The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

# Problem of the Week Problems and Solutions 2022 - 2023

## Problem B (Grade 5/6)

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# Themes

(Click on a theme name to jump to that section.)

**Number Sense (N)**

**Geometry & Measurement (G)**

**Algebra (A)**

**Data Management (D)**

**Computational Thinking (C)**

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The problems in this booklet are organized into themes. A problem often appears in multiple themes.

# Number Sense (N)

TAKE ME TO THE COVER

# Problem of the Week

## Problem B

## Into the Wild Blue Yonder!

Canadian Actor William Shatner travelled on the Blue Origin rocket in October 2021. He was in the rocket for 10 minutes and 17 seconds after liftoff, before landing back on the desert floor in Texas. The rocket rose to an altitude of 105.9 km.

(a) If his flight was straight up and down, what was his mean speed, to the nearest kilometre per hour, over the course of the whole journey?

(b) The length of the Trans-Canada Highway between the east and west coasts of Canada is 7821 km. If the rocket travels a distance of 7821 km at the mean speed found in part (a), approximately how long (in hours and minutes) would that trip take?

Themes Geometry & Measurement, Number Sense

# Problem of the Week
## Problem B and Solution
## Into the Wild Blue Yonder!

### Problem

Canadian Actor William Shatner travelled on the Blue Origin rocket in October 2021. He was in the rocket for 10 minutes and 17 seconds after liftoff, before landing back on the desert floor in Texas. The rocket rose to an altitude of 105.9 km.

(a) If his flight was straight up and down, what was his mean speed, to the nearest kilometre per hour, over the course of the whole journey?

(b) The length of the Trans-Canada Highway between the east and west coasts of Canada is 7821 km. If the rocket travels a distance of 7821 km at the mean speed found in part (a), approximately how long (in hours and minutes) would that trip take?

## Solution

(a) The total distance William Shatner travelled was $105.9 \times 2 = 211.8\,\text{km}$.

His travel time was 10 minutes and 17 seconds. Since there are 60 seconds in one minute, his travel time was $10 \times 60 + 17 = 617$ seconds. Since there are $60 \times 60 = 3600$ seconds in each hour, his travel time in hours was $617 \div 3600 \approx 0.1714$ hr.

Thus, his mean speed was $211.8$ km $\div\ 0.1714$ hr $\approx 1236$ km/hr.

(b) Travelling a distance of 7821 km at a mean speed of 1236 km/hr would take the rocket $7821 \div 1236 \approx 6.328$ hr. Since there are 60 minutes in each hour, this is equal to $6.328 \times 60 \approx 380$ minutes, or approximately 6 hours and 20 minutes.

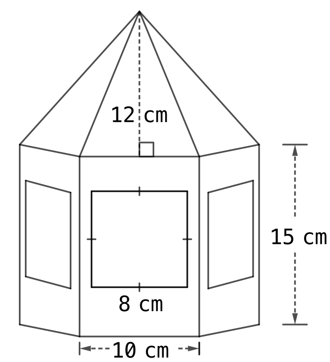
NOTE: Calculations here were carried out with four significant digits. Answers may vary if fewer are used at each stage.

# Problem of the Week

## Problem B

## Painting a Birdhouse

Bird feeders come in many shapes and sizes. Meera has one with a pentagonal base, five identical rectangular sides, and five identical triangles that meet at a point forming the roof. Each rectangular side has a width of 10 cm, a height of 15 cm, and a square window of side length 8 cm. Each triangle has a height of 12 cm and its base lines up with the top width of one of the rectangular sides.




(a) What is the total area of the five windows in the feeder?

(b) Meera has decided to paint the outer faces of the triangular roof segments and the outer sides of the feeder (except the windows), but not the base. What is the total surface area of the parts of the feeder Meera intends to paint?

(c) Suppose you can purchase a 100 mL can of paint for \$3.50 which will cover 10 000 $\text{cm}^2$ of surface area. If Meera does two coats of paint on each pentagonal bird feeder, how many complete pentagonal bird feeders can be painted by one of these cans of paint?

Themes Geometry & Measurement, Number Sense

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## Painting a Birdhouse

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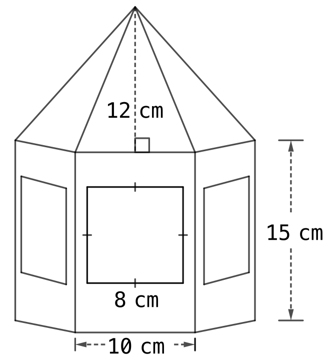


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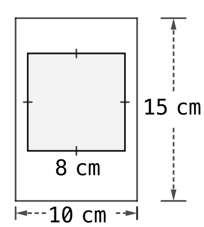
## Solution

(a) Since each of the five windows is an 8 cm square of area $8 \times 8 = 64\,\text{cm}^2$, the total area of the windows is $5 \times 64 = 320\,\text{cm}^2$.

(b) The parts of the feeder to be painted are the five rectangular borders around the windows plus the five triangular roof segments.

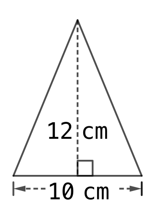
The area of one rectangular border is the area of the outer rectangle minus the area of the square window. Since the area of the outer rectangle is $10 \times 15 = 150\,\text{cm}^2$, and the area of the square window is $64\,\text{cm}^2$, the area of one rectangular border is $150 - 64 = 86\,\text{cm}^2$.




There are five of these borders and so their total area is $5 \times 86 = 430\,\text{cm}^2$.

The area of one triangular roof segment is $\frac{1}{2} \times 10 \times 12 = 60\,\text{cm}^2$. There are five of these triangles and so their total area is $5 \times 60 = 300\,\text{cm}^2$.

Thus, the total area to be painted is $430+300 = 730\,\text{cm}^2$.




(c) Two coats of paint on one feeder will require paint for $2 \times 730 = 1460\,\text{cm}^2$. Thus, Meera can paint $10\,000 \div 1460 \approx 6.8$ birdhouses. Therefore, Meera can paint 6 complete birdhouses using one can of paint.

# Problem of the Week
## Problem B
## A Stoney Problem

Sela is doing some landscaping, and needs to pave a rectangular space with an area of 53.5 $m^2$. She plans to use paving stones which are 10 cm by 20 cm, and so each has an area of 200 $cm^2$ each. Note that only whole paving stones will be used.

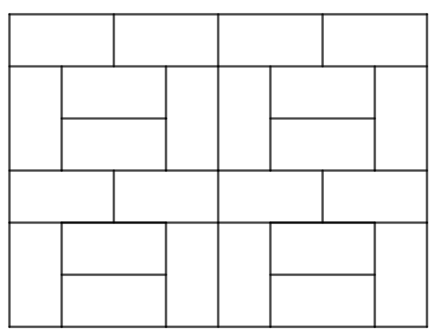
At the Home Shop, Sela learns that these pavers are sold on pallets of 1000 stones, and she must buy complete pallets at $499 each.

(a) How many stones will she need to cover the 53.5 $m^2$ area?

(b) How many pallets will she need to buy?

(c) How many stones will be left on the last pallet Sela uses?

(d) If Sela is able to buy partial pallets, how much would she save if she only bought the paving stones she needed?



Themes Geometry & Measurement, Number Sense

# Problem of the Week
## Problem B and Solution
## A Stoney Problem

### Problem

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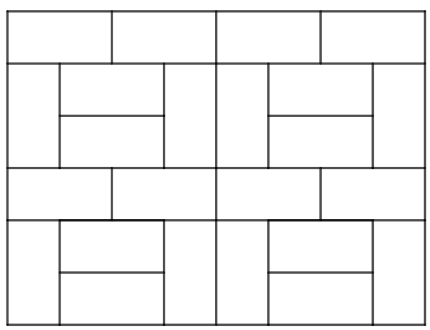

### Solution

(a) One square metre is equivalent to $100 \times 100 = 10\,000\,\text{cm}^2$, the area Sela needs to pave has area $53.5 \times 10\,000 = 535\,000\,\text{cm}^2$. Since each paving stone has area $200\,\text{cm}^2$, Sela will need $535\,000 \div 200 = 2675$ stones.

(b) Since each pallet has 1000 paving stones, Sela needs $2675 \div 1000 = 2.675$ pallets. However, she must buy complete pallets, so Sela will need to buy 3 pallets, or 3000 paving stones.

(c) On the last pallet Sela uses, there will be $3000 - 2675 = 325$ paving stones.

(d) Sela would not need to buy the extra 325 paving stones. The 325 paving stones as a fraction of a pallet is $\frac{325}{1000} = 0.325$.
Thus, she would save $0.325 \times \$499 \approx \$162.18$.

# Problem of the Week

## Problem B

## Joey Prepares for Winter

Joey the chipmunk will soon be hibernating, so he's gathering acorns, his food supply for the long winter months.

Joey has four acorns remaining from the previous day, and has gathered acorns over the last few hours as shown in the following table.

| **Hour** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Total Number of Acorns** | 4 | 20 | 36 | 52 | 68 | 84 |

(a) Is the total number of acorns a linear growing pattern? Verify your answer by creating a graph.

(b) Suppose Joey continues collecting acorns at this same rate.

(i) How many acorns would Joey have collected by the end of Hour 12?

(ii) How many hours would it take him to collect at least 330 acorns?

(iii) Write an algebraic expression to represent the total number of acorns Joey would have after collecting for $n$ hours.

Themes Algebra, Number Sense

# Problem of the Week
## Problem B and Solution
## Joey Prepares for Winter

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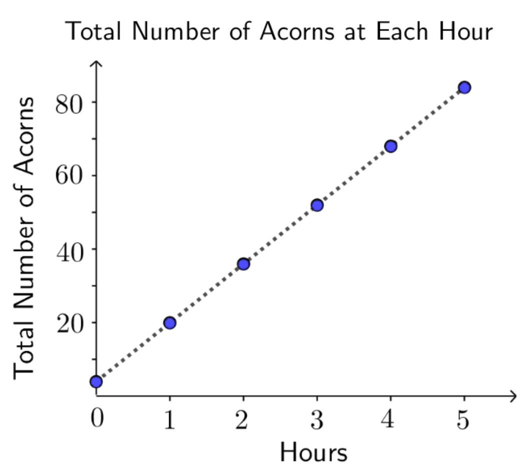
(iii) Write an algebraic expression to represent the total number of acorns Joey would have after collecting for $n$ hours.

## Solution

(a) Looking at the data, we see that the number of acorns increases by the same amount each hour; Joey is collecting acorns at a rate of 16 per hour. So we expect that the pattern of the total number of acorns is a growing linear pattern. This is verified by the following graph.



(b) (i) Hour 12 is 7 more hours after Hour 5. Since Joey will collect 16 acorns in each of those hours, he will have $7 \times 16 = 112$ more acorns, giving a total of $84 + 112 = 196$ acorns by the end of Hour 12.

