost if he fills up at Station B. Which is the more economical?



Solution

If his vehicle has a 60 L gas tank and will travel 640 000 m or 640 km on one full tank, then he is using $60 \div 640 = 0.09375$ L of gas per km.

Since the distance to the beach is 263 km, then this trip will take $263 \times 0.09375 \approx 24.656$ L of gas.

For Station A:

The cost is \$40 for 25 L. Therefore, the gas will cost $\frac{40}{25} = \$1.60$ per L.

Thus, the cost of the trip for Station A is $24.656 \times \$1.60 = \$39.45$.

For Station B:

The cost is \$51 for 30 L. Therefore, the gas will cost $\frac{51}{30} = \$1.70$ per L.

Thus, the cost of the trip for Station B is $24.656 \times \$1.70 = \$41.92$.

Therefore, Station A is more economical than Station B.

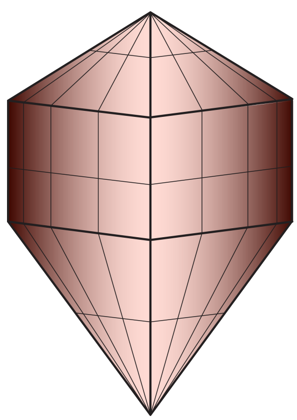
NOTE: Since the gas at Station A costs less per L than at Station B, then using gas from Station A will always cost less than using gas from Station B.

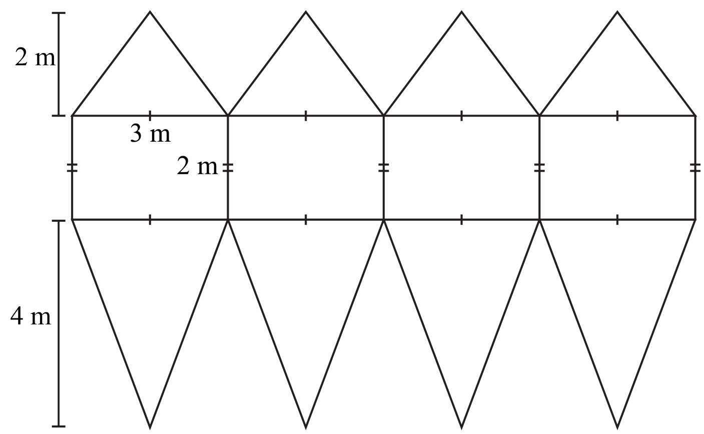
Problème de la semaine

Problème B

Tissage aérien : un défi de ballon

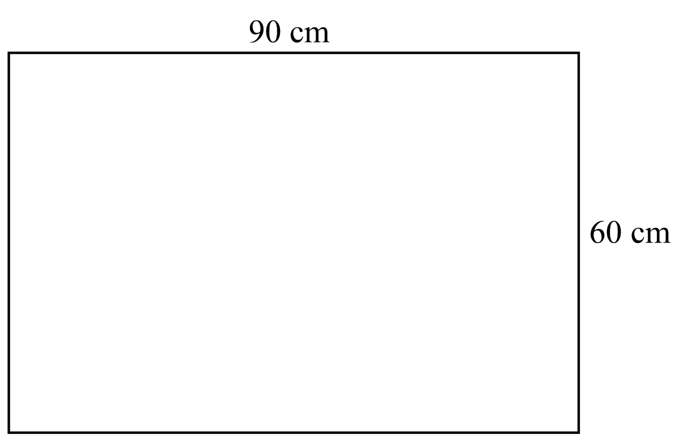
En 1783, les frères Montgolfier réalisèrent le premier vol en montgolfière de l'histoire à l'aide d'un ballon qu'ils avaient fabriqué à partir de tissu et de papier. Bien que le vol n'ait été que de courte durée, il fut un véritable succès. Inspiré par cette prouesse, Vijay a décidé dans le cadre d'une exposition d'art de créer un modèle du ballon en utilisant du carton. Dans les figures ci-dessous, on voit le modèle de Vijay, ainsi que son développement. Note bien que ces illustrations ne sont pas représentées à l'échelle.





(a) Détermine l'aire du carton que Vijay a utilisé pour son modèle, autrement dit, l'*aire totale* de son modèle.

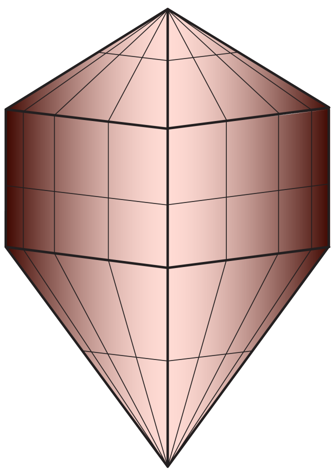
(b) Imaginons que tu disposes d'une feuille de papier de 90 cm $\times$ 60 cm. Dessine un développement qui te permettrait de créer un ballon avec cette feuille et indique toutes les dimensions.

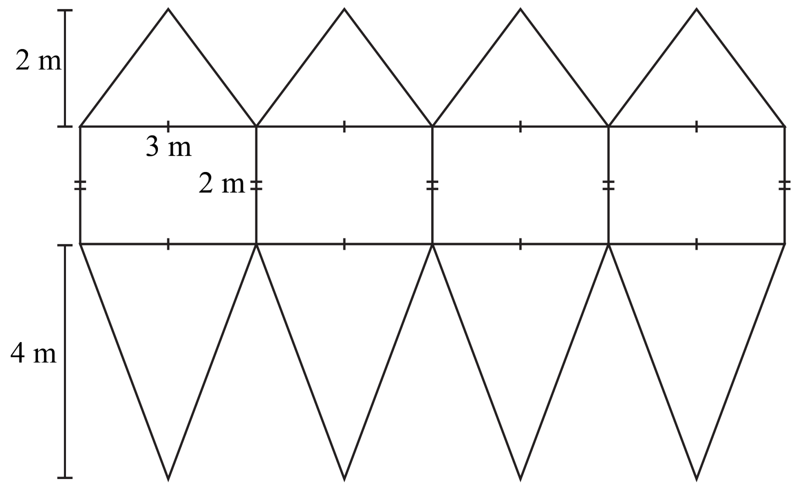


Problem of the Week
Problem B and Solution
How to Net a Balloon

Problem

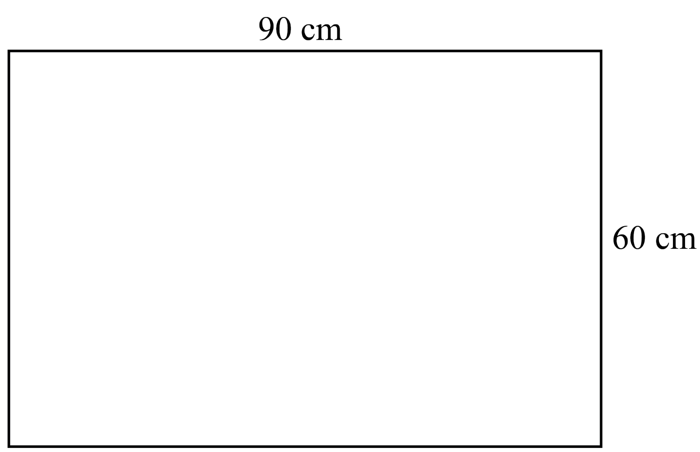
In 1783, the Montgolfier brothers launched the first hot air balloon flight in history, using a balloon that they created out of fabric and paper. The flight was short but successful. For an art exhibit, Vijay used cardboard to create a model of a balloon inspired by the Montgolfier brothers' balloon. Vijay's model, as well as its net, are shown below. Note that these diagrams are not drawn to scale.





(a) Calculate the total area of cardboard Vijay used in his model. This is also called the *surface area* of Vijay's model.

(b) Suppose you have a sheet of paper measuring 90 cm by 60 cm. Draw a net for a balloon that you could make using this sheet of paper. Write the dimensions for each shape on your net.



Solution

(a) To calculate the total area we notice that the net has four triangles on top, four rectangles, and four triangles on the bottom. We will calculate the area of each of these shapes separately and then add them together.

$$\begin{aligned}\text{Area of one top triangle} &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 3 \times 2\\ &= \frac{1}{2} \times 6\\ &= 3 \text{ m}^2\end{aligned}$$

Since there are four top triangles, the total area is $4 \times 3 = 12$ m^2.

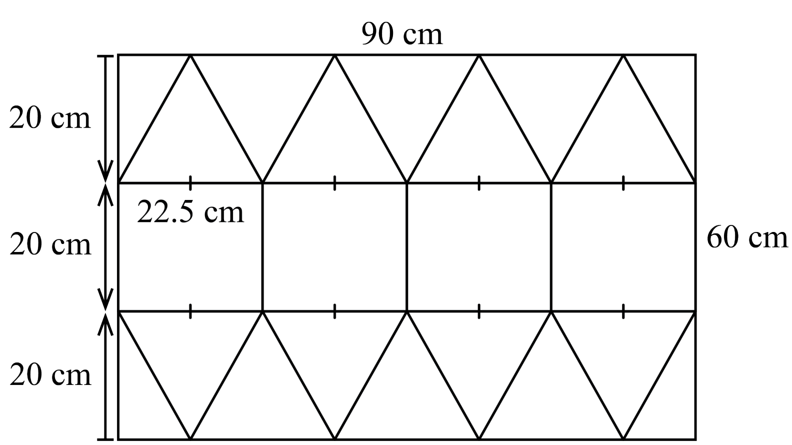
$$\begin{aligned}\text{Area of one rectangle} &= \text{length} \times \text{width}\\ &= 3 \times 2\\ &= 6 \text{ m}^2\end{aligned}$$

Since there are four rectangles, the total area is $4 \times 6 = 24$ m^2.

$$\begin{aligned}\text{Area of one bottom triangle} &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 3 \times 4\\ &= \frac{1}{2} \times 12\\ &= 6 \text{ m}^2\end{aligned}$$

Since there are four bottom triangles, the total area is $4 \times 6 = 24$ m^2.
Therefore, the total area of cardboard used is $12 + 24 + 24 = 60$ m^2.

(b) There are many possible nets. Here is one.



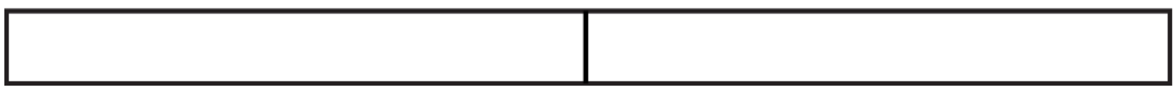
Problème de la semaine

Problème B

Évolution rectangulaire

Gaby a dessiné un rectangle et l'a nommé *Schéma 1*.

Ensuite, elle a dessiné un rectangle divisé en deux parties égales et l'a nommé *Schéma 2*.



Elle a ensuite compté le nombre total de rectangles dans *Schéma 2*. Il y a 1 rectangle à gauche, 1 rectangle à droite et le rectangle entier initial, soit un total de 3 rectangles.

Gaby a ensuite dessiné un rectangle divisé en trois parties égales, qu'elle a nommé *Schéma 3*.

Gaby a compté un total de 6 rectangles dans *Schéma 3*. Peux-tu le confirmer ?

(a) Gaby a continué à dessiner des schémas en divisant un rectangle en parties égales. *Schéma 4* est divisé en quatre parties égales, *Schéma 5* est divisé en cinq parties égales et ainsi de suite. Remplis le tableau en déterminant le nombre total de rectangles dans chaque schéma. Dessine les schémas pour t'aider, puis cherche une régularité dans le nombre total de rectangles.

Numéro du schéma	Nombre total de rectangles
1	1
2	3
3	6
4	
5	
6	

(b) Utilise la régularité que tu as trouvée dans la partie (a) pour prédire le nombre total de rectangles dans *Schéma 12*.

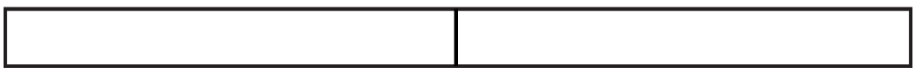
THÈMES ALGÈBRE, GÉOMÉTRIE ET MESURE

Problem of the Week
Problem B and Solution
Wrecked Tangles

Problem

Gaby drew a rectangle and called it *Diagram 1*.

She then drew a rectangle divided into two equal parts, and called *Diagram 2*.



She then counted the total number of rectangles in *Diagram 2*. There is 1 rectangle on the left, 1 rectangle on the right, and the original whole rectangle, which makes 3 rectangles in total.

Gaby then drew a rectangle divided into three equal parts, called *Diagram 3*.

Gaby counted a total of 6 rectangles in *Diagram 3*. Can you confirm this?

(a) Gaby continued drawing diagrams by dividing a rectangle into equal parts. *Diagram 4* is divided into four equal parts, *Diagram 5* is divided into five equal parts, and so on. Complete the table by determining the total number of rectangles in each diagram. Draw the diagrams to help you, and then look for a pattern in the total number of rectangles.

Diagram Number	Total Number of Rectangles
1	1
2	3
3	6
4	
5	
6	

(b) Use the pattern you found in part (a) to predict the total number of rectangles in *Diagram 12*.

Solution

(a) For each rectangle, we will assign the smallest rectangle a length of one unit.

Diagram 4 is a rectangle divided into 4 equal parts. In this diagram, there are 4 rectangles of length one unit, 3 of length two units, 2 of length three units, and 1 of length four units. This is a total of $4 + 3 + 2 + 1 = 10$ rectangles.

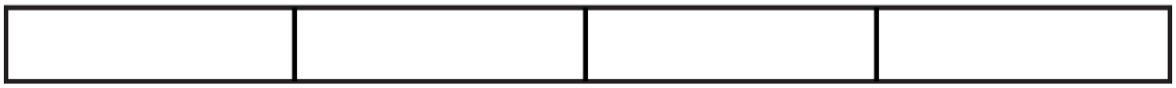
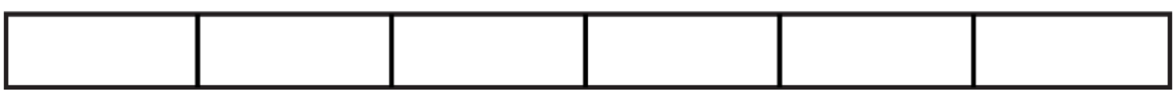


Diagram 5 is a rectangle divided into 5 equal parts. In this diagram, there are 5 rectangles of length one unit, 4 of length two units, 3 of length three units, 2 of length four units, and 1 of length five units. This is a total of $5 + 4 + 3 + 2 + 1 = 15$ rectangles.

Diagram 6 is a rectangle divided into 6 equal parts. In this diagram, there are 6 rectangles of length one unit, 5 of length two units, 4 of length three units, 3 of length four units, 2 of length five units, and 1 of length six units. This is a total of $6 + 5 + 4 + 3 + 2 + 1 = 21$ rectangles.



Now we see a pattern. The total number of rectangles for each diagram is equal to the sum of the diagram number and all the whole numbers smaller than it. Alternatively, the total number of rectangles for each diagram is equal to the diagram number plus the previous number of rectangles. So, the total number of rectangles in *Diagram 7* is equal to
$7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$, or $21 + 7 = 28$.

(b) Using the pattern from part (a), the total number of rectangles in *Diagram 12* is equal to $12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 78$, or $28 + 8 + 9 + 10 + 11 + 12 = 78$.

Problème de la semaine

Problème B

Conversion de température

Les degrés Celsius et Fahrenheit représentent les deux unités les plus utilisées pour mesurer la température. Il est souvent nécessaire de convertir des températures de Celsius en Fahrenheit.

(a) Pour convertir exactement des degrés Celsius en Fahrenheit, il faut:
Étape 1: Multiplier la température en degrés Celsius par $1,8$.
Étape 2: Ajouter 32 au résultat de l'étape 1.
En utilisant cette méthode de conversion exacte, convertis les températures en degrés Celsius suivantes en degrés Fahrenheit. La première température a déjà été convertie à titre d'exemple.

Température en degrés Celsius	Température en degrés Fahrenheit
100	212
30	
20	
10	
0	

(b) Il arrive parfois qu'on souhaite convertir des températures de Celsius en Fahrenheit sans avoir sous la main les outils habituels tels qu'un crayon, du papier ou une calculatrice. Dans ces situations, l'approximation et le calcul mental peuvent s'avérer utiles. Voici comment procéder pour une estimation rapide de la conversion des degrés Celsius en Fahrenheit :
Étape 1: Multiplier la température en degrés Celsius par 2.
Étape 2: Ajouter 30 au résultat de l'étape 1.
En utilisant cette méthode de conversion, convertis les températures en degrés Celsius suivantes en degrés Fahrenheit. La première température a déjà été convertie à titre d'exemple.

Température en degrés Celsius	Température approximative en degrés Fahrenheit
100	230
30	
20	
10	
0	

(c) Est-ce que certaines des conversions approximatives de la partie (b) correspondent exactement aux températures obtenues dans la partie (a) ?

Extension:
Soit C la température en degrés Celsius et F la température en degrés Fahrenheit. Peux-tu écrire des formules pour les conversions effectuées dans les parties (a) et (b) ?

Thèmes Géométrie et mesure, Sens du nombre

Problem of the Week

Problem B and Solution

Temperature Conversions

Problem

Two common units to measure temperature are degrees Celsius and degrees Fahrenheit. From time to time, we need to convert temperatures from degrees Celsius to degrees Fahrenheit.

(a) The exact conversion from degrees Celsius to degrees Fahrenheit is as follows:

Step 1: Take the temperature in degrees Celsius and multiply by 1.8.

Step 2: Take the result from Step 1 and add 32.