(ii) To collect at least 330 acorns in total, Joey needs $330 - 196 = 134$ more acorns than he has after 12 hours. After 8 more hours, he would have $8 \times 16 = 128$ more acorns. After 9 more hours, he would have $9 \times 16 = 144$ more acorns. Therefore, he will need to collect acorns for 9 more hours to get to at least 330 acorns.
Thus, he will need a total of $12 + 9 = 21$ hours to collect at least 330 acorns.

ALTERNATIVELY: Joey initially has 4 acorns, so to get to 330 acorns, he needs to collect 326 more acorns. Since he collects 16 acorns per hour, this would take him $326 \div 16 = 20\frac{3}{8}$ hours. This means he will have 330 acorns during the $21^{\text{st}}$ hour. That is, he will need to collect for 21 hours to get at least 330 acorns.

(iii) After $n$ hours of collecting 16 acorns each hour, Joey would have $16 \times n$ acorns. Given that he starts with four leftover acorns, Joey would have a total of
$(16 \times n) + 4$ acorns.

# Problem of the Week

## Problem B

## Water, Water, Everywhere...

Very little of Earth's fresh water is accessible for human consumption, particularly in dry countries, making alternative sources necessary.

(a) The per capita (per person) daily water consumption for nine different countries is given below.

$$155\,\text{L},\ 251\,\text{L},\ 200\,\text{L},\ 147\,\text{L},\ 135\,\text{L},\ 235\,\text{L},\ 373\,\text{L},\ 145\,\text{L},\ 380\,\text{L}$$

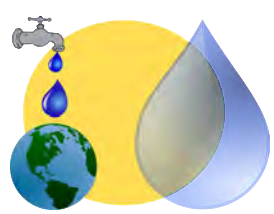
What is the average per capita daily water consumption for these countries? Round your answer to the nearest whole number.

(b) A small city of 110 000 people in an arid (very dry) country obtains its fresh water by desalination of sea water. If the per capita consumption in this city is equal to the average from part (a), how much fresh water must be produced each day by the city's desalination plant?

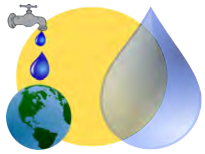
(c) Sea water is 3.5% salt; the remaining 96.5% is fresh water. Thus, if 1000 L of sea water was desalinated, the amount of fresh water produced would be $0.965 \times 1000 = 965$ L. In general, we can use the following equation to show the relationship between the amount of sea water and fresh water in the desalination process.

$$0.965 \times \text{amount of sea water} = \text{amount of fresh water}$$

Use this equation and your answer from part (b) to find the amount of sea water that must be processed by the desalination plant every day in order to fulfill the city's fresh water needs.



**Themes** Data Management, Number Sense



# Problem of the Week

## Problem B and Solution

## Water, Water, Everywhere...

### Problem

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## Solution

(a) Adding the nine countries' daily consumption figures gives 2021 L. Thus, the average daily consumption per capita is $2021 \div 9 = 224.555... \approx 225\,\text{L}$.

(b) If each of the 110 000 people consumes 225 litres of water per day, then the city's desalination plant must produce $110\,000 \times 225 = 24\,750\,000$ litres of fresh water per day.

(c) Once we substitute our answer from part (b), the equation becomes $0.965 \times \text{amount of sea water} = 24\,750\,000$. We can find the amount of sea water by trial and error, but a more efficient method is to notice that $\text{amount of sea water} = 24\,750\,000 \div 0.965 \approx 25\,647\,668$. Thus the amount of sea water needed each day is approximately 25 647 668 L, or about 25.65 million litres.

# Problem of the Week

## Problem B

## A Little Rain Must Fall

Excessive rainfall may occur during weather events such as hurricanes, or in some places, simply as a part of everyday life.

(a) During Hurricane Harvey in 2017, almost 75 mm of rain fell in one hour in Houston. If rain continued to fall at that rate for 24 hours, how much rain would fall? Express your answer in metres.

(b) Mawsynram, India is recognized as one of the wettest places on Earth, with an average annual rainfall of 11 872 mm, most of which falls during the monsoon season. If the rain was spread out evenly over the whole year, how much rain, in mm, would fall each day? Round your answer to one decimal place.

(c) Find the average annual rainfall in your community. How many times more is Mawsynram's average annual rainfall than the average annual rainfall in your community? Round your answer to one decimal place.

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## A Little Rain Must Fall

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(c) Find the average annual rainfall in your community. How many times more is Mawsynram's average annual rainfall than the average annual rainfall in your community? Round your answer to one decimal place.

## Solution

(a) If 75 mm of rain fell in each hour for 24 hours, the total amount of rain would be $75 \times 24 = 1800\,\text{mm}$, or 1.8 metres.

(b) If a total of 11 872 mm of rain was spread out evenly over the whole year, then over 365 days, the daily average would be $11\,872 \div 365 \approx 32.5$ mm.

(c) Answers will vary. The average annual rainfall for the city of Toronto is 831 mm*. Therefore, the average annual rainfall in Mawsynram is $11\,872 \div 831 \approx 14.3$ times more than the average annual rainfall of Toronto.
*Source: https://www.currentresults.com/Weather/Canada/Cities/precipitation-annual-average.php

# Problem of the Week

## Problem B

## Sarah's Bakery

Sarah is opening her own bakery, and she needs help pricing her giant chocolate chip cookies. In order to price her cookies, she first needs to know what the ingredient cost is for each cookie.

The following table provides the list of ingredients used, the cost to purchase the ingredients, and the amount of each ingredient required for a batch of 12 cookies.

| **Ingredient** | **Cost of Ingredient** | **Amount per Batch** | **Cost per Batch** |
|---|---|---|---|
| Brown Sugar | \$2.90 for 5 cups | 1 cup | |
| Eggs | \$3.00 for 12 | 1 | |
| Chocolate Chips | \$9.48 for 3 cups | $\frac{1}{2}$ cup | |
| White Sugar | \$2.48 for 10 cups | $\frac{1}{4}$ cup | |
| Butter | \$4.98 for 2 cups | $\frac{3}{4}$ cup | |
| Flour | \$9.00 for 40 cups | $2\frac{1}{4}$ cup | |

(a) Complete the information in the table by finding the cost of each ingredient for one batch of 12 giant chocolate chip cookies. Round your answers to the nearest cent.

(b) Rounded to the nearest cent, what is the cost of the ingredients for one giant chocolate chip cookie?

(c) What are some of the other costs that Sarah needs to take into consideration when pricing her cookies?

**Theme** Number Sense

# Problem of the Week
## Problem B and Solution
## Sarah's Bakery

### Problem

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(b) Rounded to the nearest cent, what is the cost of the ingredients for one giant chocolate chip cookie?

(c) What are some of the other costs that Sarah needs to take into consideration when pricing her cookies?

## Solution

(a) One way to solve this problem is to find the unit rate for each ingredient, and then multiply the unit rate by the amount per batch for the ingredient to find the cost per batch.

The completed table is below. We have added a column to the table to show the unit rate calculation for each item.

| **Ingredient** | **Cost of Ingredient** | **Unit Rate** | **Amount per Batch** | **Cost per Batch** |
|---|---|---|---|---|
| Brown Sugar | \$2.90 for 5 cups | $\frac{2.90}{5} = \$0.58$ per cup | 1 cup | \$0.58 |
| Eggs | \$3.00 for 12 | $\frac{3.00}{12} = \$0.25$ per egg | 1 | \$0.25 |
| Chocolate Chips | \$9.48 for 3 cups | $\frac{9.48}{3} = \$3.16$ per cup | $\frac{1}{2}$ cup | \$1.58 |
| White Sugar | \$2.48 for 10 cups | $\frac{2.48}{10} = \$0.248$ per cup | $\frac{1}{4}$ cup | \$0.06 |
| Butter | \$4.98 for 2 cups | $\frac{4.98}{2} = \$2.49$ per cup | $\frac{3}{4}$ cup | \$1.87 |
| Flour | \$9.00 for 40 cups | $\frac{9.00}{40} = \$0.225$ per cup | $2\frac{1}{4}$ cup | \$0.51 |

NOTE:

To find the cost per batch for the butter, we can use the fact that $\frac{3}{4}$ is the same as $3 \times \frac{1}{4}$. Therefore, the cost per batch is $\$2.49 \times 3 \times \frac{1}{4} \approx \$1.87$.

To find the cost per batch of the flour, we can use the fact that $2\frac{1}{4}$ is the same as $2 + \frac{1}{4}$. Now, we can find the cost of 2 cups of flour and the cost of $\frac{1}{4}$ cup of flour, and then add these costs together.
The cost of 2 cups of flour is $\$0.225 \times 2 = \$0.45$, and the cost of $\frac{1}{4}$ cup of flour is $\$0.225 \times \frac{1}{4} = \$0.05625$. Therefore, the total cost of the flour, rounded to the nearest cent, is \$0.51.

(b) The cost of the ingredients for one giant chocolate chip cookie is equal to the sum of the costs for one batch of 12 cookies, divided by 12. The total cost for one batch of 12 cookies is

$$\$0.58 + \$0.25 + \$1.58 + \$0.06 + \$1.87 + \$0.51 = \$4.85$$

Thus, the cost of the ingredients for one cookie is $\$4.85 \div 12 \approx \$0.40$.

(c) Some of the other costs that Sarah needs to take into consideration include labour (if she has other employees), rent (or mortgage), utilities, and equipment.

# Problem of the Week
## Problem B
## What's in a Measure?

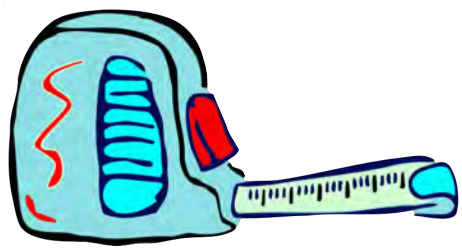
Listed below are measurements for some specific items. However, the unit of measure may not be what you are used to. Convert the measurements to a unit that makes more sense to you.

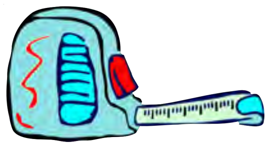
(a) The time between your 10th and 11th birthdays is 31 536 000 seconds.

(b) The distance between Montreal and Toronto is 54 160 000 centimetres.

(c) The length of my toothbrush is 0.00019 kilometres.

(d) I pour 0.25 litres of milk on my cereal in the morning.





# Problem of the Week
## Problem B and Solution
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### Solution

(a) The time between your 10th and 11th birthdays is more commonly known as 1 year. (Hopefully, students did not need to do any calculations to determine this!) If you want to do the calculations, divide 31 536 000 by 60 to get 525 600 minutes. Then divide 525 600 by 60 to get 8760 hours. Finally, divide 8760 by 24 to get 365 days, which is equal to 1 year.

(b) A more reasonable unit for the distance between Montreal and Toronto is kilometres. So we will first convert the distance to metres and then to kilometres.

That is, 54 160 000 cm is equal to $54\,160\,000 \div 100 = 541\,600$ m, and $541\,600 \div 1000 = 541.6$ km.

Therefore, the distance between Montreal and Toronto is 541.6 km.

(c) A more reasonable unit for the length of my toothbrush would be centimetres. So we will first convert the length to metres and then to centimetres.

That is, 0.00019 km is equal to $0.00019 \times 1000 = 0.19$ m, and $0.19 \times 100 = 19$ cm.

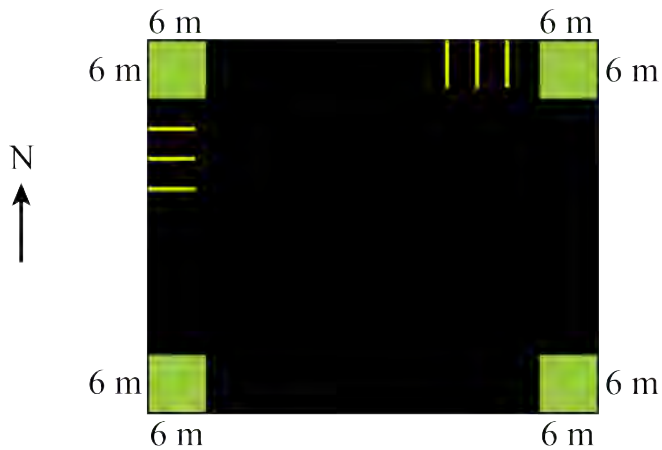
Therefore, the length of the toothbrush is 19 cm.

(d) A more reasonable unit for the amount of milk is millilitres. Therefore, the amount of milk is $0.25\,\text{L} \times 1000 = 250\,\text{mL}$.

NOTE: The units chosen may vary, since different students may find different units reasonable.

# Problem of the Week
## Problem B
## Parking by Design

KalMart has a paved, rectangular parking lot with a 6 m by 6 m curbed garden in each corner. There are parking spots along the north and west sides of the parking lot. Some of the parking spots on the north and west sides are shown in the diagram.




Each parking spot is 2.5 m wide, and the lines separating the parking spots are 7.5 cm thick.

(a) There are 25 parking spots along the north side of the parking lot. What is the length, in metres, of the north side of the parking lot, including the gardens?

(b) There are 20 parking spots along the west side of the parking lot. What is the length, in metres, of the west side of the parking lot, including the gardens?

(c) What is the total area, in square metres, of the paved portion of the parking lot, excluding the gardens?

# Problem of the Week

## Problem B and Solution

## Parking by Design

### Problem

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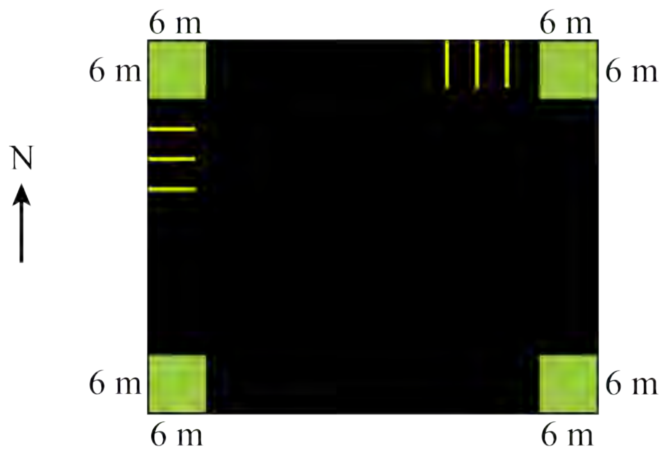


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(c) What is the total area, in square metres, of the paved portion of the parking lot, excluding the gardens?

## Solution

(a) There are 25 parking spots on the north side, plus 24 lines between them, since there are no lines at the corners next to the gardens. Since each parking spot is 2.5 m wide, the parking spots occupy a total of $25 \times 2.5 = 62.5$ m. Since each line is 7.5 cm $= 0.075$ m thick, the lines occupy a total of $24 \times 0.075 = 1.8$ m. The corner gardens occupy a total of $2 \times 6 = 12$ m. Thus, the total length of the north side is $62.5 + 1.8 + 12 = 76.3$ m.

(b) Similarly, there are 20 parking spots on the west side, plus 19 lines between them. Since each parking spot is 2.5 m wide, the parking spots occupy a

total of $20 \times 2.5 = 50$ m. Since each line is $7.5$ cm $= 0.075$ m thick, the lines occupy a total of $19 \times 0.075 = 1.425$ m. The corner gardens occupy a total of $2 \times 6 = 12$ m. Thus, the total length of the west side is
$50 + 1.425 + 12 = 63.425$ m.

(c) The total area of the parking lot is $76.3 \times 63.425 = 4839.3275$ m$^2$. Each corner garden has an area of $6 \times 6 = 36$ m$^2$. The total garden area is then $4 \times 36 = 144$ m$^2$. Thus, excluding the four gardens, the area of the paved portion of the lot is $4839.3275 - 144 = 4695.3275$ m$^2$.

# Problem of the Week

## Problem B

## Yukon Do It!

The Yukon Quest is one of the most famous endurance sled dog races in the world. The total distance the race covers is 1635 km.

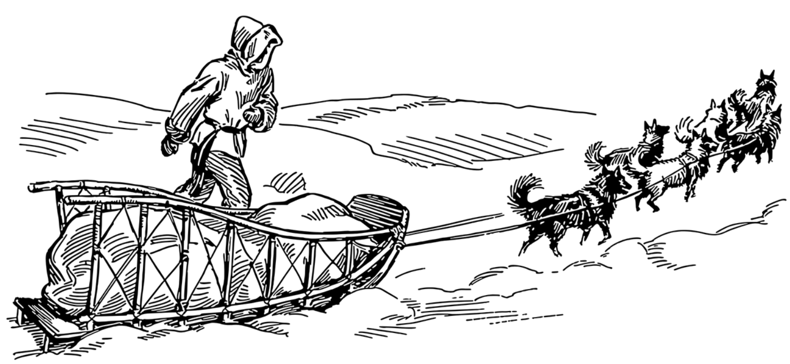
(a) If the average distance travelled per day for each team is 145 km, how many days will it take, on average, for a team to complete the Yukon Quest?

(b) A team of Alaskan huskies travels at 15 km per hour, with an 18 minute rest after every three hours. A team of Siberian huskies runs more quickly at 20 km per hour, but requires a 30 minute rest after every two hours.

On a certain day both teams travel 145 km. Create a broken-line graph of distance versus time for each team for that day.

Suggestion: You may find it helpful to first construct a table for each team, matching the total distance travelled with the elapsed time for each interval of travel and rest.

(c) Suppose that the weight of some additional equipment slows the average speed of the Siberian huskies by 5 km per hour. If the team still travels 145 km in a day, by how many minutes will this increase their travel time for the day?



Themes Data Management, Number Sense

# Problem of the Week
## Problem B and Solution
## Yukon Do It!

### Problem

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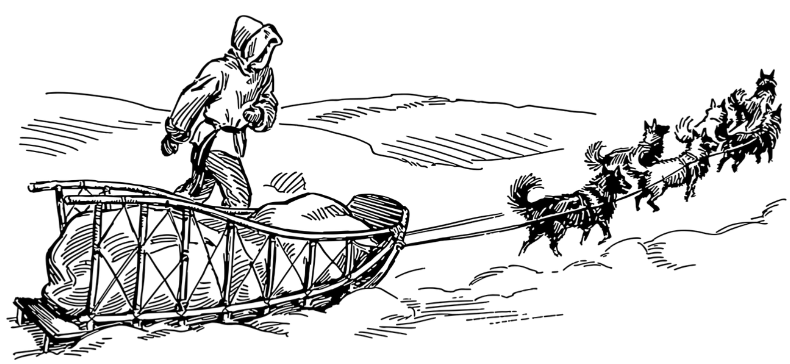
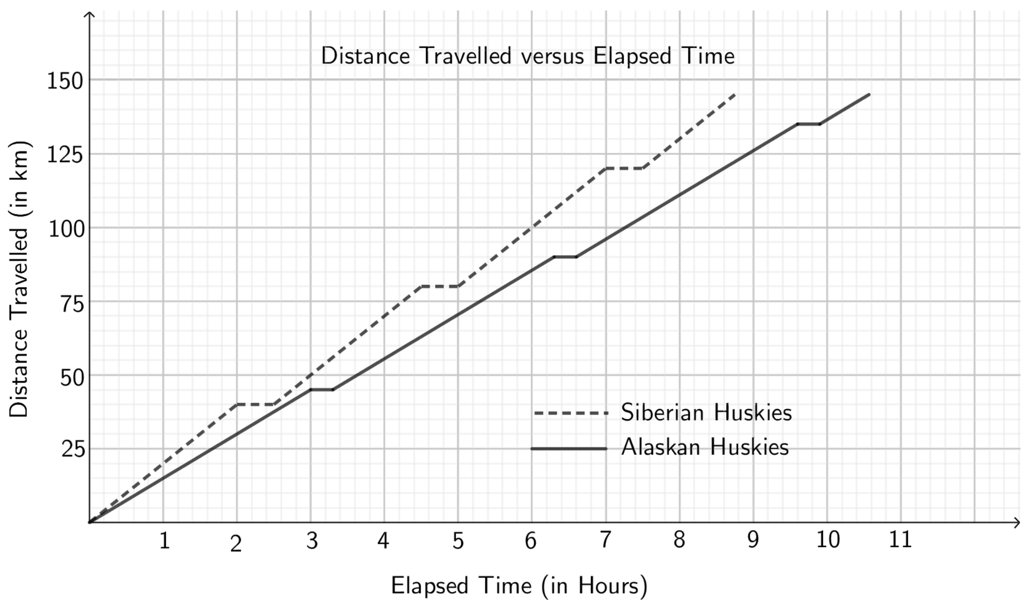


## Solution

(a) Since the average (mean) distance traveled each day is 145 km, and the total distance is 1635 km, the number of days to complete the race is $1635 \div 145 \approx 11.276$ days. Thus, on average, the teams would finish on the 12th day.

(b) A broken-line graph of distance versus time for each team is shown below.




Here is the table for the Alaskan husky team.

| Interval Type | Start Time | End Time | Interval Distance Travelled (km) | Total Distance Travelled (km) |
|---|---|---|---|---|
| Travel | 0 | 3 hrs | 45 | 45 |
| Rest | 3 hrs | 3 hrs 18 min | 0 | 45 |
| Travel | 3 hrs 18 min | 6 hrs 18 min | 45 | 90 |
| Rest | 6 hrs 18 min | 6 hrs 36 min | 0 | 90 |
| Travel | 6 hrs 36 min | 9 hrs 36 min | 45 | 135 |
| Rest | 9 hrs 36 min | 9 hrs 54 min | 0 | 135 |
| Travel | 9 hrs 54 min | 10 hrs 34 min | 10 | 145 |

Here is the table for the Siberian husky team.