Using this exact conversion, convert the following temperatures in degrees Celsius to degrees Fahrenheit. The first has been done for you.

Temperature in degrees Celsius	Temperature in degrees Fahrenheit
100	212
30	
20	
10	
0	

(b) Sometimes when we want to convert between degrees Celsius and degrees Fahrenheit, we don't have a pencil and paper or calculator nearby. In that case, using an approximation and mental math can be helpful. One way to approximate the conversion from degrees Celsius to degrees Fahrenheit is as follows:

Step 1: Take the temperature in degrees Celsius and multiply by 2.

Step 2: Take the result from Step 1 and add 30.

Using this approximate conversion, convert the following temperatures in degrees Celsius to degrees Fahrenheit. The first has been done for you.

Temperature in degrees Celsius	Approximate temperature in degrees Fahrenheit
100	230
30	
20	
10	
0	

(c) Did any of the approximate conversions in part (b) give the same temperature as the exact conversion in part (a)?

Extension:

If you let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit, can you write formulas for the conversions in parts (a) and (b)?

Solution

(a) The completed table is below.

Temperature in degrees Celsius	Temperature in degrees Fahrenheit
100	212
30	86
20	68
10	50
0	32

(b) The completed table is below.

Temperature in degrees Celsius	Approximate temperature in degrees Fahrenheit
100	230
30	90
20	70
10	50
0	30

(c) The conversion of 10°C to 50°F gave the same temperature when using both the approximate and exact conversions.

Extension:

In part (a), we have $F = 1.8 \times C + 32$.

In part (b), we have $F = 2 \times C + 30$.

Problème de la semaine

Problème B

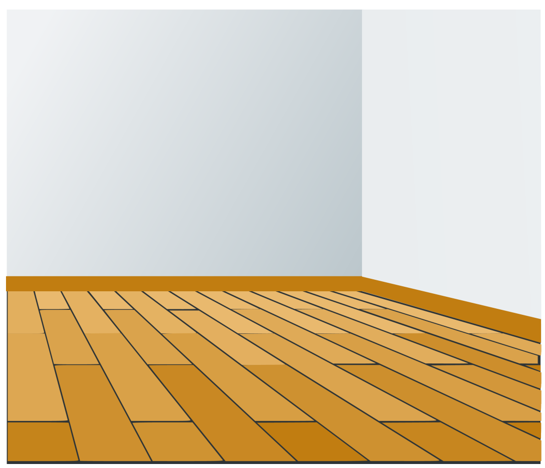
Nouveau parquet

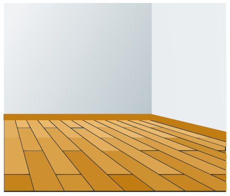
Jafar pose un nouveau parquet dans son salon rectangulaire d'une superficie de 66 m^2. Chaque boîte de parquet contient 8 planches identiques et chaque planche a une aire de $0,2$ m^2.

(a) En supposant qu'il n'y a pas de pertes liées aux découpes, combien de planches lui faudra-t-il pour couvrir entièrement le sol de son salon ?

(b) Si Jafar souhaite acheter 10 % de plus pour compenser les pertes liées aux découpes, combien de boîtes de parquet devrait-il acheter ?

(c) Si chaque boîte coûte $74,50$ \$ et que la taxe de vente est de 15 %, quel sera le coût total du parquet dans la partie (b) ?





Problem of the Week

Problem B and Solution

Jafar's New Floor

Problem

Jafar is laying new hardwood flooring in his rectangular living room, which has an area of 66 m^2. Each box of flooring has 8 identical wooden planks, and each plank has an area of 0.2 m^2.

(a) Assuming that there is no waste, how many planks will he need to cover the floor of his living room?

(b) If Jafar wants to buy an extra 10% for waste, how many boxes of flooring does he need to buy?

(c) If each box costs \$74.50 and sales tax is 15%, what will be the total cost of the flooring in part (b)?

Solution

(a) The area of the living room floor is 66 m^2 and each plank has an area of 0.2 m^2. So the total number of planks needed is $66 \div 0.2 = 330$.

(b) The extra amount is 10% of 330, which is $0.10 \times 330 = 33$ planks.

So in total Jafar wants to buy $330 + 33 = 363$ planks. Since there are 8 planks in each box, the number of boxes required is $363 \div 8 = 45.375$. Therefore, he should buy 46 boxes.

(c) Jafar wants to buy 46 boxes, and each box costs \$74.50. The total cost before tax is $46 \times \$74.50 = \$3427$.

The amount of tax is 15% of \$3427, which is $0.15 \times \$3427 = \$514.05$.

Therefore, the total cost of the flooring is $\$3427 + \$514.05 = \$3941.05$.

Problème de la semaine

Problème B

Pas un jeu Tetris^{TM}

Le 6 juin 2024, le célèbre jeu de casse-tête Tetris^{TM} fêtera ses 40 ans! Le jeu Tetris^{TM} utilise des pièces appelées « tétriminos ». Ces dernières sont des formes composées de quatre carrés identiques, comme celles qui sont présentées en bas de cette page. Ce problème s'inspire de Tetris^{TM}.

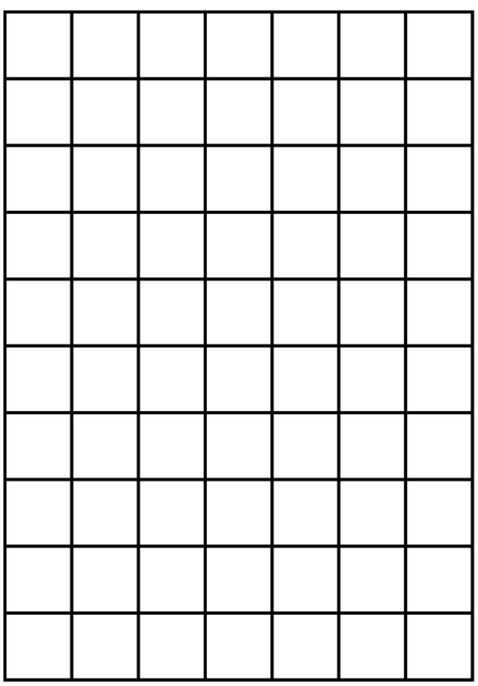
Dans ce problème, les pièces de tétrimino doivent être placées dans une grille selon les règles suivantes :

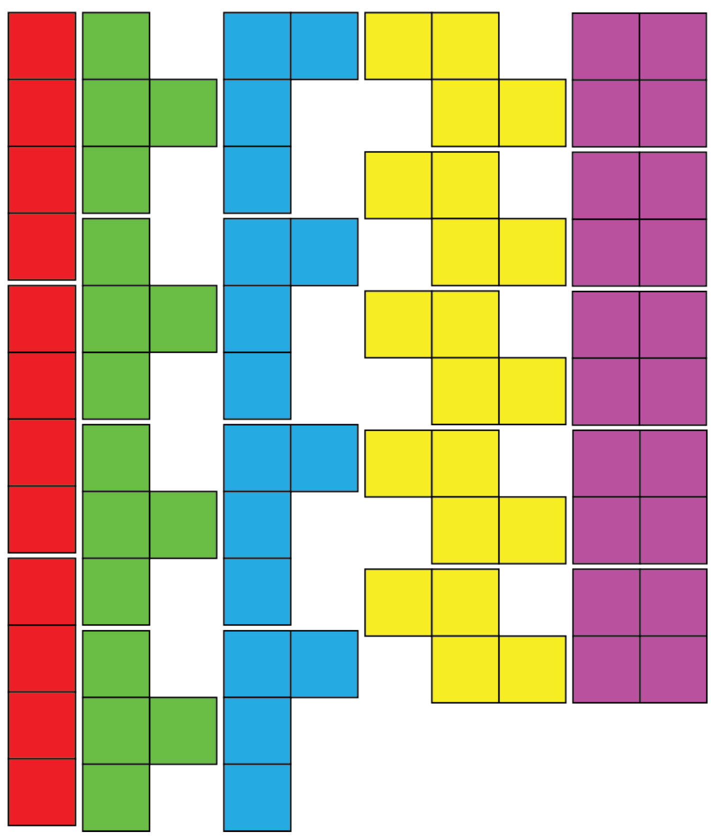
1. Les pièces peuvent être tournées ou réfléchies (retournées).
2. Les pièces ne doivent **pas** se chevaucher et chaque carré d'une pièce doit être placé directement au-dessus d'un carré de la grille.
3. Il faut utiliser uniquement les pièces présentées en bas de cette page. Cependant, il n'est pas nécessaire de toutes les utiliser.

L'objectif est de couvrir autant de carrés que possible dans la grille avec les tétriminos qui sont présentés ci-dessous. Est-il possible de couvrir tous les carrés de la grille ? Justifie ta réponse.

Pour répondre à cette question, il pourrait être utile de découper les tétriminos et de les placer sur la grille.

Tu n'arrives pas à imprimer cette page? Essaie notre feuille de travail interactive.





Problem of the Week

Problem B and Solution

Not a Tetris™ Game

Problem

On June 6, 2024, the puzzle game Tetris™ will be 40 years old! The game of Tetris™ uses pieces called "tetrominoes", which are shapes composed of four identical squares, like the ones given at the bottom of this page. This problem is inspired by Tetris™.

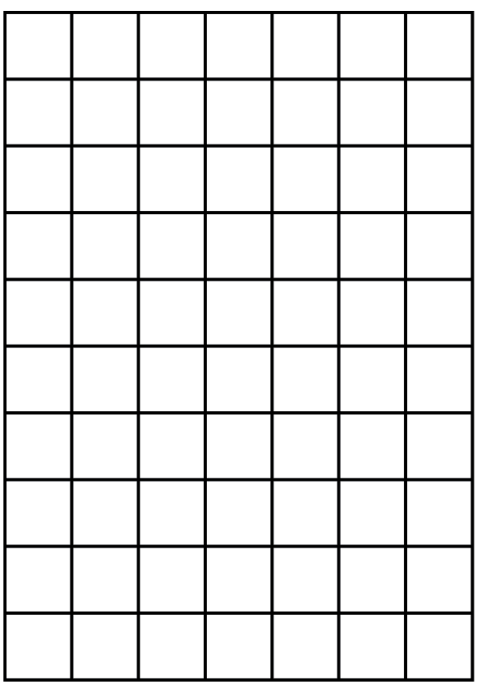
In this problem, tetromino pieces are to be placed in a grid according to the following rules:

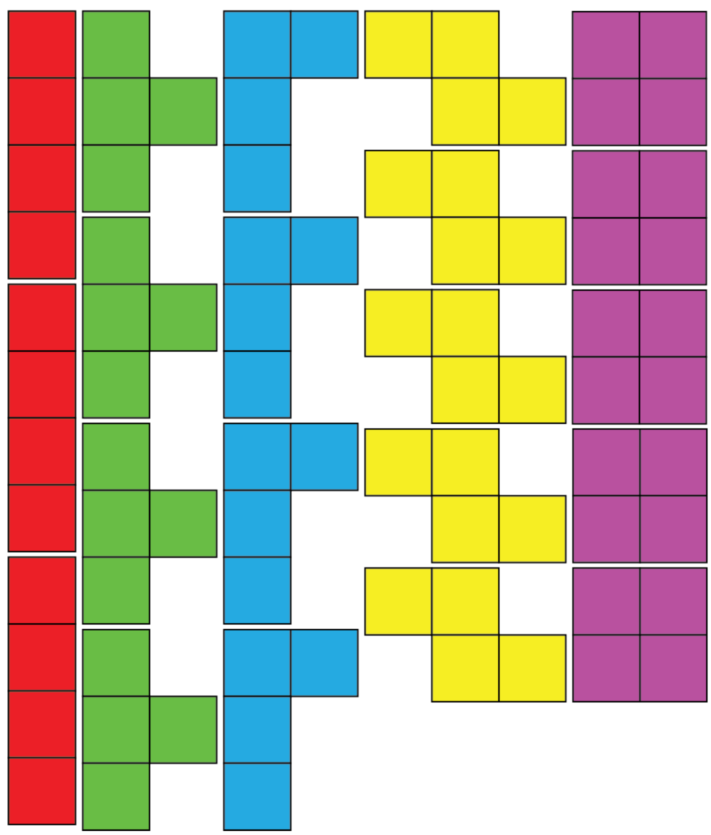
1. Pieces may be rotated or reflected (flipped over).
2. Pieces may **not** overlap each other and each square in a piece must be placed directly on top of a square in the grid.
3. Only the given pieces may be used, but you do not need to use all of them.

The goal is to cover as many squares in the grid as possible with the pieces. Is it possible to cover all the squares in the given grid? Explain why or why not.

When answering this question, you may find it helpful to cut out the given tetrominoes and place them on the grid.

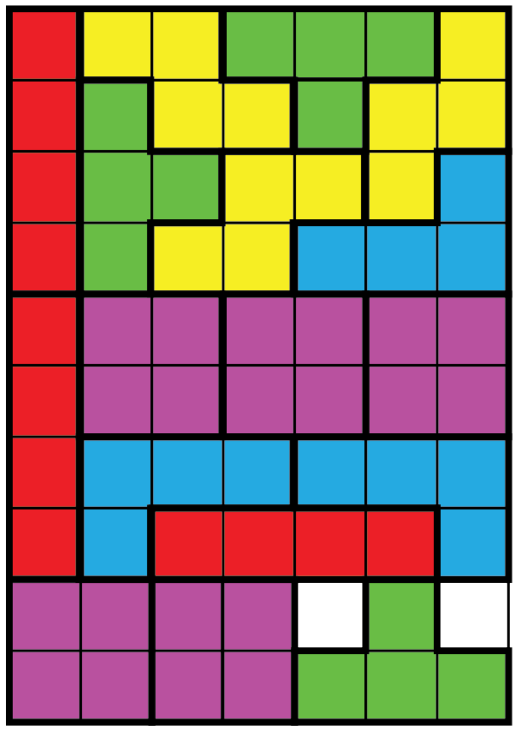
Not printing this page? You can use our interactive worksheet.





Solution

The grid has a total of $7 \times 10 = 70$ squares and each piece has 4 squares. Since $70 \div 4 = 17.5$, which is not a whole number, that tells us that 70 is not a multiple of 4. So it is not possible to cover all the squares in the grid. At most, we would be able to cover $17 \times 4 = 68$ of the squares. One such possibility is shown.



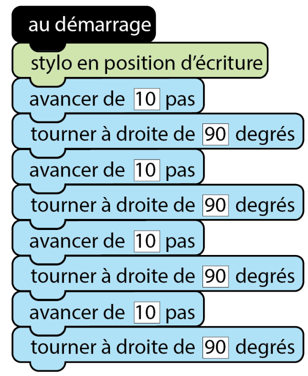
Problème de la semaine

Problème B

Le code de Shelby

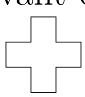
Shelby utilise le codage par blocs pour dessiner des formes.

(a) Le premier programme de Shelby est présenté dans la figure ci-contre. Quelle forme sera produite à l'issue de l'exécution de ce programme?



(b) En te servant des blocs fournis, conçois un programme qui dessine la même forme que celle de Shelby, mais en utilisant moins de blocs.

Blocs	Programme
avancer de ☐ pas tourner à droite de ☐ degrés répéter ☐ fois	au démarrage stylo en position d'écriture

(c) En te servant des blocs fournis, conçois un programme qui dessine la forme suivante: 
Un bloc peut être utilisé plus d'une fois.

Blocs	Programme
avancer de ☐ pas tourner à droite de ☐ degrés tourner à gauche de ☐ degrés répéter ☐ fois	au démarrage stylo en position d'écriture

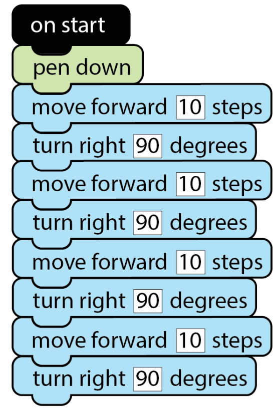
Thèmes Pensée computationnelle, Géométrie et mesure

Problem of the Week
Problem B and Solution
Shelby's Code

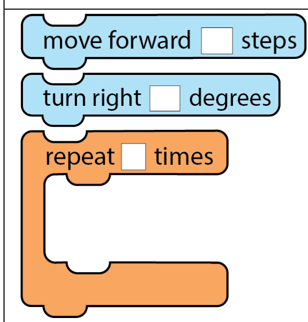
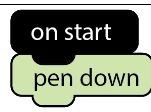
Problem

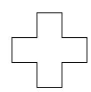
Shelby is using block coding to draw different shapes.

(a) Her first program is shown. What shape will be drawn after running this program?

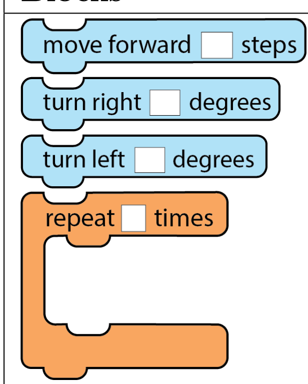
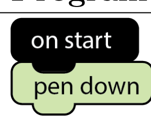


(b) Using the given blocks, write a program to draw the same shape as Shelby's program, using fewer blocks. Notice that some blocks contain an empty box to be filled with a number.

Blocks	Program
	

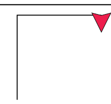
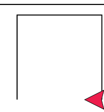
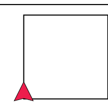
(c) Using the given blocks, write a program to draw the following shape: 

You may use a block more than once.

Blocks	Program
	

Solution

(a) This program will draw a square. The table below shows the drawing progress and position of the pen as we trace through the program.