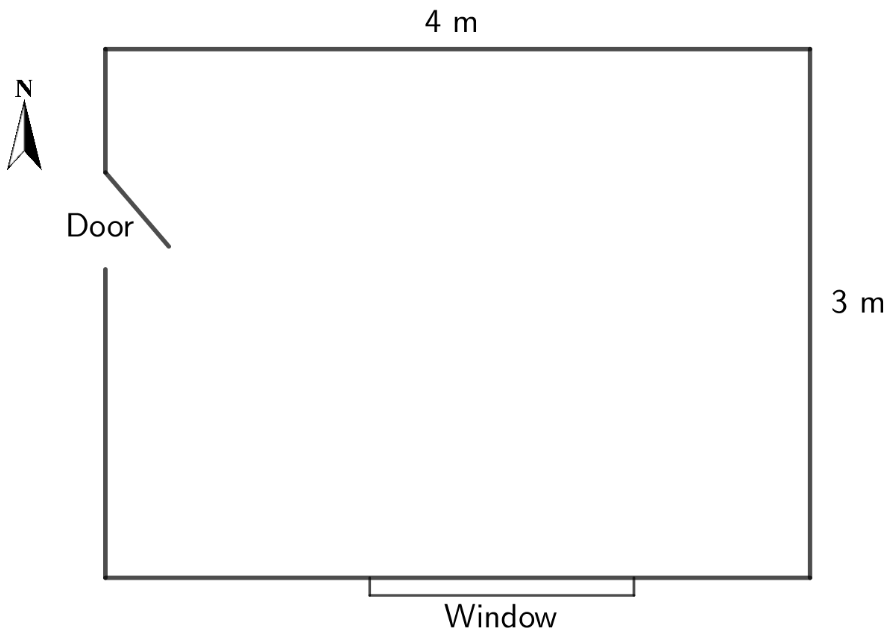
| Interval Type | Start Time | End Time | Interval Distance Travelled (km) | Total Distance Travelled (km) |
|---|---|---|---|---|
| Travel | 0 | 2 hrs | 40 | 40 |
| Rest | 2 hrs | 2 hrs 30 min | 0 | 40 |
| Travel | 2 hrs 30 min | 4 hrs 30 min | 40 | 80 |
| Rest | 4 hrs 30 min | 5 hrs | 0 | 80 |
| Travel | 5 hrs | 7 hrs | 40 | 120 |
| Rest | 7 hrs | 7 hrs 30 min | 0 | 120 |
| Travel | 7 hrs 30 min | 8 hrs 45 min | 25 | 145 |

Note that the Alaskan huskies travel 135 km in three segments of 3 hours and 18 minutes each, plus 10 km in a final segment of 40 minutes. This gives a total time of 10 hours and 34 minutes. The Siberian huskies travel 120 km in three segments of 2 hours and 30 minutes each, plus 25 km in a final segment of 1 hour and 15 minutes. This gives a total time of 8 hours and 45 minutes.

(c) Since the average speed of the Siberian huskies is now only 15 km per hour, they will travel only 30 km in each 2 hour segment. Thus, they will now travel 120 km in four segments of 2 hours and 30 minutes each. That is, they will travel 120 km in 10 hours. They will travel the last 25 km in a final segment of 100 min, or 1 hour and 40 minutes. This gives a total time of 11 hours and 40 minutes. Thus, their time for the day has increased by 2 hours and 55 minutes, or 175 minutes.

# Problem of the Week
## Problem B
## Redecoration Station

Nimrat wants to redecorate her bedroom. The floor plan for her bedroom is shown below.




The walls in Nimrat's bedroom are 2.5 m high.

(a) Nimrat wants nice, plush, wall-to-wall carpet in her bedroom. How many square metres of carpet will she need to buy? If the carpet she buys costs \$20 per square metre, how much will her carpet cost in total?

(b) Wallpaper costs \$8 per square metre. How much wallpaper will she need to cover the north and east walls? How much will it cost for the wallpaper for those two walls?

(c) She decides to paint the south and west walls, and the cost for paint to do so is \$75. If her total budget is \$500 for carpet, wallpaper, and paint, how much over or under her budget is she?

**Themes** Geometry & Measurement, Number Sense

# Problem of the Week

## Problem B and Solution

## Redecoration Station

### Problem

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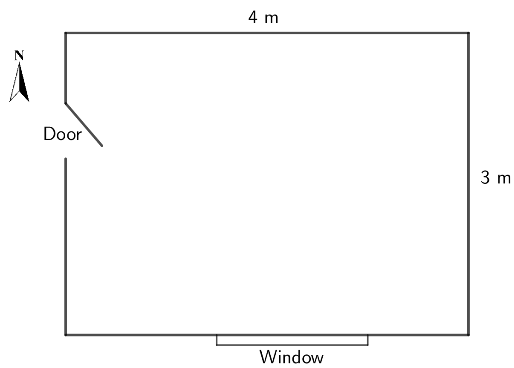


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### Solution

(a) Since the total floor area of Nimrat's bedroom is $4 \times 3 = 12\,\text{m}^2$, and the carpet costs \$20 per square metre, the cost of her wall-to-wall carpet will be $\$20 \times 12 = \$240$.

(b) Since the north wall is $4\,\text{m}$ long and $2.5\,\text{m}$ high, its area is $4 \times 2.5 = 10\,\text{m}^2$. Since the east wall is $3\,\text{m}$ long and $2.5\,\text{m}$ high, its area is $3 \times 2.5 = 7.5\,\text{m}^2$. Thus, the total area to be wallpapered is $10 + 7.5 = 17.5\,\text{m}^2$. Therefore, the cost of the wallpaper at \$8 per square metre will be $17.5 \times \$8 = \$140$.

(c) The total cost of wallpaper, carpet, and paint will be $\$240 + \$140 + \$75 = \$455$. Since her total budget is \$500, she will be $\$500 - \$455 = \$45$ under her budget.

# Problem of the Week
## Problem B
## Up and Down

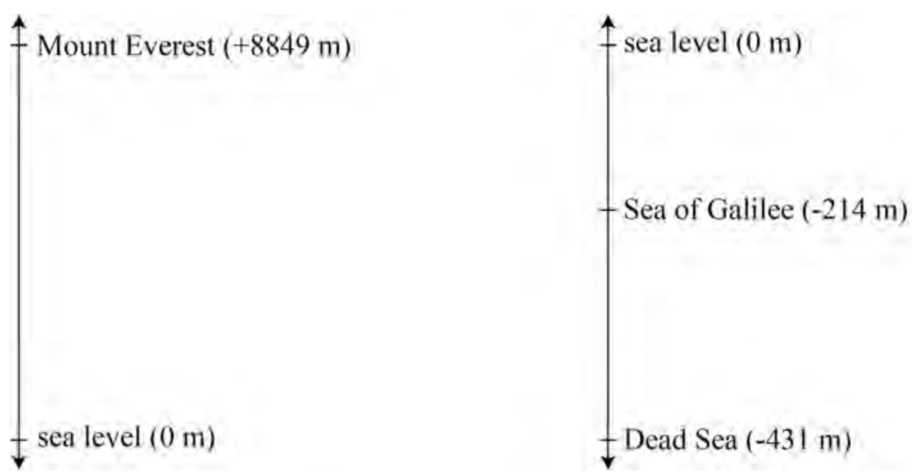
Sea level is the level of the sea's surface along a coast of land; it is often taken as the midpoint between average low and high tide levels. The elevation of a location is measured as its vertical distance above or below sea level.

(a) The table shows some geographical locations as well as their elevation. List the locations in order from highest elevation to lowest elevation.

| **Location** | **Elevation** |
|---|---|
| New Orleans, Louisiana, USA | 2 m above sea level |
| Mount Fuji, Japan | 3776 m above sea level |
| Caspian Sea, Eastern Europe | 28 m below sea level |
| Badwater Basin, Death Valley, California, USA | 86 m below sea level |
| Laguna del Carbón, Argentina (lowest point in the Americas) | 105 m below sea level |
| Mount Kilimanjaro, Tanzania | 5895 m above sea level |
| Veryovkina Cave entrance, Abkhazia (deepest known cave) | 2285 m above sea level |
| Ryfast Tunnel, Norway | 292 m below sea level |
| Lake Assal, Djibouti | 155 m below sea level |
| The Matterhorn, a mountain in the Alps | 4478 m above sea level |

(b) The highest point on Earth is Mount Everest, which is approximately 8849 m above sea level. The lowest land point on Earth is the Dead Sea, which is 431 m below sea level. The nearby Sea of Galilee is 214 m below sea level.

In the number lines below, we have written elevations above sea level as positive numbers (+) and elevations below sea level as negative numbers (−). Place the locations from the table in their approximate positions on the number lines.

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THEME NUMBER SENSE

# Problem of the Week

## Problem B and Solution

## Up and Down

### Problem

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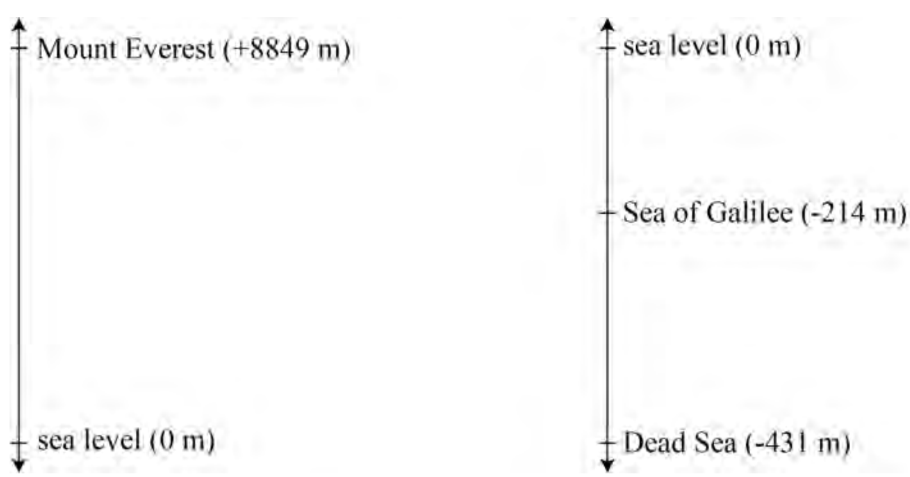


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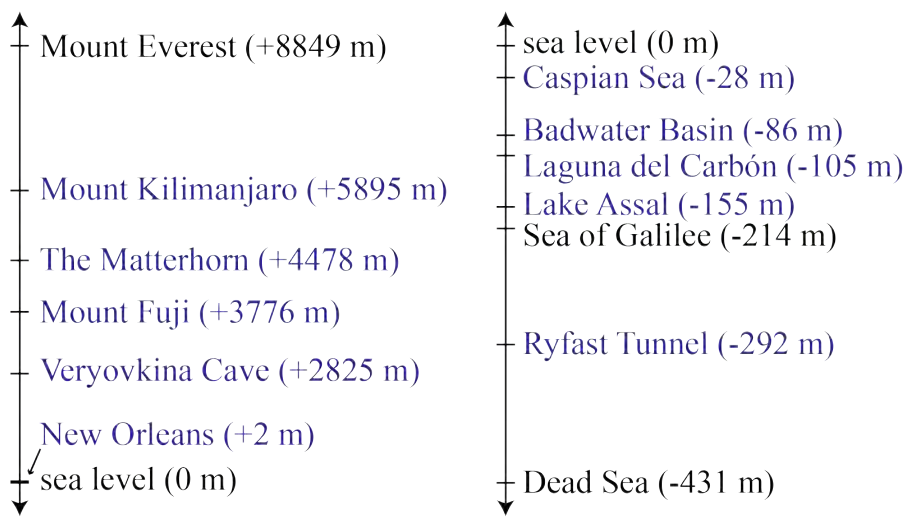
## Solution

(a) The locations are listed below in order from highest elevation to lowest elevation. Elevations above sea level are written as positive numbers (+) and elevations below sea level are written as negative numbers (−).

Mount Kilimanjaro (+5895 m), The Matterhorn (+4478 m), Mount Fuji (+3776 m), Veryovkina Cave (+2285 m), New Orleans (+2 m), Caspian Sea (−28 m), Badwater Basin (−86 m), Laguna del Carbón (−105 m), Lake Assal (−155 m), Ryfast Tunnel (−292 m)

(b) The completed number lines are shown below. Note that New Orleans is not distinguishable from sea level on the number line because it is so much closer to sea level than to any of the other locations above sea level.




## Extension:

Add scales to the number lines you drew in part (b). You will need to use different scales for each number line because the distance between Mount Everest and sea level is much larger than the distance between sea level and the Dead Sea.

# Problem of the Week

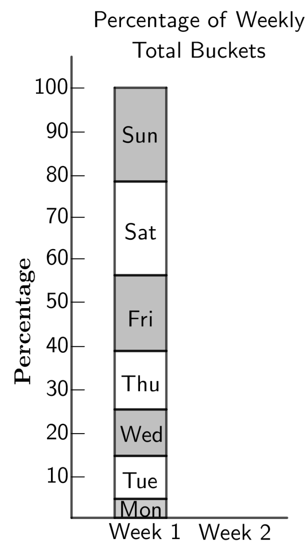
## Problem B

## Buckets of Golf Balls

Golfers will practice their golf game at a driving range. At a driving range, they hit practice balls by the bucket.

Annie works at a local driving range. Over a period of two weeks, she records the number of buckets of balls that she hands out each day. The table below displays her data.

| Day | Week 1 | Week 2 |
|---|---|---|
| Monday | 11 | 14 |
| Tuesday | 25 | 32 |
| Wednesday | 27 | 34 |
| Thursday | 34 | 37 |
| Friday | 44 | 50 |
| Saturday | 57 | 70 |
| Sunday | 52 | 63 |




(a) A *stacked bar graph* is given for Week 1, showing the percentage of each day's buckets relative to the total (250 buckets) for that week. For example, on Monday Annie gives out 11 buckets, which is $\frac{11}{250} = 4.4\%$ of the total; on Tuesday she gives out 25 buckets, which is $\frac{25}{250} = 10.0\%$ of the total.
Verify that the remaining blocks of the graph accurately portray the given data for Week 1 by calculating the remaining daily percentages.

(b) Calculate the daily percentages for Week 2, and create a similar stacked bar graph for Week 2. Round percentages to one decimal place.

(c) By examining the bar graphs, what conclusions could you draw about the number of buckets given out each day?

THEMES DATA MANAGEMENT, NUMBER SENSE

# Problem of the Week

## Problem B and Solution

## Buckets of Golf Balls

### Problem

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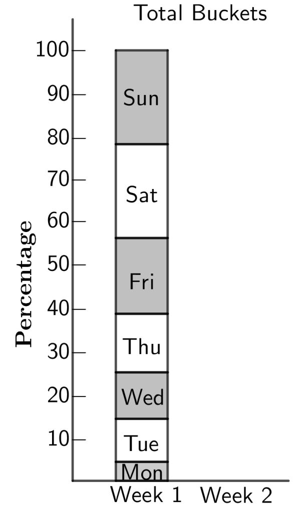
Percentage of Weekly Total Buckets

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(b) Calculate the daily percentages for Week 2, and create a similar stacked bar graph for Week 2. Round percentages to one decimal place.

(c) By examining the bar graphs, what conclusions could you draw about the number of buckets given out each day?

## Solution

(a) The remaining days' percentages are:
Wednesday: $\frac{27}{250} = 10.8\%$
Thursday: $\frac{34}{250} = 13.6\%$
Friday: $\frac{44}{250} = 17.6\%$
Saturday: $\frac{57}{250} = 22.8\%$
Sunday: $\frac{52}{250} = 20.8\%$

Note: We can find each percentage by rewriting the fraction as an equivalent fraction with a denominator of 100. We will look at the data for Wednesday and show two ways to do this.

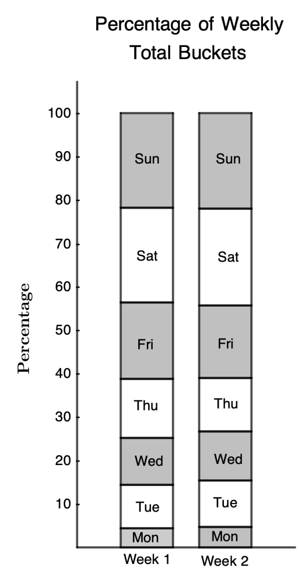
(i) We will get the denominator to be 1000 by multiplying numerator and denominator by 4. Then, we divide each by 10 to get a fraction with a denominator of 100.

$$\frac{27}{250} = \frac{108}{1000} = \frac{10.8}{100} = 10.8\%$$

(ii) Since $250 \div 100 = 2.5$, we can divide both numerator and denominator by 2.5 to get $\frac{10.8}{100} = 10.8\%$.

The heights of the remaining blocks of the graph do portray the given data for Week 1.

(b) During Week 2, Annie handed out a total of 300 buckets. The daily percentages and completed bar graph are below.
Monday: $\frac{14}{300} \approx 4.7\%$
Tuesday: $\frac{32}{300} \approx 10.7\%$
Wednesday: $\frac{34}{300} \approx 11.3\%$
Thursday: $\frac{37}{300} \approx 12.3\%$
Friday: $\frac{50}{300} \approx 16.7\%$
Saturday: $\frac{70}{300} \approx 23.3\%$
Sunday: $\frac{63}{300} = 21.0\%$




(c) The tallest rectangular boxes are for Saturday and Sunday. Therefore, we can say that the most buckets are given out on either Saturday or Sunday. The data in the table shows that it is in fact on Saturday when the most buckets are given out.

The shortest rectangular box is for Monday. Therefore, we can say that the fewest number of buckets are given out on Monday. This is verified by the table.

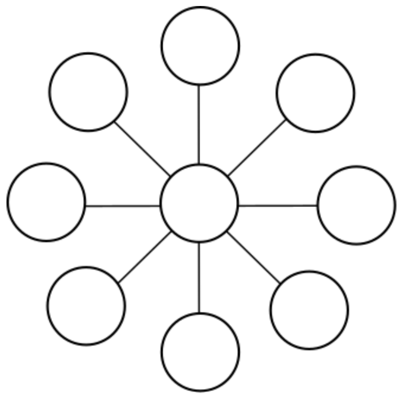
# Problem of the Week

## Problem B

## The Puzzler Returns

Our superhero The Puzzler is back, seeking your help once more to solve the following number puzzle.

Place each of the numbers 2, 3, 4, 5, 6, 7, 8, 9, and 10 in a different circle in the diagram so that each line of three circled numbers has the same sum.

Can you find more than one possibility for the number that can go in the middle circle?

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# Problem of the Week
## Problem B and Solution
## The Puzzler Returns

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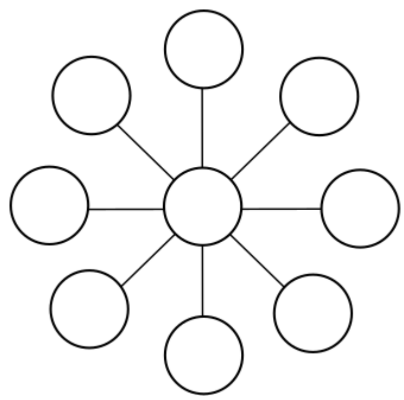


Can you find more than one possibility for the number that can go in the middle circle?

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### Solution

The key discovery for this puzzle is that the middle circle is on every line of three circled numbers. If we were to remove the number in the middle circle from the diagram, then we would be left with four pairs of numbers that each have the same sum. So after choosing the middle number, it must be possible to pair up the remaining eight numbers so that each pair has the same sum.

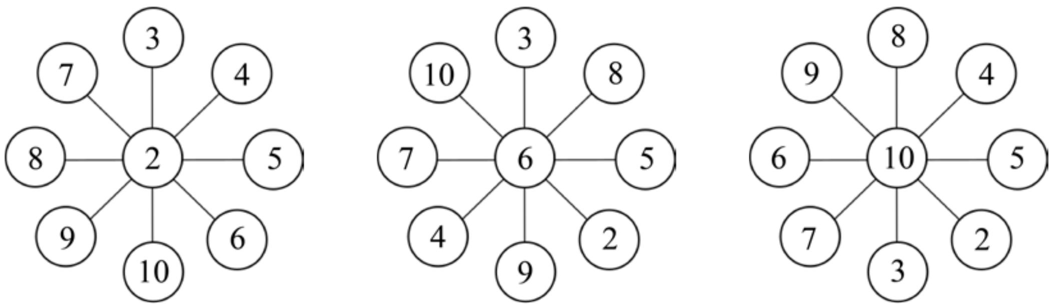
If the middle number is 2, the other numbers can be paired as follows: $3 + 10$, $4 + 9$, $5 + 8$, and $6 + 7$. Each of these sums is 13, so the sum of any line of three circled numbers would be $13 + 2 = 15$.

Similarly, if the middle number is 10, the other numbers can be paired as follows: $2 + 9$, $3 + 8$, $4 + 7$, and $5 + 6$. Each of these sums is 11, so the sum of any line of three circled numbers would be $11 + 10 = 21$.