Program Section	Drawing Progress
on start pen down	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	
move forward 10 steps turn right 90 degrees	

(b) By using the repeat block, we can use fewer blocks in the program, as shown.

```
on start
pen down
repeat 4 times
    move forward 10 steps
    turn right 90 degrees
```

(c) There are several possible programs, depending on where the pen starts, and how many repeat blocks are used. Two programs are shown.

```
on start
pen down
repeat 4 times
    move forward 10 steps
    turn right 90 degrees
    move forward 10 steps
    turn right 90 degrees
    move forward 10 steps
    turn left 90 degrees
```

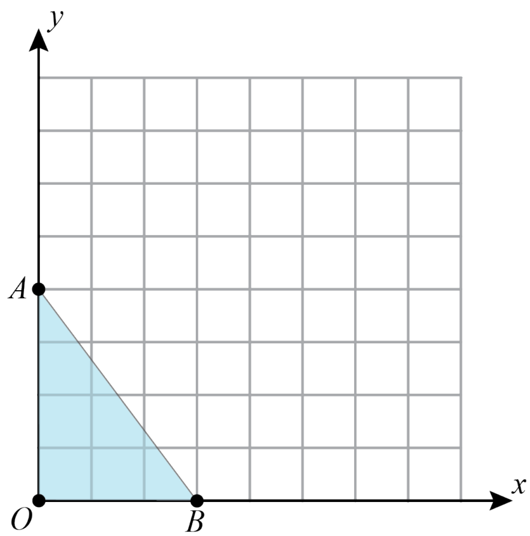
```
on start
pen down
repeat 4 times
    repeat 2 times
        move forward 10 steps
        turn right 90 degrees
    move forward 10 steps
    turn left 90 degrees
```

Problème de la semaine

Problème B

Un jeu triangulaire

Utilise le plan cartésien ci-dessous pour compléter les exercices. Les intervalles entre les lignes de la grille sont de 1 unité.



(a) Indique les coordonnées des points A, O et B.

(b) Place le point C sur l'axe des ordonnées de sorte que la longueur de OC soit le double de la longueur de OA. Ensuite, place le point D sur l'axe des abscisses de sorte que la longueur de OD soit le double de la longueur de OB. Indique les coordonnées des points C et D.

(c) Démontre que l'aire du triangle COD est quatre fois plus grande que l'aire du triangle AOB. Pour ce faire, tu peux utiliser ton diagramme ou une formule d'aire.

Extension : De manière générale, si tu doubles les longueurs des deux côtés perpendiculaires de n'importe quel triangle rectangle, l'aire du nouveau triangle sera-t-elle quatre fois plus grande que celle du triangle initial ? Justifie ta réponse.

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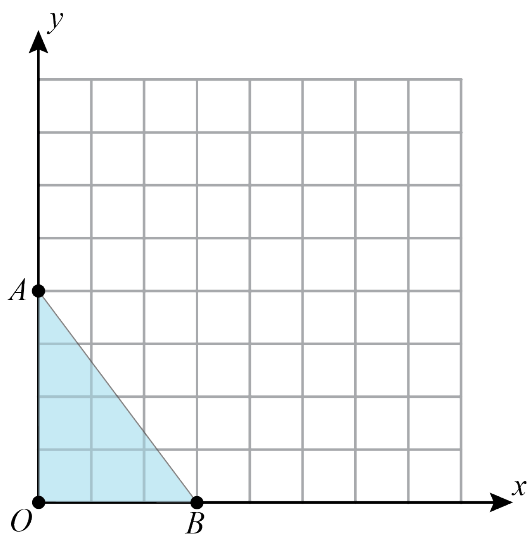
Problem of the Week
Problem B and Solution
Triangular Fun

Problem

Work through the parts that follow using the following coordinate plane, where grid lines are spaced 1 unit apart.



(a) Label the coordinates of the points A, O, and B.

(b) Plot point C on the y-axis so that OC is twice the length of OA. Then plot point D on the x-axis so that OD is twice the length of OB. Label the coordinates of points C and D.

(c) Show that the area of $\triangle COD$ is four times the area of $\triangle AOB$. To show this, you may use your diagram or an area formula.

Extension: In general, if you double the lengths of the two perpendicular sides of any right-angled triangle, will the area of the new triangle be four times the area of the original triangle? Explain.

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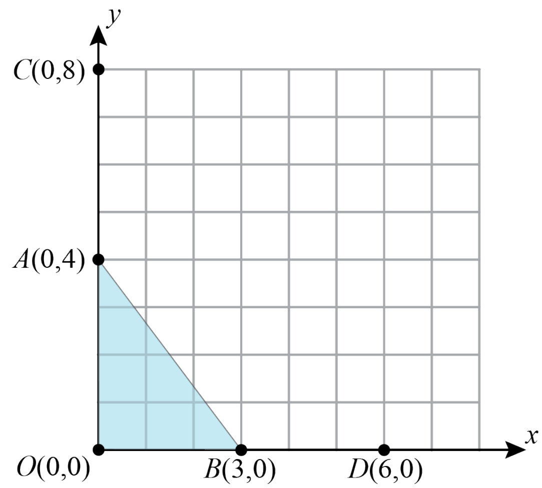
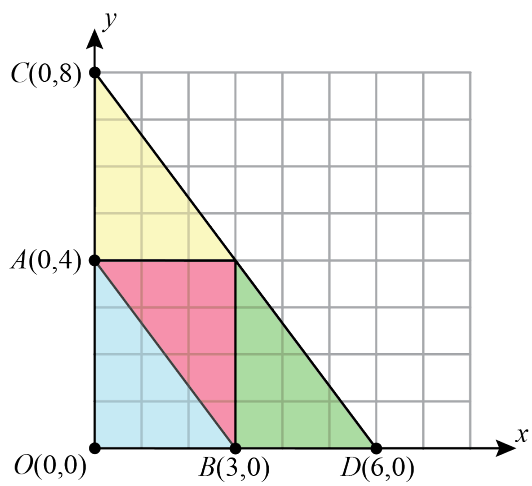
Solution

(a) The coordinates are $A(0,4)$, $O(0,0)$, and $B(3,0)$.

(b) Points C and D are plotted on the diagram, and their coordinates are $C(0,8)$ and $D(6,0)$, as shown.



(c) The diagram shows $\triangle COD$ divided into four smaller right-angled triangles, each congruent to $\triangle AOB$, with perpendicular sides of length 3 and 4. Therefore, the area of $\triangle COD$ is four times the area of $\triangle AOB$.



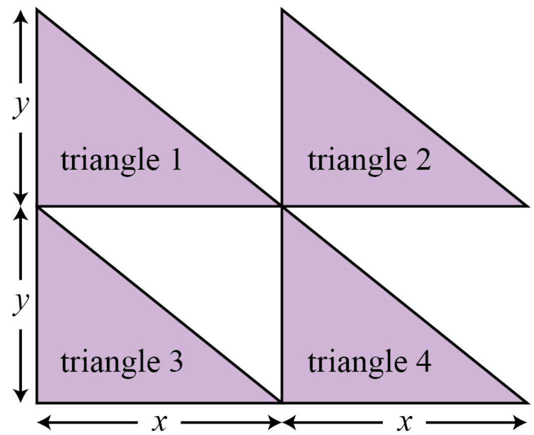
Alternatively, we can calculate the areas of $\triangle AOB$ and $\triangle COD$ using the area formula: Area = base × height ÷ 2.

$$\begin{aligned}\text{Area of } \triangle AOB &= 3 \times 4 \div 2 \\ &= 12 \div 2 \\ &= 6\end{aligned} \qquad \begin{aligned}\text{Area of } \triangle COD &= 6 \times 8 \div 2 \\ &= 48 \div 2 \\ &= 24\end{aligned}$$

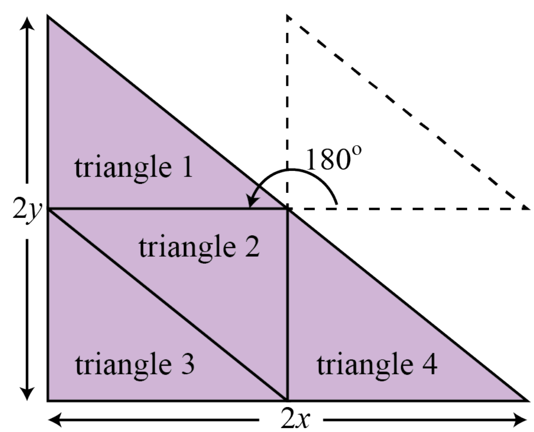
Since $6 \times 4 = 24$, the area of $\triangle COD$ is four times the area of $\triangle AOB$.

Extension Solution:

We will start with a right-angled triangle where the two perpendicular sides have lengths of x and y. We then create four copies of this triangle, numbered from 1 to 4, and arrange them as shown. The total area of the four triangles is four times the area of the original triangle.



Now, if we rotate triangle 2 by 180°, the four triangles will be in the shape of a larger right-angled triangle where the lengths of the two perpendicular sides are $2x$ and $2y$. Thus, if you double the lengths of the two perpendicular sides of any right-angled triangle, the area of the new triangle will be four times the area of the original triangle.

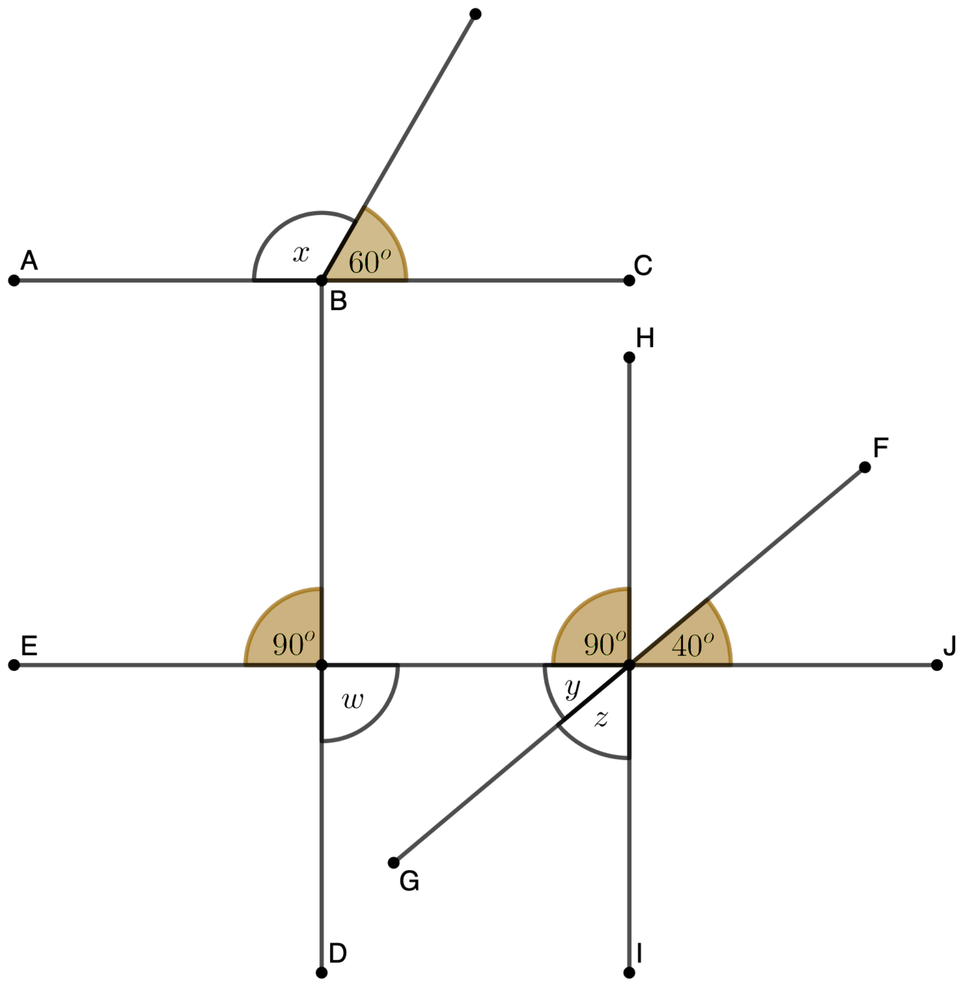


Problème de la semaine

Problème B

Aventures d'angles

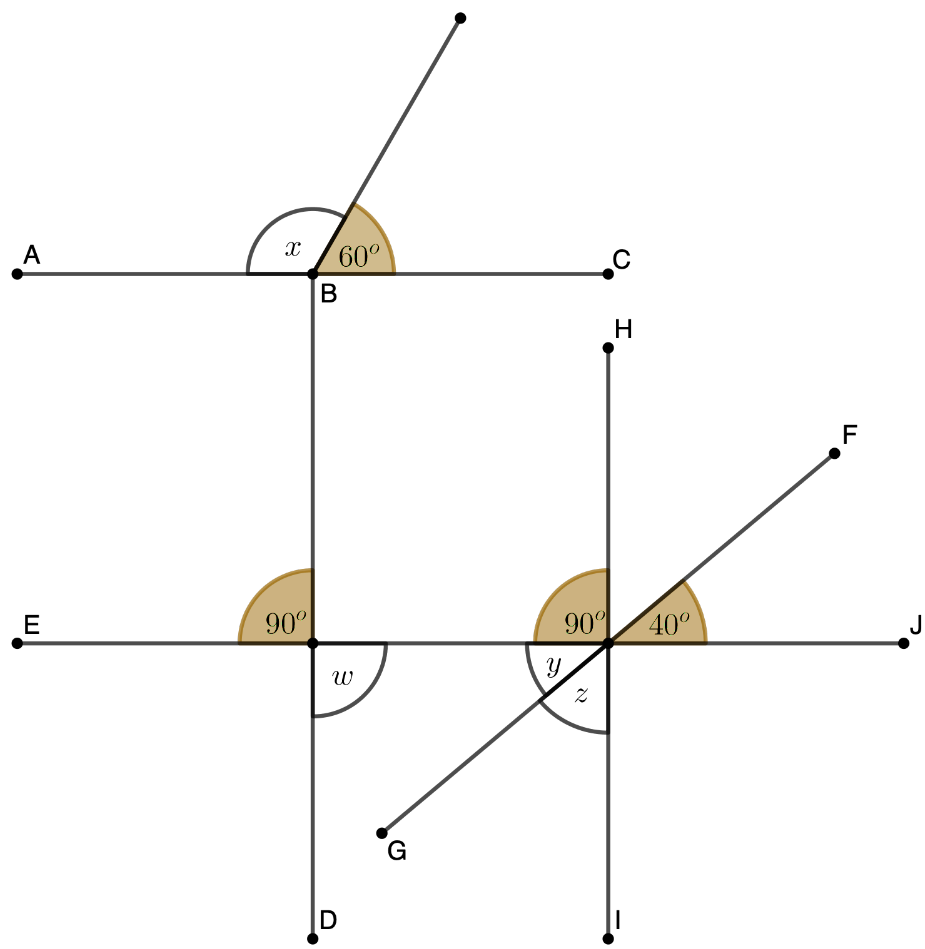
Dans la figure ci-dessous, AC, BD, EJ, HI et FG sont des segments de droites. Détermine la mesure de chaque angle inconnu w, x, y et z.



Problem of the Week
Problem B and Solution
Angle Adventures

Problem

In the diagram below, AC, BD, EJ, HI, and FG are line segments. Determine the measure of each unknown angle w, x, y, and z.



Solution

Solution 1

Since $\angle w$ is opposite to $90°$, we know $\angle w = 90°$.

Since $\angle x$ supplementary to $60°$, we know that $\angle x = 180° - 60° = 120°$.

Since $\angle y$ is opposite to $40°$, we know $\angle y = 40°$.

We know that $90° + \angle y + \angle z = 180°$, so we must have $\angle y + \angle z = 90°$. Since $\angle y = 40°$, we have $\angle z = 50°$.

Solution 2

If we measure the given angles using a protractor, we will notice that the diagram is drawn to scale. Since the diagram is drawn to scale, you may use a protractor to find the angles.

Problème de la semaine

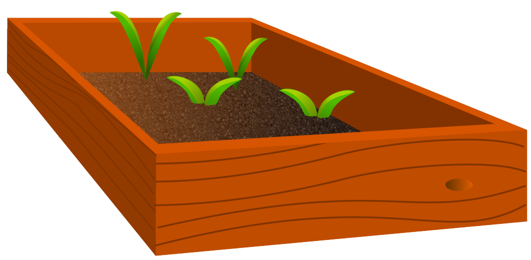
Problème B

Le jardin de Jordyn

Les habitants du voisinage de Jordyn construisent un jardin communautaire pour cultiver des légumes. Ils aimeraient que la plate-bande ait une aire de 48 mètres carrés et prévoient de mettre des planches de clôture en bois autour des bords de la plate-bande. Pour réduire les coûts du projet, ils aimeraient que le périmètre de la plate-bande soit aussi petit que possible.

(a) En supposant que les dimensions de la plate-bande soient des nombres entiers, détermine la longueur et la largeur de la plate-bande en mètres.

(b) Les habitants du voisinage ont décidé de doubler l'aire de la plate-bande, mais souhaitent toujours que son périmètre soit le plus petit possible. Supposons à nouveau que les dimensions de la plate-bande soient des nombres entiers. Détermine la longueur et la largeur de la plate-bande en mètres.



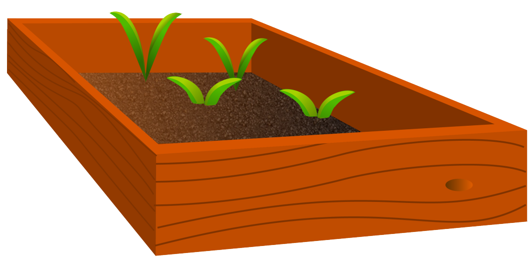
Problem of the Week
Problem B and Solution
Jordyn's Garden

Problem

Jordyn's neighbourhood is building a community garden to grow some vegetables. They would like the garden bed to have an area of 48 square metres, and plan to put wooden fence boards around the edges of the garden bed. To reduce the cost of the project, they would like the garden bed to have the smallest possible perimeter.