We can also choose 6 as the middle number. Then the other numbers can be paired as follows: $2 + 10$, $3 + 9$, $4 + 8$, and $5 + 7$. Each of these sums is 12, so the sum of any line of three circled numbers would be $12 + 6 = 18$.

It is not possible to choose any other number as the middle number. In each case, if you try pairing up the remaining numbers, you will find that you cannot do so in a way such that each pair has the same sum.

With middle numbers of 2, 6, or 10, there are many possible ways to place the remaining numbers in the diagram. The only condition is that the pairs of numbers with the same sum must be placed on the same line. Some examples are shown.



# Problem of the Week

## Problem B

## Dollars for College

Cassie starts college in the fall and has decided to live in an apartment. Her monthly living expenses are as follows:

Rent with utilities: \$800
Hydro: \$40
Phone/internet/TV: \$129
Groceries: \$300

(a) If she attends school for 18 months, what will be her total living expenses?

(b) She has learned that her total college fees for the program will be \$6769. If she has \$10 400 saved, how much more money will she need to pay her living expenses plus college fees?

(c) Instead of taking out a loan to pay for her additional costs found in part (b), Cassie has decided to work part-time at the local bakery. If she earns \$16/hr, for how many hours will she need to work to pay for her additional costs? Round your answer to the nearest hour.

(d) If Cassie works every week for the 18 months she attends college, for how many hours per week will she have to work to pay for her additional costs? When answering this question, assume that there are four weeks in each month. Round your answer to the nearest hour.

# Problem of the Week

## Problem B and Solution

## Dollars for College

### Problem

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(d) If Cassie works every week for the 18 months she attends college, for how many hours per week will she have to work to pay for her additional costs? When answering this question, assume that there are four weeks in each month. Round your answer to the nearest hour.

## Solution

(a) Cassie's monthly living expenses total $\$800 + \$40 + \$129 + \$300 = \$1269$. Thus, if she attends school for 18 months, her total living expenses will be $\$1269 \times 18 = \$22\,842$.

(b) Her total cost for college fees and living expenses will be $\$22\,842 + \$6769 = \$29\,611$. Thus, in addition to the \$10 400 she has saved, she will need $\$29\,611 - \$10\,400 = \$19\,211$.

(c) Working part-time at the local bakery at \$16/hr, she will need to put in $19\,211 \div 16 \approx 1201$ hours to pay for her additional costs.

(d) Since she will work a total of $18 \times 4 = 72$ weeks, and needs to put in 1201 hours, she will need to work $1201 \div 72 \approx 17$ hours per week.

# Problem of the Week

## Problem B

## What's Beneath the Surface?

In each problem below, use the information given about part of the object's mass to determine the unknown mass.

(a) Contrary to what you may have heard, ostriches do not bury their heads in the sand. But, if one decided to do so just for fun, and its 2000 g head was 2% of its total body mass, then what would be the mass of its entire body, in kilograms?

(b) Generally, about 90% of an iceberg's mass is below water level. If the mass of the visible portion of a certain iceberg is 50 000 tonnes, then what is the mass of the whole iceberg, in tonnes?

(c) Only a small portion of a growing mushroom is visible; most of the fungus is below the ground. If 5% of a mushroom is above the ground, and this portion has a mass of 100 g, then what is the mass of the mushroom below the ground, in kilograms?

# Problem of the Week

## Problem B and Solution

## What's Beneath the Surface?

### Problem

In each problem below, use the information given about part of the object's mass to determine the unknown mass.

(a) Contrary to what you may have heard, ostriches do not bury their heads in the sand. But, if one decided to do so just for fun, and its 2000 g head was 2% of its total body mass, then what would be the mass of its entire body, in kilograms?

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(c) Only a small portion of a growing mushroom is visible; most of the fungus is below the ground. If 5% of a mushroom is above the ground, and this portion has a mass of 100 g, then what is the mass of the mushroom below the ground, in kilograms?

## Solution

(a) We're given that 2% of the ostrich's mass is 2000 g. Since $2\% \times 50 = 100\%$, the total mass of the ostrich must be $2000 \times 50 = 100\,000$ g, or 100 kg.

(b) Given that 90% of an iceberg is hidden, the visible mass must be $100\% - 90\% = 10\%$ of its total mass. Thus, if the visible portion is 50 000 tonnes, and since $10\% \times 10 = 100\%$, the total mass must be $50\,000 \times 10 = 500\,000$ tonnes.

(c) If 5% of the mushroom is above the ground, then $100\% - 5\% = 95\%$ of the mushroom is below the ground. Since $5\% \times 19 = 95\%$, the portion of the mushroom below the ground must have a mass of $100 \times 19 = 1900$ g, or 1.9 kg.

Alternatively, the visible portion of the mushroom has a mass of 100 g, which is 5% of its total mass. Since $5\% \times 20 = 100\%$, the total mass of the mushroom must be $100 \times 20 = 2000$ g. Then the portion of the mushroom below the ground must have a mass of $2000 - 100 = 1900$ g, or 1.9 kg.

# Problem of the Week

## Problem B

## Saphyr Goes to a Concert

Saphyr and her friends are going to their local concert hall to see their favourite singer, Tinker Leisurely.

The rectangular hall is to be set up so that people sit in rows of chairs facing the stage.

Each row will have the same number of seats, with each row having at most 50 seats. The hall has a capacity for at most 115 rows. Saphyr has discovered that there are to be 4032 seats for the audience.

(a) Using what you know about the area of a rectangle and about the whole numbers that divide evenly into 4032, determine the possible configurations of seats in the hall.

(b) If the stage is 36 m wide and the chairs are to be spaced 1 m apart (from centre to centre), which of your answers in part (a) do you think gives the most reasonable dimensions for the seating? Why?

# Problem of the Week

## Problem B and Solution

## Saphyr Goes to a Concert

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## Solution

(a) We are looking for pairs of factors which have product 4032, thinking of the product as the number of seats in each row multiplied by the number of rows. We are given that each row must have at most 50 seats and we also know that there must be no more than 115 rows. Thus, one factor must be less than or equal to 50 and the other factor must be less than or equal to 115. The only possible such pairs with product 4032 are $36 \times 112$, $42 \times 96$, and $48 \times 84$. Therefore, the hall could have 112 rows with 36 chairs in each row, or have 96 rows with 42 chairs in each row, or 84 rows with 48 chairs in each row.

Some students may list all possible products before checking constraints. They are:
$1 \times 4032$, $2 \times 2016$, $3 \times 1344$, $4 \times 1008$, $6 \times 672$, $7 \times 576$, $8 \times 504$, $9 \times 448$, $12 \times 336$, $14 \times 288$, $16 \times 252$, $18 \times 224$, $21 \times 192$, $24 \times 168$, $28 \times 144$, $32 \times 126$, $36 \times 112$, $42 \times 96$, $48 \times 84$, $56 \times 72$, and $63 \times 64$.

(b) Answers will vary.

Of the three possible arrangements, 112 rows of 36 seats would optimize the audience view of the stage, since the stage is 36 m wide. On the other hand, 84 rows of 48 seats would minimize the audience's distance from the stage, but give a less optimal viewing angle for those on the outer edges of the seating. Thus, a good compromise with reasonable stage view and distance may be 96 rows of 42 seats.

**Extension:** If the rows did not have to have the same number of seats, how would your answer to part (b) change?

# Problem of the Week

## Problem B

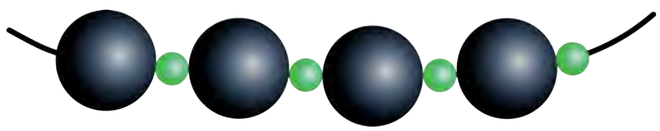
## A String of Beads

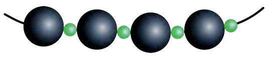
Aurora is making a beaded necklace using black and green beads. The black beads are all 1.2 cm wide and the green beads are all 4 mm wide. Aurora will make her necklace by alternating the black and green beads.

(a) If Aurora wants her necklace to be 80 cm long, how many beads will she need in total?

(b) If the black beads cost \$0.10 each and the green beads cost \$0.03 each, how much will it cost for Aurora to buy all the beads she needs for her necklace?

(c) Would it cost more or less for Aurora to buy the beads if instead of alternating the black and green beads, she put two green beads after each black bead? Explain.





# Problem of the Week

## Problem B and Solution

## A String of Beads

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## Solution

(a) First we need to write the widths of the beads with the same unit of measurement. If we choose centimetres, then the green beads are $4 \div 10 = 0.4$ cm wide. Since Aurora is alternating black and green beads, the necklace will be made up of pairs of black and green beads. Each pair of black and green beads is $1.2 + 0.4 = 1.6$ cm wide. We need to determine how many pairs of black and green beads will fit on the necklace. Since $80 \div 1.6 = 50$, there will be 50 pairs of black and green beads on the necklace. So Aurora will need 50 black beads and 50 green beads, which is a total of 100 beads.

(b) Aurora needs 50 black beads. Since the black beads cost \$0.10 each, it will cost $50 \times \$0.10 = \$5$ to buy them all. Aurora needs 50 green beads. Since the green beads cost \$0.03 each, it will cost $50 \times \$0.03 = \$1.50$ to buy them all. Therefore, in total, it will cost $\$5 + \$1.50 = \$6.50$ to buy all the beads for the necklace.

(c) If Aurora puts two green beads after each black bead, then the necklace will be made up of groups of one black bead and two green beads. Each of these groups is $1.2 + 0.4 + 0.4 = 2$ cm wide. Since the necklace is 80 cm long, and $80 \div 2 = 40$, it follows that 40 of these groups will fit on the necklace. So the necklace will have 40 black beads and $40 \times 2 = 80$ green beads. Since the black beads cost \$0.10 each, it will cost $40 \times \$0.10 = \$4$ to buy them all. Since the green beads cost \$0.03 each, it will cost $80 \times \$0.03 = \$2.40$ to buy them all. Therefore, in total, it will cost $\$4 + \$2.40 = \$6.40$ to buy all the beads for the necklace. Since $\$6.40 < \$6.50$, it will be cheaper to buy the beads if Aurora puts two green beads after each black bead.

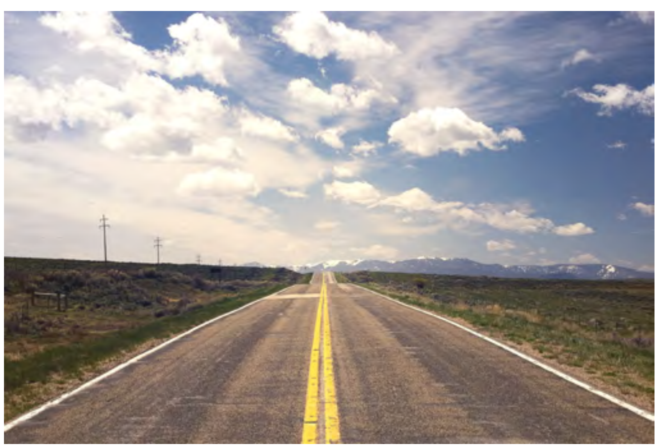
Alternatively, we could have justified this without doing all the calculations. Notice that the width of three green beads is $3 \times 0.4 = 1.2$ cm, which is the width of one black bead. However, the cost of three green beads is $3 \times \$0.03 = \$0.09$, but the cost of one black bead is \$0.10. So three green beads take up the same space as one black bead, but are \$0.01 cheaper to buy. If Aurora puts two green beads after each black bead instead of alternating the black and green beads, then she will end up using more green beads and fewer black beads in her necklace. Every time she replaces one black bead with three green beads she will save \$0.01, so it will be cheaper to buy the beads if Aurora puts two green beads after each black bead.

# Problem of the Week
## Problem B
## On the Road Again!

This problem looks at the epic journeys of two young men whose fortitude knew no bounds.

(a) In 1980, Terry Fox set out to run across Canada in order to raise money for cancer research, in what is called the Marathon of Hope. He planned to run the entire length of the Trans-Canada Highway, which is 7821 km. Terry Fox ran an average of 42 km every day, but had to stop after 143 days and 5373 km. If he had been able to complete his journey and had continued at the same pace, how many days would it have taken him to complete the remainder of his run across Canada?

(b) In 1985, Rick Hansen, the Man in Motion, wheeled around the world in his wheelchair in order to help people understand the importance of a world without barriers for people with disabilities. Starting on March 21, 1985, and finishing on May 22, 1987, he went through 34 countries and travelled a total of 40 075 km. On average, how many kilometres did he travel on each day of his world tour?



Themes Geometry & Measurement, Number Sense

# Problem of the Week
## Problem B and Solution
## On the Road Again!

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## Solution

(a) The Trans-Canada Highway is 7821 km long and Terry Fox completed 5373 km. Thus, $7821 - 5373 = 2448$ km remain. If Terry Fox travelled at 42 km per day, then since $2448 \div 42 \approx 58.286$, it would have taken him 59 days to complete his run across Canada.

(b) To calculate the number of days between March 21, 1985 and May 22, 1987, we will first calculate the number of days between March 21, 1985 and March 20, 1987, inclusive, and then calculate the number of days between March 21, 1987 and May 22, 1987, inclusive.

- Since neither 1986 nor 1987 were leap years, the number of days between March 21, 1985 and March 20, 1987 is $2 \times 365 = 730$.
- To calculate the number of days between March 21, 1987 and May 22, 1987, we will look at the number of days in each month. There are 11 days from March 21 to March 31. April has 30 days, and there are 22 days from May 1 to May 22. This is a total of $11 + 30 + 22 = 63$ days.

Thus, in total, Rick Hansen travelled for $730 + 63 = 793$ days. Since he travelled 40 075 km in total, this means he travelled on average $40\,075 \div 793 \approx 50.5$ km per day.

# Problem of the Week
## Problem B
## Fields of Flowers

Sadie has garden beds that are 11 m by 14 m. She wants to grow giant sunflowers in one of her garden beds and dwarf sunflowers in another garden bed.

(a) Sadie spaces the giant sunflower seeds 50 cm apart, in rows that are 50 cm apart, leaving a 100 cm border on all sides of the garden bed. How many giant sunflower seeds can she plant in one garden bed?

(b) In another garden bed, Sadie plants dwarf sunflower seeds. She spaces the seeds 25 cm apart, in rows that are 25 cm apart, leaving a 100 cm border on all sides of the garden bed. How many dwarf sunflowers can she plant in this garden bed?

(c) All of Sadie's sunflowers have germinated and matured, but then in a cold early frost one evening, she loses 20% of the giant sunflowers and 10% of the dwarf sunflowers. If Sadie sells all the surviving sunflowers at \$5.00 each for the giants and \$3.00 each for the dwarfs, which crop will provide the greater income?

**Themes** Geometry & Measurement, Number Sense

# Problem of the Week
## Problem B and Solution
## Fields of Flowers

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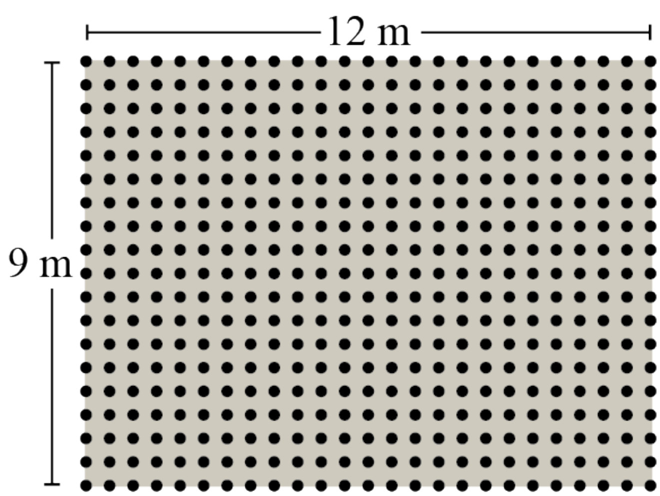
### Solution

(a) Since 100 cm is equal to 1 m, the border around Sadie's garden bed is 1 m on each side. That means the planting area inside the garden is 9 m by 12 m. If she plants rows of seeds starting right on the edge of the planting area, and plants them 50 cm (or $\frac{1}{2}$ m) apart, then in each row she can plant 2 seeds per metre, plus 1 more seed at the end of the row. So, along the 9 m width she can plant $9 \times 2 + 1 = 18 + 1 = 19$ seeds. Along the 12 m length she can plant $12 \times 2 + 1 = 24 + 1 = 25$ seeds. Thus, Sadie can plant a total of $19 \times 25 = 475$ giant sunflower seeds in one garden bed, as shown.



(b) As in part (a), we can conclude that the planting area inside this garden bed is also 9 m by 12 m. If Sadie plants rows of seeds starting right on the edge of the planting area, and plants them 25 cm (or $\frac{1}{4}$ m) apart, then in each row she can plant 4 seeds per metre, plus 1 more seed at the end of the row. So, along the 9 m width she can plant $9 \times 4 + 1 = 36 + 1 = 37$ seeds. Along the 12 m length she can plant $12 \times 4 + 1 = 48 + 1 = 49$ seeds. Thus, Sadie can plant a total of $37 \times 49 = 1813$ dwarf sunflower seeds in this garden bed.

(c) After the loss of 20% of the giant sunflowers, Sadie will have 80% of 475, or $0.8 \times 475 = 380$ flowers left. These giant sunflowers will provide an income of $380 \times \$5.00 = \$1900$.

After the loss of 10% of the dwarf sunflowers, Sadie will have 90% of 1813, or $0.9 \times 1813 \approx 1632$ flowers left. These dwarf sunflowers will provide an income of $1632 \times \$3.00 = \$4896$. Thus, the income from the dwarf sunflowers is more than double that of the giant sunflowers.

# Geometry & Measurement (G)

TAKE ME TO THE COVER

# Problem of the Week

## Problem B

## Into the Wild Blue Yonder!

Canadian Actor William Shatner travelled on the Blue Origin rocket in October 2021. He was in the rocket for 10 minutes and 17 seconds after liftoff, before landing back on the desert floor in Texas. The rocket rose to an altitude of 105.9 km.

(a) If his flight was straight up and down, what was his mean speed, to the nearest kilometre per hour, over the course of the whole journey?

(b) The length of the Trans-Canada Highway between the east and west coasts of Canada is 7821 km. If the rocket travels a distance of 7821 km at the mean speed found in part (a), approximately how long (in hours and minutes) would that trip take?

# Problem of the Week

## Problem B and Solution

## Into the Wild Blue Yonder!

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## Solution

(a) The total distance William Shatner travelled was $105.9 \times 2 = 211.8$ km.

His travel time was 10 minutes and 17 seconds. Since there are 60 seconds in one minute, his travel time was $10 \times 60 + 17 = 617$ seconds. Since there are $60 \times 60 = 3600$ seconds in each hour, his travel time in hours was $617 \div 3600 \approx 0.1714$ hr.

Thus, his mean speed was 211.8 km $\div$ 0.1714 hr $\approx$ 1236 km/hr.

(b) Travelling a distance of 7821 km at a mean speed of 1236 km/hr would take the rocket $7821 \div 1236 \approx 6.328$ hr. Since there are 60 minutes in each hour, this is equal to $6.328 \times 60 \approx 380$ minutes, or approximately 6 hours and 20 minutes.

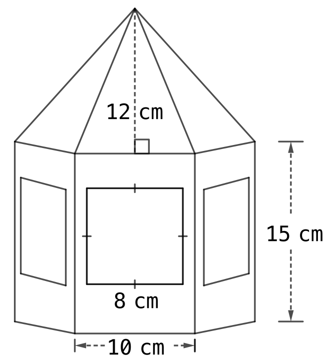
NOTE: Calculations here were carried out with four significant digits. Answers may vary if fewer are used at each stage.

# Problem of the Week

## Problem B

## Painting a Birdhouse

Bird feeders come in many shapes and sizes. Meera has one with a pentagonal base, five identical rectangular sides, and five identical triangles that meet at a point forming the roof. Each rectangular side has a width of 10 cm, a height of 15 cm, and a square window of side length 8 cm. Each triangle has a height of 12 cm and its base lines up with the top width of one of the rectangular sides.




(a) What is the total area of the five windows in the feeder?

(b) Meera has decided to paint the outer faces of the triangular roof segments and the outer sides of the feeder (except the windows), but not the base. What is the total surface area of the parts of the feeder Meera intends to paint?

(c) Suppose you can purchase a 100 mL can of paint for \$3.50 which will cover 10 000 $\text{cm}^2$ of surface area. If Meera does two coats of paint on each pentagonal bird feeder, how many complete pentagonal bird feeders can be painted by one of these cans of paint?

Themes Geometry & Measurement, Number Sense

# Problem of the Week

## Problem B and Solution

## Painting a Birdhouse

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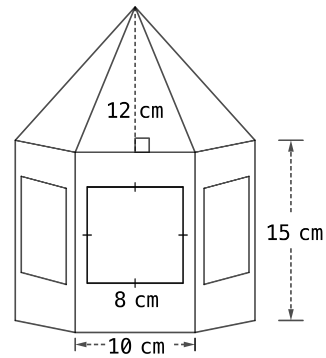


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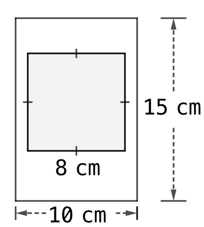
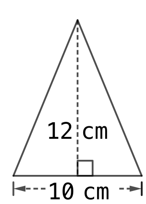
## Solution

(a) Since each of the five windows is an 8 cm square of area $8 \times 8 = 64\,\text{cm}^2$, the total area of the windows is $5 \times 64 = 320\,\text{cm}^2$.