(a) Determine the length and width of Jordyn's community garden bed. Assume the side lengths are whole numbers, in metres.

(b) The community decided to double the area of the garden bed, but would still like it to have the smallest possible perimeter. Again, assume the side lengths are whole numbers, in metres. Determine the length and width of the garden bed now.



Solution

(a) Since the garden bed is in the shape of a rectangle and has an area of 48 square metres, it follows that length $\times$ width $= 48$. To determine the length and width, we need to find pairs of whole numbers that multiply to 48. These are called the factor pairs of 48, and are as follows: 1 and 48, 2 and 24, 3 and 16, 4 and 12, and 6 and 8. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

Width (metres)	Length (metres)	Perimeter (metres)
1	48	$2 \times (1 + 48) = 2 \times 49 = 98$
2	24	$2 \times (2 + 24) = 2 \times 26 = 52$
3	16	$2 \times (3 + 16) = 2 \times 19 = 38$
4	12	$2 \times (4 + 12) = 2 \times 16 = 32$
6	8	$2 \times (6 + 8) = 2 \times 14 = 28$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 8 metres and the width should be 6 metres.

(b) After they double the area of the garden bed it will have an area of $2 \times 48 = 96$ square metres. Using a similar approach to part (a), we need to find the factor pairs of 96, which are: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, and 8 and 12. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

Width (metres)	Length (metres)	Perimeter (metres)
1	96	$2 \times (1 + 96) = 2 \times 97 = 194$
2	48	$2 \times (2 + 48) = 2 \times 50 = 100$
3	32	$2 \times (3 + 32) = 2 \times 35 = 70$
4	24	$2 \times (4 + 24) = 2 \times 28 = 56$
6	16	$2 \times (6 + 16) = 2 \times 22 = 44$
8	12	$2 \times (8 + 12) = 2 \times 20 = 40$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 12 metres and the width should be 8 metres.

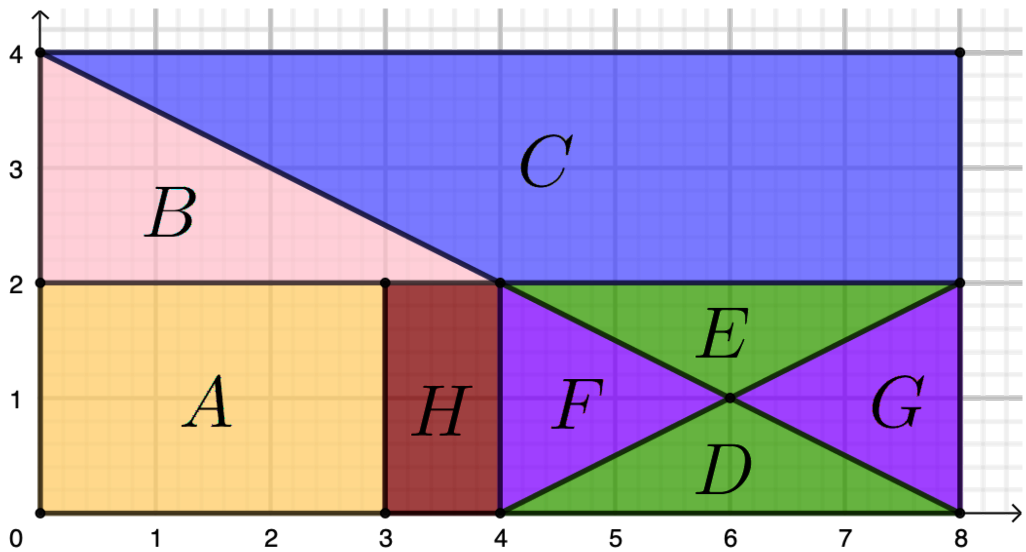
Extension: Note that in each case, the minimum perimeter occurs for the factor pair whose positive difference is the smallest. Will this always happen? Why or why not?

Problème de la semaine

Problème B

Fractions fascinantes

Un grand rectangle se trouve dans le premier quadrant du plan cartésien et a pour sommets $(0, 0)$, $(0, 8)$, $(8, 4)$ et $(4, 0)$. Le rectangle est divisé en huit régions, soit les regions A, B, C, D, E, F, G et H, comme dans la figure ci-dessous.



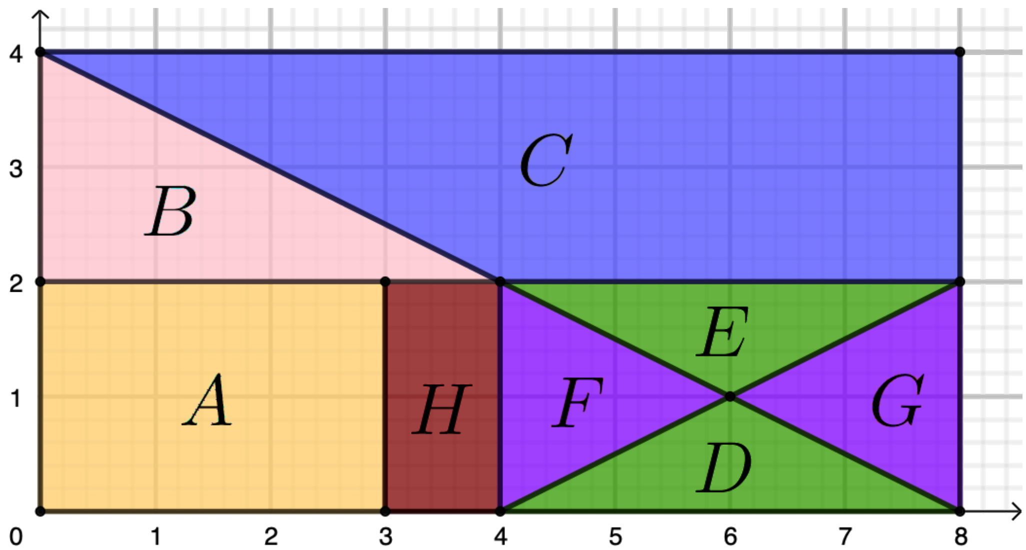
Quelle fraction de l'aire du grand rectangle est occupée par la région A ? Par la région B ? Par la région C ? Par la région D ?

Thèmes Géométrie et mesure, Sens du nombre

Problem of the Week
Problem B and Solution
Fraction Fun

Problem

A large rectangle is in the first quadrant of the Cartesian plane with its four vertices at $(0, 0)$, $(8, 0)$, $(8, 4)$, and $(0, 4)$. It is divided into eight regions labelled A, B, C, D, E, F, G, and H, as shown.



What fraction of the area of the large rectangle is the area of region A? the area of region B? the area of region C? the area of region D?

Solution

Solution 1

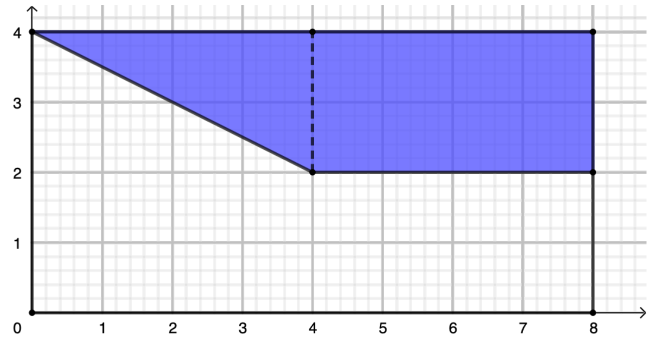
The large rectangle has a length of 8 units and a width of 4 units. Therefore, the area of the large rectangle is $8 \times 4 = 32$ square units.

Region A is a rectangle with a length of 3 units and a width of 2 units. Hence, its area is $3 \times 2 = 6$ square units. So, the area of region A is $\frac{6}{32} = \frac{3}{16}$ of the area of the large rectangle.

Region B is a triangle with a base of 4 units and height of 2 units. Hence, its area is $\frac{1}{2} \times 4 \times 2 = 4$ square units. So, the area of region B is $\frac{4}{32} = \frac{1}{8}$ of the area of the large rectangle.

Region D is a triangle with a base of 4 units and a height of 1 unit. Hence, its area is $\frac{1}{2} \times 1 \times 4 = 2$ square units. So the area of region D is $\frac{2}{32} = \frac{1}{16}$ of the area of the large rectangle.

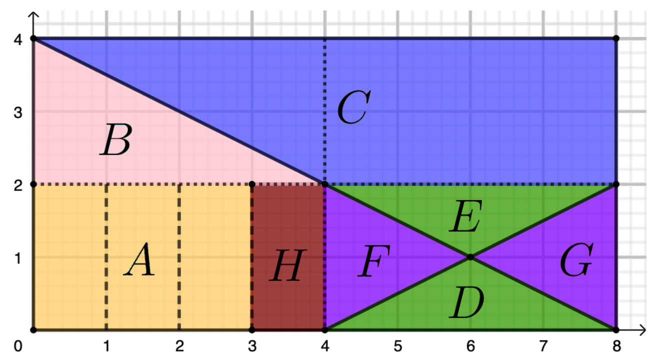
Region C is made up of a rectangle and a triangle as shown by the dashed line in the diagram below.



The rectangle has a length of 4 units and a width of 2 units. So, the area of the rectangle is $4 \times 2 = 8$ square units. The triangle has a base of 4 units and height of 2 units. So, the area of the triangle is $\frac{1}{2} \times 4 \times 2 = 4$ square units. Therefore, the area of region C is $8 + 4 = 12$ square units. Thus, the area of region C is $\frac{12}{32} = \frac{3}{8}$ of the area of the large rectangle.

Solution 2

We draw in dotted lines which divide the large rectangle into four equal parts, or quarters, and draw in dashed lines divide the lower left quarter further into quarters.

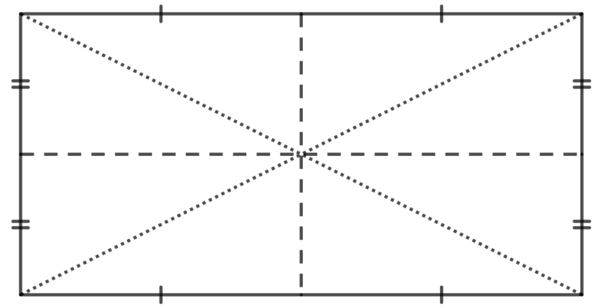


Since the dashed lines divide the lower left quarter of the rectangle further into quarters, the area of each of those four rectangles is $\frac{1}{4}$ of $\frac{1}{4}$ of the area the large rectangle, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle. Thus, the area of region A is $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$ of the area of the large rectangle.

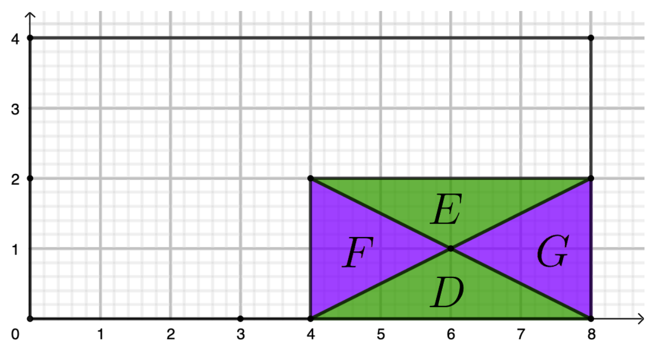
The area of region B is half of the area of the top left quarter, and so is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ of the area of the large rectangle.

The area of region C is the area of the top half of the large rectangle, minus the area of region B, which is $\frac{1}{8}$ of the large rectangle. So in total, the area of region C is $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ of the area of the large rectangle.

Note that we can divide any rectangle into 4 smaller rectangles of equal area by joining the midpoints of opposite sides of the rectangles. When we construct the two diagonals of the large rectangle, we further divide each smaller rectangle into two triangles of equal areas. So, in the diagram below, the eight smaller triangles have equal area.



In our problem, the area of region D is equal to $\frac{2}{8}$ or $\frac{1}{4}$ of the area of the lower right rectangle. Therefore, the area of the region D is $\frac{1}{4}$ of $\frac{1}{4}$, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle.



Sens du nombre (N)

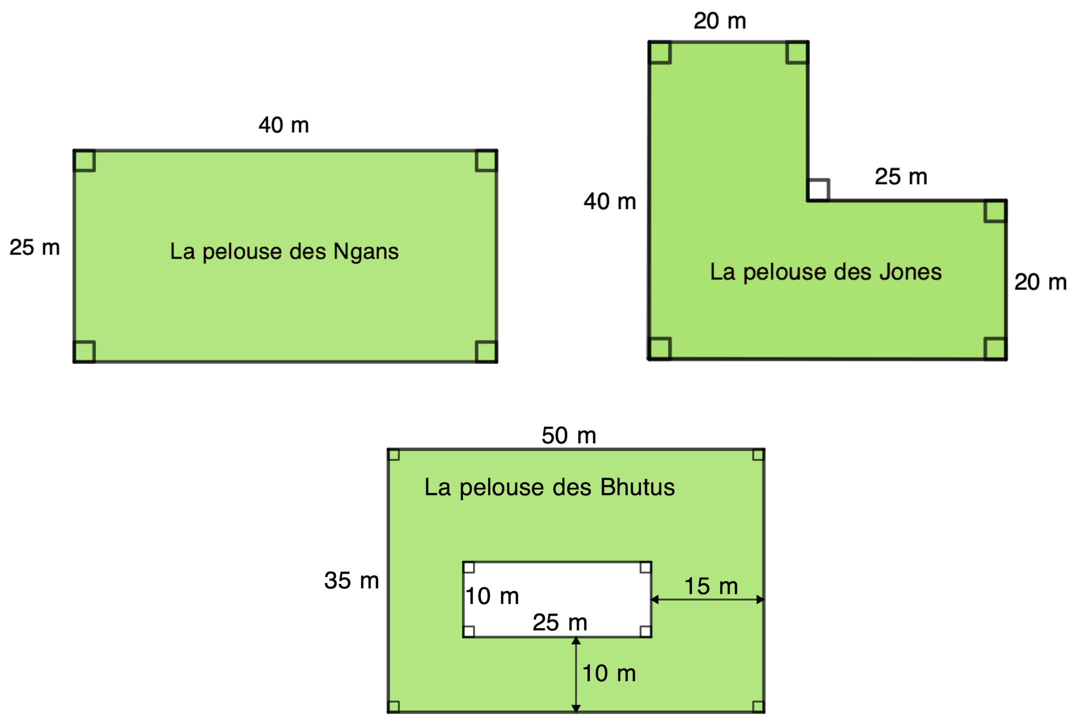
Amène-moi à la couverture

Problème de la semaine

Problème B

Tondre des pelouses

Jon a une entreprise de tonte de pelouse. Trois des pelouses qu'il tond sont représentées dans les figures ci-dessous. Les pelouses sont ombrées en vert et tous les angles sont des angles droits. Sa tondeuse peut couper une rangée d'herbe d'une largeur de 1 mètre.



(a) Laquelle des pelouses nécessitera le plus de temps à tondre? Explique ton raisonnement.

(b) Sachant que sa tondeuse se déplace à une vitesse de 3 km par heure, quelle est la superficie, en m^2, qu'il peut tondre en une heure?

(c) Combien de temps, en minutes, lui faudra-t-il pour tondre chaque pelouse?

THÈMES Géométrie et mesure, Sens du nombre

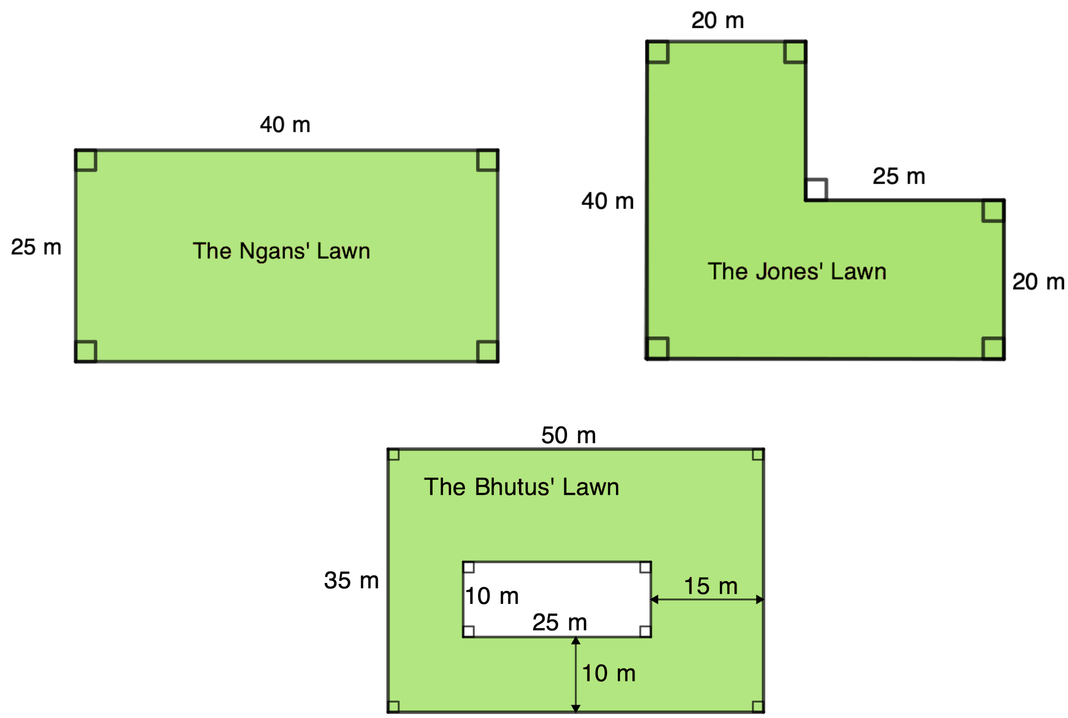
Problem of the Week

Problem B and Solution

Don't Get Mowed Over!

Problem

Jon has a grass cutting business. Three of the lawns that he cuts are shown. The lawns are shaded in green and all angles are right angles. His lawnmower can cut a swath of width 1 metre.



(a) Which lawn will take the longest to cut? Explain your reasoning.

(b) His lawnmower travels at 3 km per hour. What area of lawn, in m^2, can he cut in one hour?

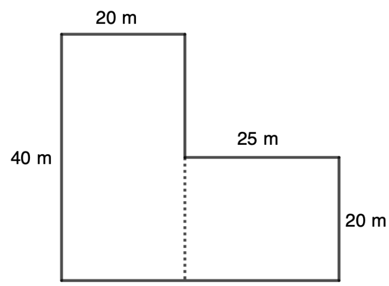
(c) How long, in minutes, will it take him to cut each lawn?

Solution

(a) Since each lawn is composed of regions that have integer lengths, in metres, and since the lawnmower can cut a swath of width 1 metre, we can compare the areas of the lawns to determine which will take the longest to cut.

The Ngans' lawn is a rectangle measuring 25 m $\times$ 40 m. Its total area is therefore 25 m $\times$ 40 m $= 1000\,\text{m}^2$.

The Jones' lawn can be divided into two smaller rectangles as shown.



The total area of the Jones' lawn is equal to the sum of the area of the rectangle on the left and the area of the rectangle on the right. The rectangle on the left has area equal to $40 \text{ m} \times 20 \text{ m} = 800 \text{ m}^2$. The rectangle on the right has area equal to $20 \text{ m} \times 25 \text{ m} = 500 \text{ m}^2$. Thus, the total area of the Jones' lawn is equal to $800 \text{ m}^2 + 500 \text{ m}^2 = 1300 \text{ m}^2$.

The area of the Bhutus' lawn can be found by finding the area of the outer rectangle and subtracting the area of the inner rectangle. The area of the outer rectangle is equal to $50 \text{ m} \times 35 \text{ m} = 1750 \text{ m}^2$. The area of the inner rectangle is equal to $10 \text{ m} \times 25 \text{ m} = 250 \text{ m}^2$. Thus, the total area of the Bhutus' lawn is equal to $1750 \text{ m}^2 - 250 \text{ m}^2 = 1500 \text{ m}^2$.

Since the Bhutus' lawn has the largest area, it will take the longest to cut.

NOTE: To determine the area of the Bhutus' lawn, we could have alternatively divided the lawn into smaller rectangles, and summed the areas of those rectangles.

(b) Jon's mower is 1 m wide and it travels at 3 km/h, or 3000 m/h. Therefore, he can cut $1 \text{ m} \times 3000 \text{ m} = 3000 \text{ m}^2$ in one hour.

(c) The Ngans' lawn has area equal to 1000 m^2. Thus, it would take $1000 \div 3000 = 0.333$ (or $\frac{1}{3}$) of an hour, which is $\frac{1}{3} \times 60 = 20$ minutes to cut the lawn.

The Jones' lawn has area equal to 1300 m^2. It would take $1300 \div 3000 = 0.4333$ (or $\frac{13}{30}$) of an hour, which is $\frac{13}{30} \times 60 = 26$ minutes to cut the lawn.

The Bhutus' lawn has area equal to 1500 m^2. It would take $1500 \div 3000 = 0.5$ (or $\frac{1}{2}$) of an hour, which is $\frac{1}{2} \times 60 = 30$ minutes to cut the lawn.

Problème de la semaine

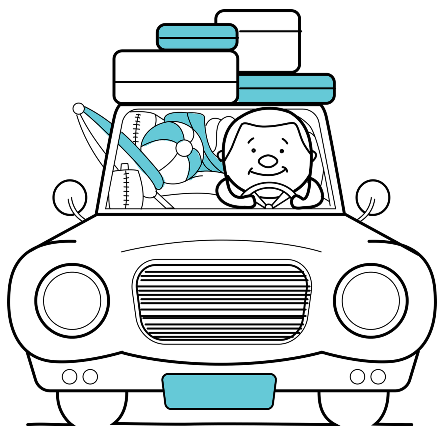
Problème B

Virée en voiture

M. Sand prévoit un voyage à la plage. La distance totale jusqu'à la plage est de 263 km. Sa voiture possède un réservoir de 60 L et peut parcourir 640 000 m avec ce plein.

Supposons que M. Sand a accès à deux stations-service. À la station A, 25 L d'essence coûtent 40 $, tandis qu'à la station B, 30 L d'essence coûtent 51 $.

Détermine le coût de l'essence pour son voyage s'il fait le plein à la station A par rapport au coût s'il fait le plein à la station B. Laquelle des stations est la plus économique ?



Thèmes Géométrie et mesure, Sens du nombre

Problem of the Week

Problem B and Solution

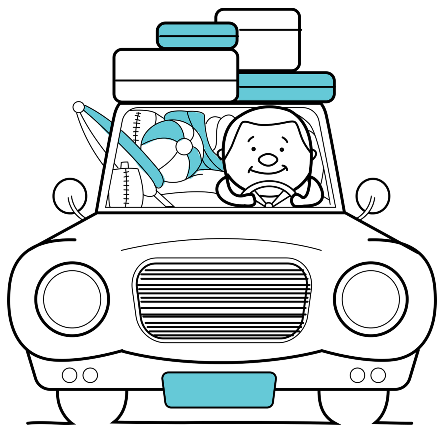
Road Trip

Problem

Mr. Sand is going on a trip to the beach. The total distance to the beach is 263 km. His car has a 60 L gas tank and can travel 640 000 m on that tank of gas.

Suppose that there are two service stations available to Mr. Sand. Station A charges \$40 for 25 L of gas, while Station B charges \$51 for 30 L of gas.

Determine the cost of the gas for his trip if he fills up at Station A versus the cost if he fills up at Station B. Which is the more economical?



Solution

If his vehicle has a 60 L gas tank and will travel 640 000 m or 640 km on one full tank, then he is using $60 \div 640 = 0.09375$ L of gas per km.

Since the distance to the beach is 263 km, then this trip will take $263 \times 0.09375 \approx 24.656$ L of gas.

For Station A:

The cost is \$40 for 25 L. Therefore, the gas will cost $\frac{40}{25} = \$1.60$ per L.

Thus, the cost of the trip for Station A is $24.656 \times \$1.60 = \$39.45$.

For Station B:

The cost is \$51 for 30 L. Therefore, the gas will cost $\frac{51}{30} = \$1.70$ per L.

Thus, the cost of the trip for Station B is $24.656 \times \$1.70 = \$41.92$.

Therefore, Station A is more economical than Station B.

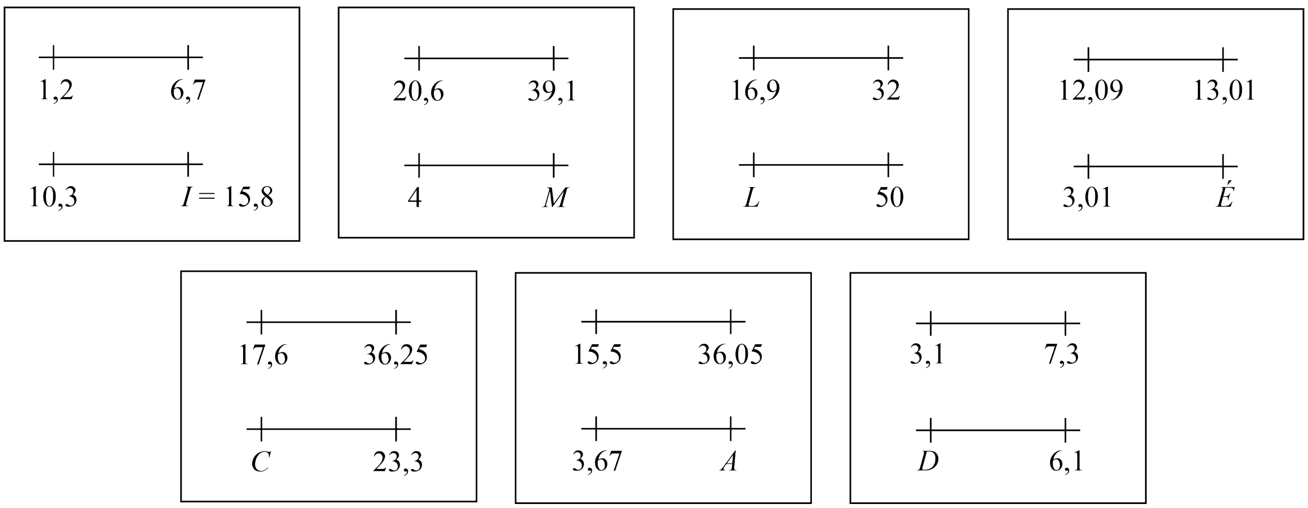
NOTE: Since the gas at Station A costs less per L than at Station B, then using gas from Station A will always cost less than using gas from Station B.

Problème de la semaine

Problème B

Lettres à la ligne

Détermine le nombre associé à chaque lettre pour que les deux droites numériques dans chaque cadre aient le même intervalle. On te présente un exemple avec la première lettre ci-dessous.



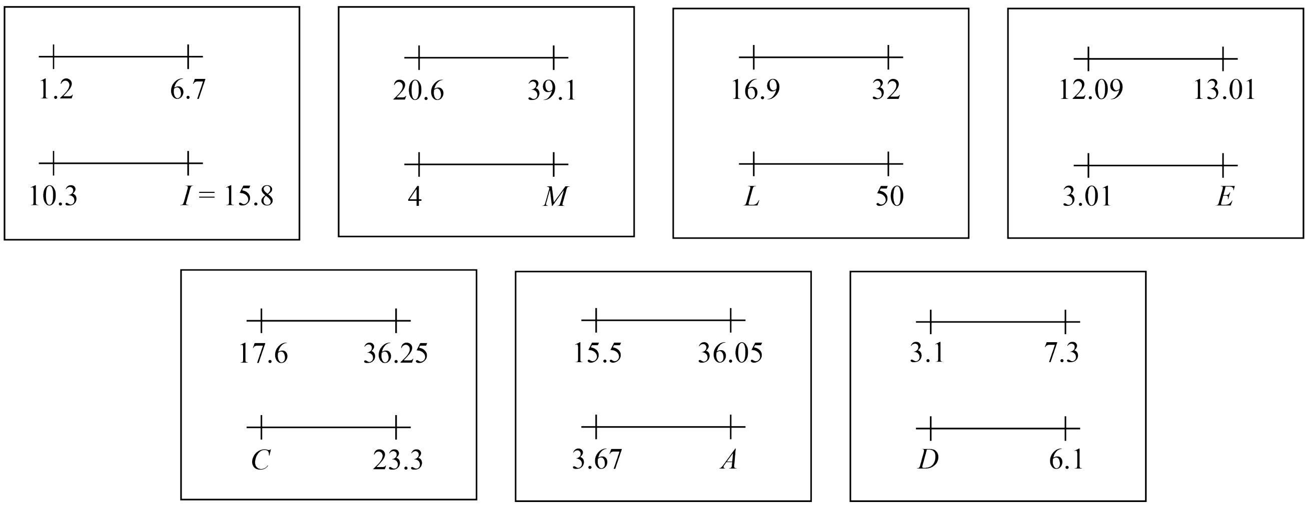
Ensuite, écris ces nombres en ordre croissant dans la première rangée du tableau ci-dessous. Dans la seconde rangée, écris les lettres correspondantes à ces nombres. Les lettres formeront un terme mathématique !

Nombres				15,8			
Lettres				I			

Problem of the Week
Problem B and Solution
Line Up These Letters!

Problem

Determine the number corresponding to each letter so that the two number lines in each box have the same range. The first letter has been done for you.



Then write the numbers in the table below, in order from least to greatest, along with their corresponding letters in the row below. The corresponding letters will spell a mathematical term!

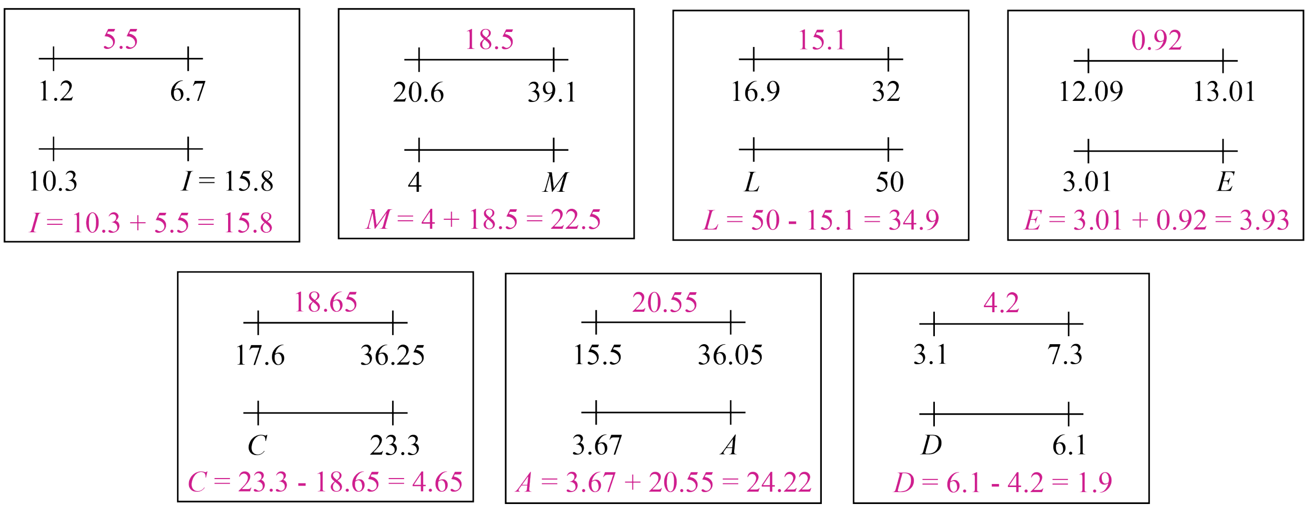
Numbers				15.8			
Letters				*I*			

Solution

In each case, the range of the number line with the given end points must first be determined. These are shown above the top number line in each box. For example, the range of the number line in the first box is $6.7 - 1.2 = 5.5$.

Then the unknown endpoint on the bottom number line can be determined by either addition or subtraction. For example, in the first box, $I = 10.3 + 5.5 = 15.8$.

The calculations for each box are shown in the following diagram.



When the numbers are written in order from least to greatest, along with their corresponding letters, the letters spell the word DECIMAL, as shown.

Numbers	1.9	3.93	4.65	15.8	22.5	24.22	34.9
Letters	D	E	C	I	M	A	L

Problème de la semaine

Problème B

Quand les feuilles nous racontent l'automne

Masha habite dans une maison entourée d'un terrain boisé. Les arbres sont magnifiques, mais en automne, il y a beaucoup de feuilles à ramasser.

Il lui a fallu 10 minutes pour ramasser suffisament de feuilles pour remplir son premier sac. Ce sac avait une masse de 11 kg. Au cours de l'automne, il a rempli 35 sacs de feuilles.

(a) S'il suppose que chaque sac a la même masse que le premier, quelle est la masse totale théorique de toutes les feuilles qu'il a ramassées ?

(b) S'il suppose qu'il lui a fallu autant de temps pour remplir chaque sac que pour le premier, combien de temps aurait-il théoriquement passé à ramasser les feuilles pendant l'automne ?

En réalité, il lui a fallu 8 heures pour ramasser toutes les feuilles. Ces dernières avaient une masse de 425 kg selon la balance au centre environnemental de tri des déchets.

(c) Quelle était la masse moyenne réelle, en kg, de chaque sac de feuilles ? Arrondis ta réponse au dixième près.

(d) Quel était le temps moyen réel, en minutes, qu'il lui a fallu pour remplir chaque sac de feuilles ? Arrondis ta réponse à l'entier près.

(e) Réflexion: Était-ce une bonne approche de prévoir la charge de travail de cet automne en se basant sur le premier sac ?

Thème Sens du nombre

Problem of the Week
Problem B and Solution
Let the Leaves Fall Where They May

Problem

Masha lives in a house on a forested lot. The trees are lovely, but in the fall there is a lot of raking that needs to be done.

It took him 10 minutes to rake and fill his first bag of leaves, which had a mass of 11 kg. Over the course of the fall, he collected 35 bags of leaves.

(a) If he assumes that each bag has the same mass as the first bag, what is the expected total mass of all the leaves he collected?

(b) If he assumes that his time to rake and fill each bag was the same as for the first bag, what is his total expected time to collect all the leaves?

It actually took him 8 hours to do all his raking, and according to the weigh scale at the Environmental Transfer Station, he had 425 kg of leaves in total.

(c) What was the actual mean (average) mass of each bag of leaves? Round your answer to the nearest tenth of a kg.

(d) What was the actual mean (average) time that it took for him to rake the leaves for each bag? Round your answer to the nearest minute.

(e) To Think About: Was predicting his raking workload based on his first bag a good approach?

Solution

(a) If he collected 35 bags that each weighed 11 kg, the total mass of leaves he collected was 35×11 kg $= 385$ kg.

(b) If he collected 35 bags and took 10 minutes to collect the leaves for each one, his total time would have been 35 bags $\times$ 10 min/bag $= 350$ minutes or 5 hours and 50 minutes.

(c) The actual mean mass of each bag was $\dfrac{425}{35} \approx 12.1$ kg.

(d) It took him 8 hours or $8 \times 60 = 480$ minutes to rake the 35 bags. The mean time was $\dfrac{480}{35} \approx 14$ min/bag, rounded to the nearest minute.

(e) Answers will vary. Estimating based on what you know is usually a good way to make predictions. He might have gotten a better estimate if he had used the first few bags, rather than just the first.

Problème de la semaine

Problème B

À grands pas

Donovan et son ami Usain se considèrent chacun comme le plus rapide. Donovan s'est beaucoup entraîné récemment, espérant surpasser Usain dans une course de 0,75 km. Lorsque Usain court, sa foulée mesure 120 cm.

(a) Si la foulée de Donovan est égale aux $\frac{2}{3}$ de celle d'Usain, combien de foulées supplémentaires Donovan devra-t-il faire pour parcourir la distance de 0,75 km par rapport à Usain ?

(b) Donovan et Usain mettent tous les deux exactement 255 secondes pour courir 0,75 km. Qui fait le plus de foulées par seconde ? Justifie ta réponse.

(c) Combien de foulées par seconde Donovan a-t-il faites ? Arrondis ta réponse au dixième près.

Problem of the Week
Problem B and Solution
Take It In Stride

Problem

Donovan and his friend Usain each think they're faster than the other. Donovan has been practicing his running, and is hoping to beat Usain in a 0.75 km race. When Usain runs, his stride length is 120 cm.

(a) If the span of Donovan's running stride length is $\frac{2}{3}$ that of Usain's, how many more strides than Usain will he have to take in order to run the 0.75 km distance?

(b) It takes both Donovan and Usain exactly 255 seconds to run 0.75 km. Who takes more strides per second? Explain your reasoning.

(c) How many strides per second did Donovan take? Round your answer to one decimal place.

Solution

(a) Since Usain's stride length is 120 cm, Donovan's stride length is

$$\frac{2}{3} \times 120 = \frac{2 \times 120}{3} = \frac{240}{3} = 80\,\text{cm}$$

So to run 0.75 km, or 75 000 cm, Donovan will take $75\,000 \div 80 = 937.5$ strides. Since he cannot take partial strides, this means he will take 938 strides.
For Usain, he will take $75\,000 \div 120 = 625$ strides.
Thus, Donovan will take $938 - 625 = 313$ more strides than Usain.

(b) Since Donovan has the smaller stride length, to run the same distance in the same time he must take more strides per second.

(c) The actual number of strides per second for Donovan was $938 \div 255 \approx 3.7$ strides per second.

Note: The actual number of strides per second for Usain was $625 \div 255 \approx 2.5$ strides per second.

Problème de la semaine

Problème B

Équivalences d'arrondissement

Parfois, le processus d'arrondissement des nombres produit des résultats intéressants. Par exemple, lorsqu'on arrondit le nombre 39,99 à la dizaine près, on obtient 40; si on l'arrondit à l'entier près, on obtient 40 et si on l'arrondit au dixième près, on obtient 40,0. Remarque que l'on obtient la même valeur numérique en arrondissant 39,99 à la dizaine près, à l'entier près et au dixième près.

(a) Trouve un nombre à deux décimales inférieur à 100 tel que, lorsqu'on l'arrondit au dixième près, on obtient la même valeur numérique que si on l'avait arrondi à l'entier près.

(b) Trouve un nombre à deux décimales inférieur à 100 tel que, lorsqu'on l'arrondit au dixième près, on obtient la même valeur numérique que si on l'avait arrondi à la dizaine près.

(c) Trouve le plus petit nombre à deux décimales entre 99 et 100 tel que l'on obtient la même valeur numérique en l'arrondissant au dixième près, à l'entier près, à la dizaine près et à la centaine près.

Thème Sens du nombre

Problem of the Week
Problem B and Solution
Rounding Equivalents

Problem

Sometimes the process of rounding numbers produces interesting results. For example, if you round the number 39.99 to the nearest ten, you get 40, if you round it to the nearest whole number, you get 40, and if you round it to the nearest tenth, you get 40.0. Notice that you get the same numerical value when rounding 39.99 to the nearest ten, whole number, and tenth.

(a) Find a number less than 100 with two decimal places such that when you round to the nearest tenth you get the same numerical value as when you round to the nearest whole number.

(b) Find a number less than 100 with two decimal places such that when you round to the nearest tenth you get the same numerical value as when you round to the nearest ten.

(c) Find the smallest number between 99 and 100 that has two decimal places that rounds to the same numerical value when you round to the nearest tenth, whole number, ten, and hundred.

Solution

(a) Answers will vary. One possible answer is 18.96.
Rounding 18.96 to the nearest tenth yields 19.0.
Rounding 18.96 to the nearest whole number yields 19.

(b) Answers will vary. One possible answer is 20.03.
Rounding 20.03 to the nearest tenth yields 20.0.
Rounding 20.03 to the nearest ten yields 20.

(c) When the number is between 99 and 100, it must be 100 when rounded to the nearest hundred.
Therefore, the number rounded to the nearest tenth would be 100.0. The numbers less than 100 that have two decimal places that round to 100.0 when rounded to the nearest tenth are

$$99.99,\ 99.98,\ 99.97,\ 99.96 \text{ and } 99.95$$

(Note that 99.94 will round to 99.9 when rounded to the nearest tenth.) Therefore, the smallest of these numbers is 99.95.
Notice that 99.95 does indeed yield 100.0 or 100 when rounded to the nearest tenth, whole number, ten, or hundred.

Problème de la semaine

Problème B

Cadeaux d'anniversaire

Paula et Quinn sont jumeaux. Leurs amis ont économisé de l'argent pour leur offrir des cadeaux. Aleta a économisé 30 $, Benji 25 $ et Carolina 28 $. Ils ont exploré différentes idées de cadeaux et voici ce qu'ils ont trouvé :

— Paula et Quinn aimeraient chacun une paire de chaussettes à 7 $ la paire.

— Paula voudrait des médiators pour sa guitare à 6 $ et un yo-yo à 7 $.

— Quinn, de son côté, souhaiterait des crayons pour dessiner à 21 $.

— Enfin, Paula et Quinn apprécieraient tous les deux des boîtes de chocolats à la menthe à 4 $ l'unité.

(a) S'ils combinent leurs économies, combien leur restera-t-il après avoir acheté ces cadeaux ? Ignore les taxes.

(b) Parmi les autres articles envisagés, il y a un chandail à capuchon à 20 $, des pantoufles à 10 $, un journal intime à 6 $ et une bouteille d'eau à 16 $. Ils doivent également acheter du papier d'emballage et des rubans, ce qui leur coûterait 5 $ au total pour les jumeaux.

S'ils souhaitent utiliser tout leur budget en veillant à dépenser une somme à peu près équivalente pour chacun des jumeaux, quels cadeaux supplémentaires pourraient-ils choisir pour Paula et Quinn ? Ignore les taxes.

Thème Sens du nombre

Problem of the Week
Problem B and Solution
Birthday Presents

Problem

Paula and Quinn are twins. Their friends have saved money to buy them some gifts. Aleta has saved \$30, Benji has saved \$25, and Carolina has saved \$28. They have done some research on possible gifts and their costs:

- Both Paula and Quinn would like a pair of warm socks that are \$7 per pair.
- Paula wants some guitar picks for \$6 and a yo-yo that is \$7.
- Quinn would like some art pencils that are \$21.
- Both like chocolate mints that are \$4 per box.

(a) If they combine their savings, how much will be left after purchasing these gifts? You may ignore taxes.

(b) Other items they are considering are a hoodie for \$20, slippers for \$10, a diary for \$6, and a water bottle for \$16. They also have to buy some wrapping paper and ribbons, which is \$5 in total for both twins.

If they want to spend all their money and also spend about the same total amount on each person, which of the additional items could they buy for each of Paula and Quinn? You may ignore taxes.

Solution

(a) The three friends have saved a total of $\$30 + \$25 + \$28 = \$83$. The gifts cost $(2 \times \$7) + \$6 + \$7 + \$21 + (2 \times \$4) = \$56$ in total. Thus, there will be $\$83 - \$56 = \$27$ left over.

(b) First we will figure out approximately how much money they should spend on each person. Subtracting \$5 for wrapping paper and ribbons, the friends have a total of $\$83 - \$5 = \$78$ to spend on gifts. So they should spend about $\$78 \div 2 = \$39$ on each person.

- After buying the socks, art pencils, and chocolate mints for Quinn, they will have spent $\$7 + \$21 + \$4 = \$32$. Thus, they should spend approximately $\$39 - \$32 = \$7$ more on gifts for Quinn.
- After buying the socks, guitar picks, yo-yo, and chocolates for Paula, they will have spent $\$7 + \$6 + \$7 + \$4 = \$24$. Thus, they should spend approximately $\$39 - \$24 = \$15$ more on gifts for Paula.

After buying the wrapping paper and ribbons, they will have $\$27 - \$5 = \$22$ left, and want to spend it all on gifts. From the additional items, they can buy a diary and a water bottle for $\$6 + \$16 = \$22$, or two diaries and a pair of slippers for $\$6 + \$6 + \$10 = \$22$. No other combination of items equals \$22.

So they could buy a diary for \$6 for Quinn, and either a water bottle for \$16 or a diary and a pair of slippers for $\$6 + \$10$ for Paula. Either way they will have spent a total of $\$32 + \$6 = \$38$ on Quinn and $\$24 + \$16 = \$40$ on Paula.

Problème de la semaine

Problème B

Conversion de température

Les degrés Celsius et Fahrenheit représentent les deux unités les plus utilisées pour mesurer la température. Il est souvent nécessaire de convertir des températures de Celsius en Fahrenheit.

(a) Pour convertir exactement des degrés Celsius en Fahrenheit, il faut:
Étape 1: Multiplier la température en degrés Celsius par $1,8$.
Étape 2: Ajouter 32 au résultat de l'étape 1.
En utilisant cette méthode de conversion exacte, convertis les températures en degrés Celsius suivantes en degrés Fahrenheit. La première température a déjà été convertie à titre d'exemple.

Température en degrés Celsius	Température en degrés Fahrenheit
100	212
30	
20	
10	
0	

(b) Il arrive parfois qu'on souhaite convertir des températures de Celsius en Fahrenheit sans avoir sous la main les outils habituels tels qu'un crayon, du papier ou une calculatrice. Dans ces situations, l'approximation et le calcul mental peuvent s'avérer utiles. Voici comment procéder pour une estimation rapide de la conversion des degrés Celsius en Fahrenheit :
Étape 1: Multiplier la température en degrés Celsius par 2.
Étape 2: Ajouter 30 au résultat de l'étape 1.
En utilisant cette méthode de conversion, convertis les températures en degrés Celsius suivantes en degrés Fahrenheit. La première température a déjà été convertie à titre d'exemple.

Température en degrés Celsius	Température approximative en degrés Fahrenheit
100	230
30	
20	
10	
0	

(c) Est-ce que certaines des conversions approximatives de la partie (b) correspondent exactement aux températures obtenues dans la partie (a) ?

Extension:
Soit C la température en degrés Celsius et F la température en degrés Fahrenheit. Peux-tu écrire des formules pour les conversions effectuées dans les parties (a) et (b) ?

Problem of the Week

Problem B and Solution

Temperature Conversions

Problem

Two common units to measure temperature are degrees Celsius and degrees Fahrenheit. From time to time, we need to convert temperatures from degrees Celsius to degrees Fahrenheit.

(a) The exact conversion from degrees Celsius to degrees Fahrenheit is as follows:

Step 1: Take the temperature in degrees Celsius and multiply by 1.8.

Step 2: Take the result from Step 1 and add 32.

Using this exact conversion, convert the following temperatures in degrees Celsius to degrees Fahrenheit. The first has been done for you.

Temperature in degrees Celsius	Temperature in degrees Fahrenheit
100	212
30	
20	
10	
0	

(b) Sometimes when we want to convert between degrees Celsius and degrees Fahrenheit, we don't have a pencil and paper or calculator nearby. In that case, using an approximation and mental math can be helpful. One way to approximate the conversion from degrees Celsius to degrees Fahrenheit is as follows:

Step 1: Take the temperature in degrees Celsius and multiply by 2.

Step 2: Take the result from Step 1 and add 30.

Using this approximate conversion, convert the following temperatures in degrees Celsius to degrees Fahrenheit. The first has been done for you.

Temperature in degrees Celsius	Approximate temperature in degrees Fahrenheit
100	230
30	
20	
10	
0	

(c) Did any of the approximate conversions in part (b) give the same temperature as the exact conversion in part (a)?

Extension:

If you let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit, can you write formulas for the conversions in parts (a) and (b)?

Solution

(a) The completed table is below.

Temperature in degrees Celsius	Temperature in degrees Fahrenheit
100	212
30	86
20	68
10	50
0	32

(b) The completed table is below.

Temperature in degrees Celsius	Approximate temperature in degrees Fahrenheit
100	230
30	90
20	70
10	50
0	30

(c) The conversion of 10°C to 50°F gave the same temperature when using both the approximate and exact conversions.

Extension:
In part (a), we have $F = 1.8 \times C + 32$.
In part (b), we have $F = 2 \times C + 30$.

Problème de la semaine

Problème B

Faire les courses

Une famille de cinq personnes planifie un repas spécial pour le dîner, tout en respectant un budget de 30 $. Les trois enfants (Sean, Siobhan, Saorise) doivent chacun proposer un menu pour le dîner, comprenant un plat principal, un légume en accompagnement et un dessert. Pour répondre aux questions suivantes, utilise les tableaux ci-dessous qui contiennent la liste des articles disponibles et leurs prix respectifs.

(a) Le menu de Sean comprend des wraps faits de lanières de poulet, de laitue, de tomates et de mayonnaise. Il choisit du céleri pour son accompagnement de légumes et des pommes pour le dessert. Son menu coûte-t-il moins de 30 $? Si ce n'est pas le cas, comment pourrait-il modifier son menu pour qu'il coûte moins de 30 $?

(b) Le menu de Siobhan comprend des sandwichs faits de pain, de viandes froides, de fromage, de mayonnaise et de moutarde. Elle choisit des carottes pour son accompagnement de légumes et une tarte aux pommes pour le dessert. Son menu coûte-t-il moins de 30 $? Si ce n'est pas le cas, comment peut-elle modifier son menu pour qu'il coûte moins de 30 $?

(c) Saorise aimerait avoir deux options différentes pour le plat principal du repas, ainsi qu'une option pour l'accompagnement de légumes et une pour le dessert. Aide-la à élaborer un menu à partir des articles proposés ci-dessous, tout en respectant un budget de 30 $.

Plats Principaux	
Article	**Prix**
Pain	2,98 $
Lanières de poulet	7,99 $
Viandes froides	6,99 $
Saucisses à hot dogs	3,49 $
Pains à hot dog	2,99 $
Pâtes	1,99 $
Sauce pour pâtes	2,98 $
Wraps	2,99 $
Fromage	4,98 $

Condiments	
Article	**Prix**
Ketchup	3,99 $
Mayonnaise	3,59 $
Moutarde	4,49 $
Relish	3,49 $

Légumes	
Article	**Prix**
Carottes	2,99 $
Céleri	2,99 $
Concombres	1,79 $
Laitue	1,97 $
Tomates	1,49 $

Dessert	
Article	**Prix**
Pommes	3,99 $
Tarte aux pommes	5,99 $
Bananes	1,45 $
Biscuits	2,99 $
Crème glacée	2,98 $

Thème Sens du nombre

Problem of the Week
Problem B and Solution
Grocery Shopping

Problem

A family of five is planning on having a special meal for lunch, while sticking to a \$30 budget. The three children (Sean, Siobhan, Saorise) are each to propose a menu for the lunch. Each menu is to consist of a main dish, along with a vegetable side and a dessert. When answering the questions that follow, use the tables of items and prices per item below.

(a) Sean's menu has wraps made up of chicken fingers, lettuce, tomatoes, and mayonnaise. He chooses celery for his vegetable side and apples for dessert. Does his menu cost less than \$30? If it does not, how can he change his menu so it costs less than \$30?

(b) Siobhan's menu has sandwiches made up of bread, cold cuts, cheese, mayonnaise, and mustard. She chooses carrots for her vegetable side and apple pie for dessert. Does her menu cost less than \$30? If it does not, how can she change her menu so that it costs less than \$30?

(c) Saorise would like to have two different options for the main part of the meal, along with one option for the vegetable side and one option for the dessert. Help her make a menu from the items below, without exceeding \$30.

Main Dish Items	
Item	**Price**
Bread	\$2.98
Chicken Fingers	\$7.99
Cold Cuts	\$6.99
Hot Dogs	\$3.49
Hot Dog Buns	\$2.99
Pasta	\$1.99
Pasta Sauce	\$2.98
Wraps	\$2.99
Cheese	\$4.98

Condiments	
Item	**Price**
Ketchup	\$3.99
Mayonnaise	\$3.59
Mustard	\$4.49
Relish	\$3.49

Vegetables	
Item	**Price**
Carrots	\$2.99
Celery	\$2.99
Cucumbers	\$1.79
Lettuce	\$1.97
Tomatoes	\$1.49

Dessert	
Item	**Price**
Apples	\$3.99
Apple Pie	\$5.99
Bananas	\$1.45
Cookies	\$2.99
Ice Cream	\$2.98

Solution

(a) The total cost for Sean's menu is
$\$2.99 + \$7.99 + \$1.97 + \$1.49 + \$3.59 + \$2.99 + \$3.99 = \$25.01$. This menu does not exceed \$30.

(b) The total cost for for Siobhan's menu is
$\$2.98 + \$6.99 + \$4.98 + \$3.59 + \$4.49 + \$2.99 + \$5.99 = \$32.01$. This menu does exceed \$30 by \$2.01.

There are many options to change the menu so that cost does not exceed \$30. For example, removing either mayonnaise or mustard will make the cost less than \$30. Or she could have bananas for dessert instead of apple pie.

(c) Answers will vary. One possible main dish could be hot dogs with hot dog buns, ketchup, and mustard, which would cost
$\$3.49 + \$2.99 + \$3.99 + \$4.49 = \$14.96$. Another main dish could be pasta with pasta sauce, which would cost $\$1.99 + \$2.98 = \$4.97$. She could have cucumbers for her vegetable side and ice cream for dessert. The total cost for this menu is $\$14.96 + \$4.97 + \$1.79 + \$2.98 = \$24.70$.

Problème de la semaine

Problème B

Joueur !

Geoff joue à un jeu dans lequel on lance deux dés réguliers à six faces, l'un noir et l'autre blanc. Pour gagner, il doit lancer les dés et obtenir deux nombres dont la somme est 11.

(a) Quelle est la probabilité qu'il obtienne un 7 en utilisant uniquement le dé noir?

(b) Quelle est la probabilité théorique qu'il obtienne un 1 avec le dé noir et un 6 avec le dé blanc ?

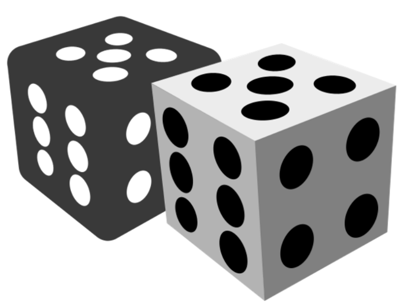
(c) S'il lance les deux dés et qu'il calcule la somme des nombres obtenus, quelle(s) somme(s) a (ont) la plus faible probabilité théorique d'être obtenue(s)?

(d) Quelle est la probabilité théorique d'obtenir une somme de 7 en lançant les deux dés ?

(e) Quelle est la probabilité théorique d'obtenir une somme de 11 en lançant les deux dés ?

(f) Lance les deux dés trente-six fois et compte le nombre de fois où tu obtiens une somme de 11. D'après tes observations, quelle était la probabilité empirique d'obtenir une somme de 11 ?

(g) Compare tes résultats de la partie (f) avec ceux de tes camarades. Pour combien d'entre eux la probabilité empirique d'obtenir une somme de 11 était-elle égale à la probabilité théorique d'obtenir une somme de 11 ?



Problem of the Week

Problem B and Solution

Gamer!

Problem

Geoff plays a game using two standard six-sided dice: a black one and a white one. To win the game, Geoff must roll the dice and have the numbers on the two top faces sum to 11.

(a) What is the probability that he rolls a 7 with just the black die?

(b) What is the theoretical probability that he rolls a 1 on the black die and a 6 on the white die?

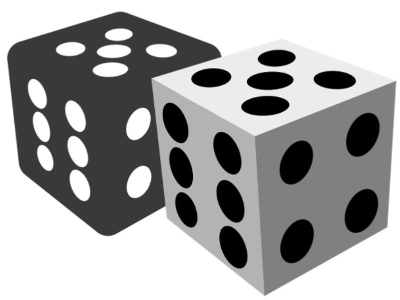
(c) If he rolls both dice and calculates the sum of the numbers on the two top faces, what sum(s) have the lowest theoretical probability of being rolled?

(d) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 7?

(e) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 11?

(f) Roll two dice thirty-six times and keep track of the number of times the numbers on the two top faces sum to 11. What was your empirical probability of rolling a sum of 11?

(g) Share your results in part (f) with your classmates. How many had their empirical probability of rolling a sum of 11 equal the theoretical probability of rolling a sum of 11?



Solution

(a) Since the numbers on the faces of a standard six-sided die are 1, 2, 3, 4, 5, and 6, it is impossible to roll a 7. So the probability is 0.

(b) For each of the 6 possible numbers he could throw with the black die there are 6 possible numbers on the white die, so the total number of possible outcomes is $6 \times 6 = 36$. Thus, the theoretical probability that he throws a 1 on the black die and a 6 on the white die is 1 in 36, or $\frac{1}{36}$.

Alternatively, to solve this problem we can create a table where the columns show the possible numbers on the top face of the white die, the rows show the possible numbers on the top face of the black die, and each cell in the body of the table gives the sum of the corresponding pair of numbers.

		White Die					
		1	2	3	4	5	6
Black Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

From this table, we can conclude that there is 1 outcome out of 36 possible outcomes where the number on the top face of the black die is a 1 and the number on the top face of the white die is a 6. We will use this table in our answers to parts (c), (d), and (e).

(c) For each of the sums of 2 and 12, there is only one possible way to obtain that outcome (two ones or two sixes). Thus, each of these sums has the lowest theoretical probability, namely 1 in 36, or $\frac{1}{36}$.

(d) A sum of 7 can be obtained in 6 possible ways (as $1+6$ or $6+1$, $2+5$ or $5+2$, $3+4$ or $4+3$). So, there are six outcomes which give the desired sum. Thus, the theoretical probability that he rolls a 7 is 6 in 36, or $\frac{6}{36}$, which is equivalent to 1 in 6, or $\frac{1}{6}$.

(e) A sum of 11 can be obtained in 2 possible ways (as $5+6$ or $6+5$). Thus, the theoretical probability of rolling an 11 is 2 in 36, or $\frac{2}{36}$, which is equivalent to 1 in 18, or $\frac{1}{18}$.

(f) Answers will vary.

(g) Answers will vary.

Problème de la semaine

Problème B

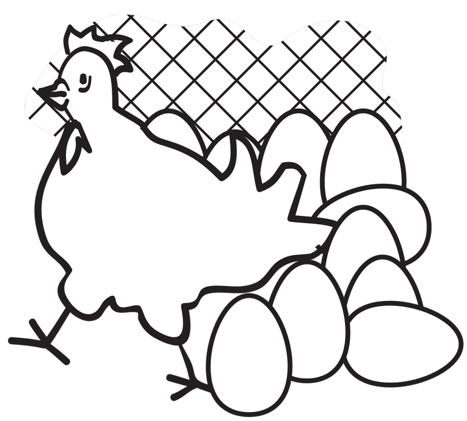
Un problème de poules et d'oeufs

Kamini élève des poules dans son jardin. Ses poules pondent en moyenne un total de 7 oeufs par jour. Elle a trois clients qui lui achètent une douzaine d'oeufs par semaine, au prix de 5 $ la douzaine. Elle garde les oeufs restants pour sa propre consommation.

(a) Environ combien d'oeufs par semaine lui restera-t-il pour sa propre consommation ?

(b) Si les poules continuent à pondre le même nombre moyen d'oeufs tout au long de l'année et que les clients continuent à acheter la même quantité d'oeufs par semaine, combien d'argent Kamini pourrait-elle gagner en une année ?

(c) De façon générale, les poules pondent des oeufs pendant environ quatre ans. Cependant, elles ne pondent pas pendant la mue (perte puis repousse des plumes), ce qu'elles font pendant 6 à 12 semaines par an. Si les poules de Kamini continuent de pondre en moyenne 7 oeufs par jour pendant les quatre prochaines années, sauf pendant la mue, et que les clients continuent à acheter la même quantité d'oeufs par semaine, quel est le montant maximal d'argent que Kamini pourrait gagner pendant ces quatre années ?



Thème Sens du nombre

Problem of the Week

Problem B and Solution

A Chicken and Egg Problem

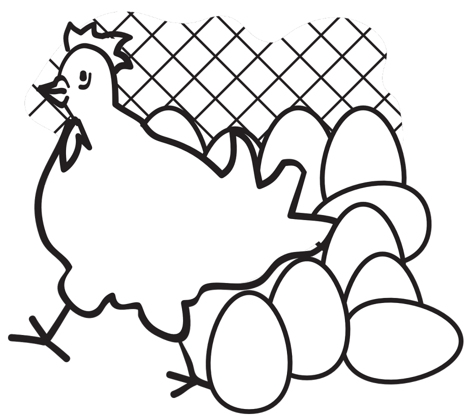
Problem

Kamini raises chickens in her backyard, which lay an average total of 7 eggs per day. She has three customers who buy a dozen eggs from her each week, paying \$5 per dozen. She keeps the remaining eggs for herself.

(a) How many eggs per week should Kamini expect to keep for herself?

(b) If the chickens continue to lay the same average number of eggs throughout the year and the customers continue to buy the same weekly amount, how much money can Kamini expect to make in one year?

(c) Chickens normally lay eggs for about four years. However, they do not lay eggs while moulting (losing then regrowing their feathers), which they do for 6 to 12 weeks per year. If Kamini's chickens maintain laying an average total of 7 eggs per day throughout the four years, except when they are moulting, and the customers continue to buy the same weekly amount, what is the maximum amount of money Kamini could make over those four years?



Solution

(a) Since her chickens lay 7 eggs per day for 7 days a week, Kamini can expect $7 \times 7 = 49$ eggs per week.

Since she has 3 customers that each buy 12 eggs per week, she has $3 \times 12 = 36$ eggs going to customers each week.

Thus, there are $49 - 36 = 13$ eggs left for Kamini to keep for herself each week.

(b) Each week she sells a dozen eggs at \$5 per dozen to 3 customers, so earns $3 \times \$5 = \$15$ each week.

Since there are 52 weeks in a year, in one year Kamini can expect to make $52 \times \$15 = \$780$.

(c) Kamini would make the maximum amount of money if the chickens only moult for 6 weeks in a year. So the chickens will be producing eggs for $52 - 6 = 46$ weeks each year.

Thus, the amount she would make in one year is $46 \times \$15 = \$690$. And the amount she would make in 4 years would be $4 \times \$690 = \$2760$.

Thus, Kamini could make a maximum of \$2760 over those 4 years.

Problème de la semaine

Problème B

À vos marques, prêts, partez !

Manish, Diana, Isebel, Ris et Ji-Yeong participent à une course de 400 m. Leur ami est venu les encourager et a pris une photo pendant la course. Dans la photo:

— Isebel mène la course.
— Ji-Yeong est devant Ris, mais derrière Manish.
— Diana a deux coureurs devant elle et deux derrière.

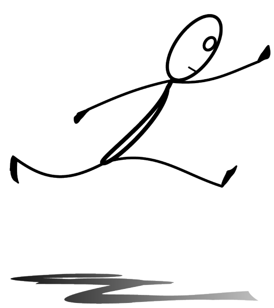
(a) À partir des indications fournies, détermine l'ordre des coureurs sur la photo. Remplis les blancs dans la liste ci-dessous.

DÉPART ________, ________, ________, ________, ________ ARRIVÉE

(b) Les cinq fractions suivantes représentent la fraction du parcours que chaque coureur avait complété sur la photo.

$$\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{3}, \frac{1}{4}$$

Associe chaque fraction au coureur correspondant. Montre ton travail à l'aide de diagrammes ou de fractions équivalentes.



Problem of the Week
Problem B and Solution
It's a Race

Problem

Manish, Diana, Isebel, Ris, and Ji-Yeong are the five runners in a 400 m race. Their friend cheered them on and took a photo partway through the race. The photo shows the following:

- Isebel is in the lead.
- Ji-Yeong has run farther than Ris, but not as far as Manish.
- Diana has two people ahead of her and two people behind her.

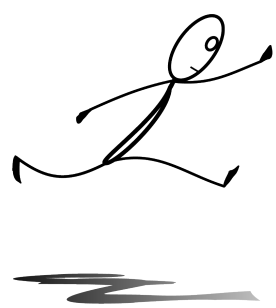
(a) Using the information given, determine the order of the runners in the photo. Fill in the blanks in the list shown below.

START ________, ________, ________, ________, ________ FINISH

(b) The following five fractions represent the fraction of the course that each runner had completed in the photo.

$$\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{3}, \frac{1}{4}$$

Which runner completed each fraction of the course? Show your work using diagrams or equivalent fractions.



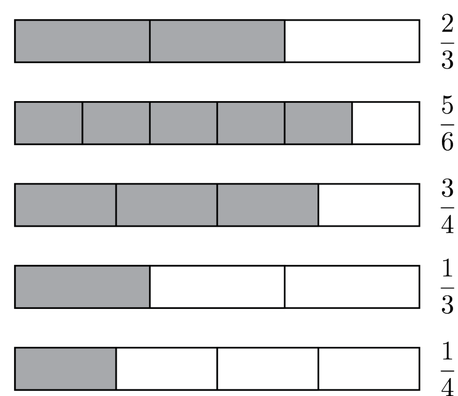
Solution

(a) We will number the positions from 1 to 5, starting on the left. Since Isebel is in the lead, she must be in position 5. Since Diana has two people ahead of her and two people behind her, she must be in position 3. That leaves us with positions 1, 2, and 4. Since Ji-Yeong has run farther than Ris but not as far as Manish, that tells us that Ris must be in position 1, Ji-Yeong must be in position 2, and Manish must be in position 4, as shown.

START Ris, Ji-Yeong, Diana, Manish, Isebel FINISH

(b) In order to determine which runner completed each fraction of the course, we must first write the fractions in order from smallest to largest. Then we can match the fractions with the runners in the order from part (a), since the runner who completed the smallest fraction of the course will be closest to the start, and the runner who completed the largest fraction of the course will be closest to the finish.

One way to compare the fractions is using diagrams, as shown.



Since each diagram is the same width, we can compare the shaded part of each diagram to place the fractions in order from smallest to largest. This gives us $\frac{1}{4}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$.

Alternatively, we can use equivalent fractions. Using a common denominator of 12, our fractions can be written as follows.

$$\frac{2}{3} = \frac{8}{12}, \quad \frac{5}{6} = \frac{10}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{1}{4} = \frac{3}{12}$$

Now we can use the equivalent fractions to place the fractions in order from smallest to largest.

$$\frac{1}{4} = \frac{3}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{5}{6} = \frac{10}{12}$$

Once we have the fractions written in order from smallest to largest, we can match each runner to the fraction of the course they completed as shown.

Runner	Ris	Ji-Yeong	Diana	Manish	Isebel
Fraction of Course Completed	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$

Problème de la semaine

Problème B

La satisfaction d'un bon service

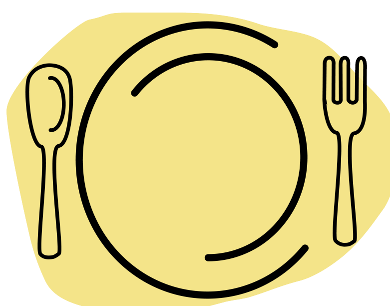
Tu viens de déguster un délicieux repas au restaurant avec une personne que tu aimes. Le coût du repas, avant les taxes et le pourboire, s'élève à 35, 10 $.

(a) Supposons que la taxe représente 15 % du coût total. Estime le montant de la taxe, en dollars, en utilisant une stratégie de calcul mental. Calcule ensuite le montant réel de la taxe et le coût total incluant la taxe.

(b) Si tu souhaites donner un pourboire qui représente 20 % du montant après taxes, combien paierais-tu au total ?

(c) Combien de monnaie te sera-t-il rendue si tu payes avec un billet de 100$?

Extension: Supposons que tu sois la personne qui fait le service. De façon générale, quand préférerais-tu recevoir un pourboire de 20 $ plutôt qu'un pourboire qui représente 20 % de la facture ? Justifie ta réponse.



Thème Sens du nombre

Problem of the Week

Problem B and Solution

Server Satisfaction

Problem

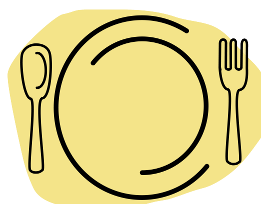
You've just enjoyed a delicious meal at a restaurant with your friend. The cost of the meal before tax and tip was \$35.10.

(a) Suppose the tax is 15% of the total cost. Estimate the dollar amount of tax using a mental math strategy. Then calculate the actual dollar amount of tax, and total cost including tax.

(b) If you want to tip the server an additional 20% after tax, how much would you pay in total?

(c) How much change would you receive if you paid with a \$100 bill?

Extension: Suppose you are a server. In general, when would you rather receive a \$20 tip instead of 20% of your bill? Justify your thinking.



Solution

(a) To estimate the dollar amount of tax, we could first round the total cost to \$35. Then we could think of 15% as $10\% + 5\%$. Since 10% of \$35 is \$3.50, and half of that is \$1.75, we can estimate that the dollar amount of tax is $\$3.50 + \$1.75 = \$5.25$.

The actual dollar amount of tax is $\$35.10 \times 0.15 = \$5.265$, which rounds to \$5.27. Thus, the total cost including tax is $\$35.10 + \$5.27 = \$40.37$.

(b) If you want to tip the server an additional 20%, then we need to calculate 20% of \$40.37. We know that 10% of \$40.37 is \$4.037. We then double that to get 20%, which is \$8.074. Rounded to the nearest cent, this is \$8.07. Finally, adding that to the total cost including tax gives $\$40.37 + \$8.07 = \$48.44$.

(c) If you paid with a \$100 bill, your change would be $\$100 - \$48.44 = \$51.56$.

Extension: When 20% of the total cost including tax is less than \$20, then a \$20 tip would likely be preferred. When 20% of the total cost including tax is greater than \$20, then a tip of 20% would likely be preferred.

Problème de la semaine

Problème B

Ces taux sont choquants

La plupart des provinces considèrent l'heure de la journée lorsqu'elles établissent les tarifs de consommation d'électricité. Ces tarifs sont souvent appelés tarifs en fonction de l'heure de consommation (FHC). En utilisant les exemples de tarifs FHC dans le tableau ci-dessous, réponds aux questions suivantes.

Période de tarif FHC	**1er novembre - 30 avril Heure du jour**	**1er mai - 31 octobre Heure du jour**	**Tarif FHC (¢ par kWh)**
Période creuse	Les jours de semaine de 19 h à 7 h, à toute heure les fins de semaine	Les jours de semaine de 19 h à 7 h, à toute heure les fins de semaine	$7,4$
Période médiane	Les jours de semaine de 11 h à 17 h	Les jours de semaine de 7 h à 11 h et de 17 h à 19 h	$10,2$
Période de pointe	Les jours de semaine de 7 h à 11 h et de 17 h à 19 h	Les jours de semaine de 11 h à 17 h	$15,1$

(a) La famille de Garret a consommé 50 kWh un samedi après-midi. Combien ces 50 kWh ont-ils coûté ?

(b) Le 10 novembre, quel serait le meilleur moment de la journée pour utiliser ta sécheuse ?

(c) À quel moment devrais-tu éviter d'utiliser ta sécheuse en été ?

(d) Quelle serait une méthode plus avantageuse (écologiquement et financièrement) pour sécher tes vêtements en été ?

(e) La famille de Ramal a consommé 1180 kWh d'électricité en un mois.

(i) Quel est le montant maximal (en dollars) qu'ils auraient pu payer pour l'électricité ce mois-là ?

(ii) Quel est le montant minimal (en dollars) qu'ils auraient pu payer pour l'électricité ce mois-là ?

Thèmes Gestion des données, Sens du nombre

Problem of the Week

Problem B and Solution

These Rates are Shocking

Problem

Most provinces take into consideration the time of day when they charge for electricity usage. The rates they charge are often referred to as Time-Of-Use (TOU) rates. Using the sample TOU rates in the table below, answer the questions that follow.

TOU Price Period	November 1 - April 30 Time of Day	May 1 - October 31 Time of Day	TOU Rate (¢ per kWh)
Off-Peak Hours	Weekdays 7 p.m. - 7 a.m., anytime on weekends	Weekdays 7 p.m. - 7 a.m., anytime on weekends	7.4
Mid-Peak Hours	Weekdays 11 a.m. - 5 p.m.	Weekdays 7 a.m. - 11 a.m. and 5 p.m. - 7 p.m.	10.2
On-Peak Hours	Weekdays 7 a.m. - 11 a.m. and 5 p.m. - 7 p.m	Weekdays 11 a.m. - 5 p.m.	15.1

(a) Garret's family used 50 kWh on a Saturday afternoon. What would be the charge for those 50 kWh?

(b) On November 10, when would be the best time of day to run your clothes dryer?

(c) When should you avoid using your clothes dryer in the summer?

(d) What might be a better way (environmentally and financially) to dry your clothes in the summer?

(e) Ramal's family used 1180 kWh hours of electricity in one month.

  (i) What is the maximum amount of money (in dollars) they could have paid for electricity that month?

  (ii) What is the minimum amount of money (in dollars) they could have paid for electricity that month?

Solution

(a) The rate for any Saturday is 7.4¢ per kWh, which is \$0.074 per kWh. Therefore, the charge for 50 kWh would be $50 \times \$0.074 = \$3.70$.

(b) If November 10 falls on a weekday, the best time to run the dryer would be anytime before 7 a.m. or after 7 p.m. If November 10 falls on the weekend, you could run it anytime from Friday after 7 p.m. until Monday morning before 7 a.m.

(c) You should avoid running your dryer from 7 a.m. to 7 p.m. on weekdays, but it is most expensive to run your dryer between 11 a.m. and 5 p.m.

(d) You could hang your clothes out to dry in the summer which would have little or no cost, both environmentally and financially.

(e) (i) Ramal's family used 1180 kWh. The most they could have paid for electricity is \$0.151 per kWh. Therefore, the maximum amount they could have paid for electricity that month is $1180 \times \$0.151 = \$178.18$.

(ii) Ramal's family used 1180 kWh. The least they could have paid for electricity is \$0.074 per kWh. Therefore, the minimum amount they could have paid for electricity that month is $1180 \times \$0.074 = \$87.32$.

Problème de la semaine

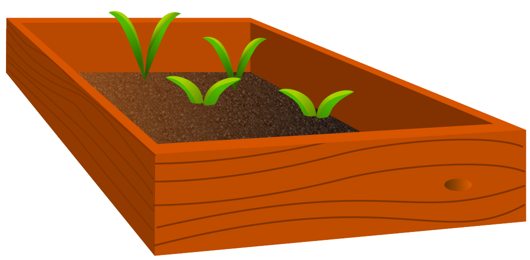
Problème B

Le jardin de Jordyn

Les habitants du voisinage de Jordyn construisent un jardin communautaire pour cultiver des légumes. Ils aimeraient que la plate-bande ait une aire de 48 mètres carrés et prévoient de mettre des planches de clôture en bois autour des bords de la plate-bande. Pour réduire les coûts du projet, ils aimeraient que le périmètre de la plate-bande soit aussi petit que possible.

(a) En supposant que les dimensions de la plate-bande soient des nombres entiers, détermine la longueur et la largeur de la plate-bande en mètres.

(b) Les habitants du voisinage ont décidé de doubler l'aire de la plate-bande, mais souhaitent toujours que son périmètre soit le plus petit possible. Supposons à nouveau que les dimensions de la plate-bande soient des nombres entiers. Détermine la longueur et la largeur de la plate-bande en mètres.



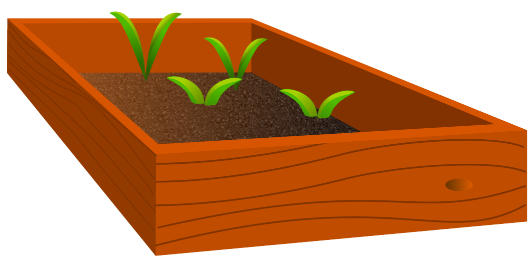
Problem of the Week
Problem B and Solution
Jordyn's Garden

Problem

Jordyn's neighbourhood is building a community garden to grow some vegetables. They would like the garden bed to have an area of 48 square metres, and plan to put wooden fence boards around the edges of the garden bed. To reduce the cost of the project, they would like the garden bed to have the smallest possible perimeter.

(a) Determine the length and width of Jordyn's community garden bed. Assume the side lengths are whole numbers, in metres.

(b) The community decided to double the area of the garden bed, but would still like it to have the smallest possible perimeter. Again, assume the side lengths are whole numbers, in metres. Determine the length and width of the garden bed now.



Solution

(a) Since the garden bed is in the shape of a rectangle and has an area of 48 square metres, it follows that length $\times$ width $= 48$. To determine the length and width, we need to find pairs of whole numbers that multiply to 48. These are called the factor pairs of 48, and are as follows: 1 and 48, 2 and 24, 3 and 16, 4 and 12, and 6 and 8. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

Width (metres)	Length (metres)	Perimeter (metres)
1	48	$2 \times (1 + 48) = 2 \times 49 = 98$
2	24	$2 \times (2 + 24) = 2 \times 26 = 52$
3	16	$2 \times (3 + 16) = 2 \times 19 = 38$
4	12	$2 \times (4 + 12) = 2 \times 16 = 32$
6	8	$2 \times (6 + 8) = 2 \times 14 = 28$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 8 metres and the width should be 6 metres.

(b) After they double the area of the garden bed it will have an area of $2 \times 48 = 96$ square metres. Using a similar approach to part (a), we need to find the factor pairs of 96, which are: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, and 8 and 12. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

Width (metres)	Length (metres)	Perimeter (metres)
1	96	$2 \times (1 + 96) = 2 \times 97 = 194$
2	48	$2 \times (2 + 48) = 2 \times 50 = 100$
3	32	$2 \times (3 + 32) = 2 \times 35 = 70$
4	24	$2 \times (4 + 24) = 2 \times 28 = 56$
6	16	$2 \times (6 + 16) = 2 \times 22 = 44$
8	12	$2 \times (8 + 12) = 2 \times 20 = 40$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 12 metres and the width should be 8 metres.

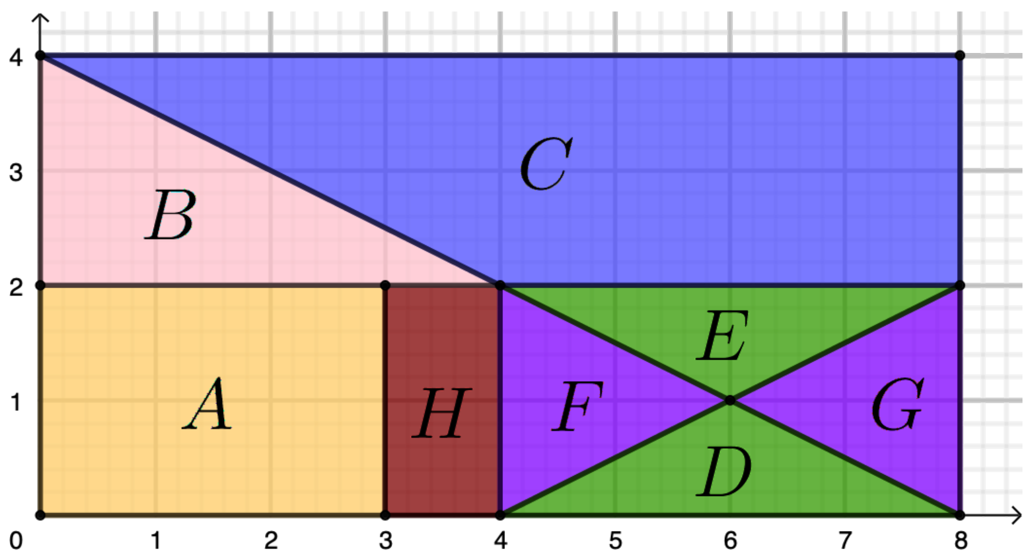
Extension: Note that in each case, the minimum perimeter occurs for the factor pair whose positive difference is the smallest. Will this always happen? Why or why not?

Problème de la semaine

Problème B

Fractions fascinantes

Un grand rectangle se trouve dans le premier quadrant du plan cartésien et a pour sommets $(0, 0)$, $(0, 8)$, $(8, 4)$ et $(4, 0)$. Le rectangle est divisé en huit régions, soit les regions A, B, C, D, E, F, G et H, comme dans la figure ci-dessous.



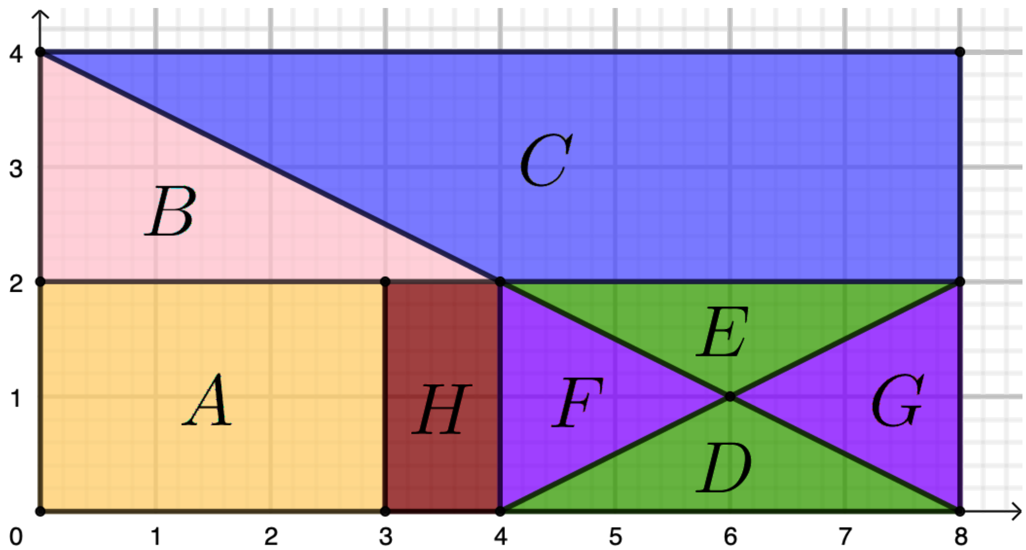
Quelle fraction de l'aire du grand rectangle est occupée par la région A ? Par la région B ? Par la région C ? Par la région D ?

Thèmes Géométrie et mesure, Sens du nombre

Problem of the Week
Problem B and Solution
Fraction Fun

Problem

A large rectangle is in the first quadrant of the Cartesian plane with its four vertices at $(0, 0)$, $(8, 0)$, $(8, 4)$, and $(0, 4)$. It is divided into eight regions labelled A, B, C, D, E, F, G, and H, as shown.



What fraction of the area of the large rectangle is the area of region A? the area of region B? the area of region C? the area of region D?

Solution

Solution 1

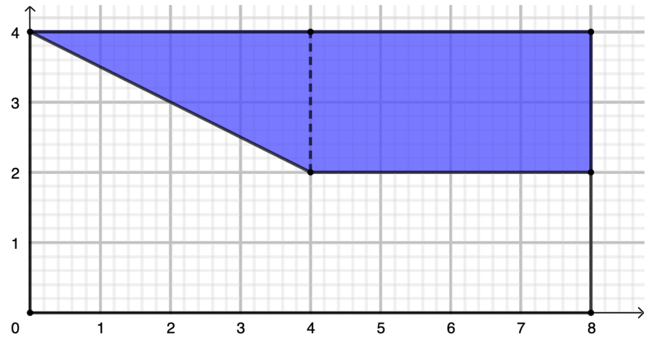
The large rectangle has a length of 8 units and a width of 4 units. Therefore, the area of the large rectangle is $8 \times 4 = 32$ square units.

Region A is a rectangle with a length of 3 units and a width of 2 units. Hence, its area is $3 \times 2 = 6$ square units. So, the area of region A is $\frac{6}{32} = \frac{3}{16}$ of the area of the large rectangle.

Region B is a triangle with a base of 4 units and height of 2 units. Hence, its area is $\frac{1}{2} \times 4 \times 2 = 4$ square units. So, the area of region B is $\frac{4}{32} = \frac{1}{8}$ of the area of the large rectangle.

Region D is a triangle with a base of 4 units and a height of 1 unit. Hence, its area is $\frac{1}{2} \times 1 \times 4 = 2$ square units. So the area of region D is $\frac{2}{32} = \frac{1}{16}$ of the area of the large rectangle.

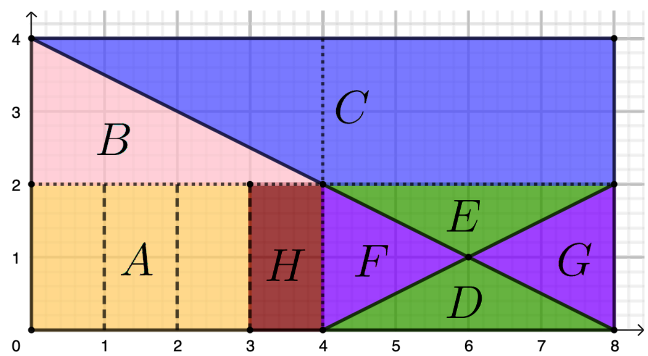
Region C is made up of a rectangle and a triangle as shown by the dashed line in the diagram below.



The rectangle has a length of 4 units and a width of 2 units. So, the area of the rectangle is $4 \times 2 = 8$ square units. The triangle has a base of 4 units and height of 2 units. So, the area of the triangle is $\frac{1}{2} \times 4 \times 2 = 4$ square units. Therefore, the area of region C is $8 + 4 = 12$ square units. Thus, the area of region C is $\frac{12}{32} = \frac{3}{8}$ of the area of the large rectangle.

Solution 2

We draw in dotted lines which divide the large rectangle into four equal parts, or quarters, and draw in dashed lines divide the lower left quarter further into quarters.

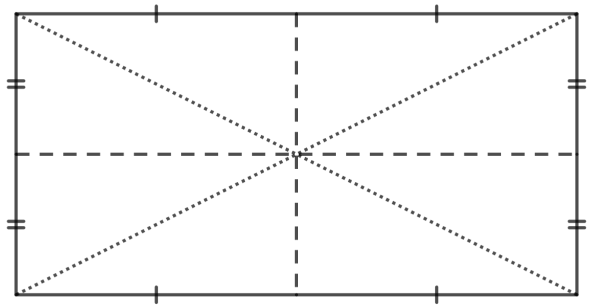


Since the dashed lines divide the lower left quarter of the rectangle further into quarters, the area of each of those four rectangles is $\frac{1}{4}$ of $\frac{1}{4}$ of the area the large rectangle, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle. Thus, the area of region A is $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$ of the area of the large rectangle.

The area of region B is half of the area of the top left quarter, and so is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ of the area of the large rectangle.

The area of region C is the area of the top half of the large rectangle, minus the area of region B, which is $\frac{1}{8}$ of the large rectangle. So in total, the area of region C is $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ of the area of the large rectangle.

Note that we can divide any rectangle into 4 smaller rectangles of equal area by joining the midpoints of opposite sides of the rectangles. When we construct the two diagonals of the large rectangle, we further divide each smaller rectangle into two triangles of equal areas. So, in the diagram below, the eight smaller triangles have equal area.



In our problem, the area of region D is equal to $\frac{2}{8}$ or $\frac{1}{4}$ of the area of the lower right rectangle. Therefore, the area of the region D is $\frac{1}{4}$ of $\frac{1}{4}$, or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ of the area of the large rectangle.