(b) The parts of the feeder to be painted are the five rectangular borders around the windows plus the five triangular roof segments.

The area of one rectangular border is the area of the outer rectangle minus the area of the square window. Since the area of the outer rectangle is $10 \times 15 = 150\,\text{cm}^2$, and the area of the square window is $64\,\text{cm}^2$, the area of one rectangular border is $150 - 64 = 86\,\text{cm}^2$.
There are five of these borders and so their total area is $5 \times 86 = 430\,\text{cm}^2$.
The area of one triangular roof segment is $\frac{1}{2} \times 10 \times 12 = 60\,\text{cm}^2$. There are five of these triangles and so their total area is $5 \times 60 = 300\,\text{cm}^2$.
Thus, the total area to be painted is $430+300 = 730\,\text{cm}^2$.




(c) Two coats of paint on one feeder will require paint for $2 \times 730 = 1460\,\text{cm}^2$. Thus, Meera can paint $10\,000 \div 1460 \approx 6.8$ birdhouses. Therefore, Meera can paint 6 complete birdhouses using one can of paint.

# Problem of the Week

## Problem B

## A Stoney Problem

Sela is doing some landscaping, and needs to pave a rectangular space with an area of 53.5 $m^2$. She plans to use paving stones which are 10 cm by 20 cm, and so each has an area of 200 $cm^2$ each. Note that only whole paving stones will be used.

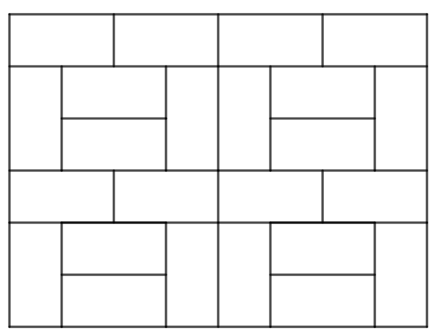
At the Home Shop, Sela learns that these pavers are sold on pallets of 1000 stones, and she must buy complete pallets at \$499 each.

(a) How many stones will she need to cover the 53.5 $m^2$ area?

(b) How many pallets will she need to buy?

(c) How many stones will be left on the last pallet Sela uses?

(d) If Sela is able to buy partial pallets, how much would she save if she only bought the paving stones she needed?



Themes Geometry & Measurement, Number Sense

# Problem of the Week
## Problem B and Solution
## A Stoney Problem

### Problem

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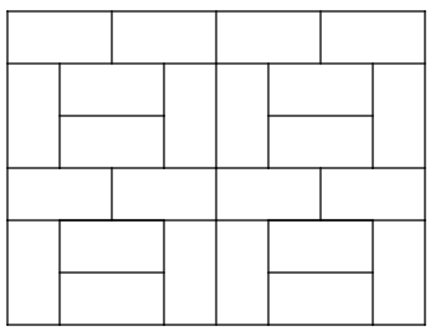

## Solution

(a) One square metre is equivalent to $100 \times 100 = 10\,000\,\text{cm}^2$, the area Sela needs to pave has area $53.5 \times 10\,000 = 535\,000\,\text{cm}^2$. Since each paving stone has area $200\,\text{cm}^2$, Sela will need $535\,000 \div 200 = 2675$ stones.

(b) Since each pallet has 1000 paving stones, Sela needs $2675 \div 1000 = 2.675$ pallets. However, she must buy complete pallets, so Sela will need to buy 3 pallets, or 3000 paving stones.

(c) On the last pallet Sela uses, there will be $3000 - 2675 = 325$ paving stones.

(d) Sela would not need to buy the extra 325 paving stones. The 325 paving stones as a fraction of a pallet is $\frac{325}{1000} = 0.325$.
Thus, she would save $0.325 \times \$499 \approx \$162.18$.

# Problem of the Week

## Problem B

## A Little Rain Must Fall

Excessive rainfall may occur during weather events such as hurricanes, or in some places, simply as a part of everyday life.

(a) During Hurricane Harvey in 2017, almost 75 mm of rain fell in one hour in Houston. If rain continued to fall at that rate for 24 hours, how much rain would fall? Express your answer in metres.

(b) Mawsynram, India is recognized as one of the wettest places on Earth, with an average annual rainfall of 11 872 mm, most of which falls during the monsoon season. If the rain was spread out evenly over the whole year, how much rain, in mm, would fall each day? Round your answer to one decimal place.

(c) Find the average annual rainfall in your community. How many times more is Mawsynram's average annual rainfall than the average annual rainfall in your community? Round your answer to one decimal place.

# Problem of the Week
## Problem B and Solution
## A Little Rain Must Fall

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(c) Find the average annual rainfall in your community. How many times more is Mawsynram's average annual rainfall than the average annual rainfall in your community? Round your answer to one decimal place.

## Solution

(a) If 75 mm of rain fell in each hour for 24 hours, the total amount of rain would be $75 \times 24 = 1800\,\text{mm}$, or 1.8 metres.

(b) If a total of 11 872 mm of rain was spread out evenly over the whole year, then over 365 days, the daily average would be $11\,872 \div 365 \approx 32.5$ mm.

(c) Answers will vary. The average annual rainfall for the city of Toronto is 831 mm*. Therefore, the average annual rainfall in Mawsynram is $11\,872 \div 831 \approx 14.3$ times more than the average annual rainfall of Toronto.
*Source: https://www.currentresults.com/Weather/Canada/Cities/precipitation-annual-average.php

# Problem of the Week
## Problem B
## Draw Me!

Using a protractor and a ruler, follow the instructions below to draw three different shapes.

(a) Steps for drawing Shape 1:

- Draw a horizontal line segment $AB$ of length 4 cm.
- From $B$, draw line segment $BC$ of length 8 cm at an angle of $32°$ to $AB$.
- Draw a line segment connecting $C$ and $A$.

What is the measure of angle $ACB$ in your shape?

What kind of triangle did you draw?

(b) Steps for drawing Shape 2:

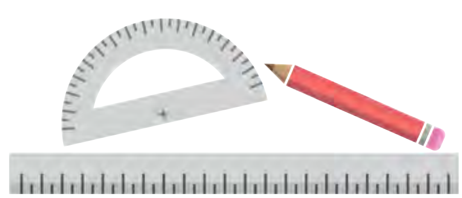
- Draw a horizontal line segment $AB$ of length 6 cm.
- From $B$, draw line segment $BC$ of length 7 cm at an angle of $135°$ to $AB$.
- From $C$, draw line segment $CD$ of length 6 cm at an angle of $45°$ to $BC$.
- Draw a line segment connecting $D$ and $A$.

What shape did you draw?

(c) Steps for drawing Shape 3:

- Draw a vertical line segment $AB$ of length 5 cm.
- From $B$, draw line segment $BC$ of length 5 cm at an angle of $90°$ to $AB$.
- Draw line segment $AD$ of length 7 cm, parallel to $BC$.
- Draw a line segment connecting $D$ and $C$.

What shape did you draw?



Theme Geometry & Measurement

# Problem of the Week

## Problem B and Solution

## Draw Me!

### Problem

Using a protractor and a ruler, follow the instructions below to draw three different shapes.

(a) Steps for drawing Shape 1:

- Draw a horizontal line segment $AB$ of length 4 cm.
- From $B$, draw line segment $BC$ of length 8 cm at an angle of 32° to $AB$.
- Draw a line segment connecting $C$ and $A$.

What is the measure of angle $ACB$ in your shape?

What kind of triangle did you draw?

(b) Steps for drawing Shape 2:

- Draw a horizontal line segment $AB$ of length 6 cm.
- From $B$, draw line segment $BC$ of length 7 cm at an angle of 135° to $AB$.
- From $C$, draw line segment $CD$ of length 6 cm at an angle of 45° to $BC$, parallel to $AB$.
- Draw a line segment connecting $D$ and $A$.

What shape did you draw?

(c) Steps for drawing Shape 3:

- Draw a vertical line segment $AB$ of length 5 cm.
- From $B$, draw line segment $BC$ of length 5 cm at an angle of 90° to $AB$.
- Draw line segment $AD$ of length 7 cm, parallel to $BC$.
- Draw a line segment connecting $D$ and $C$.

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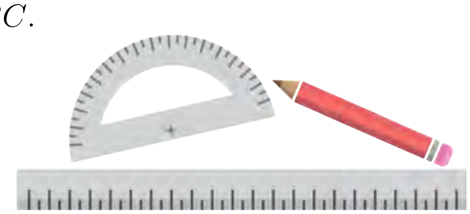
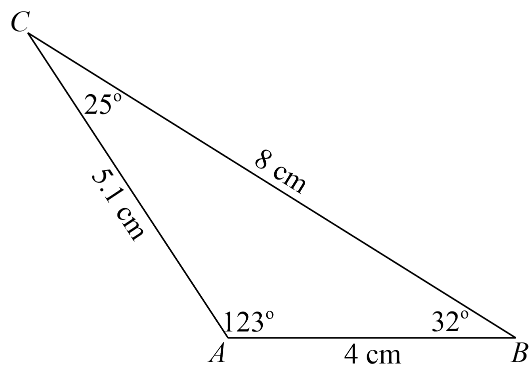
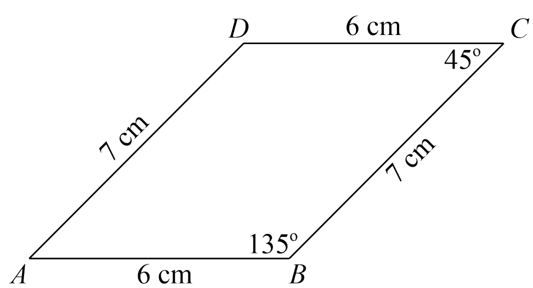


## Solution

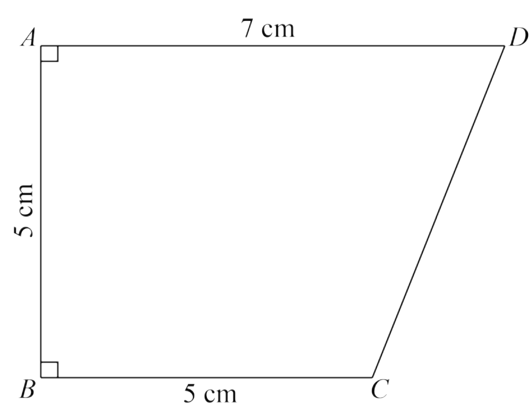
(a) Following the steps for Shape 1 creates a scalene triangle. The measure of angle $ACB$ is 25°.




(b) Following the steps for Shape 2 creates a parallelogram, which is a quadrilateral with two pairs of parallel sides.

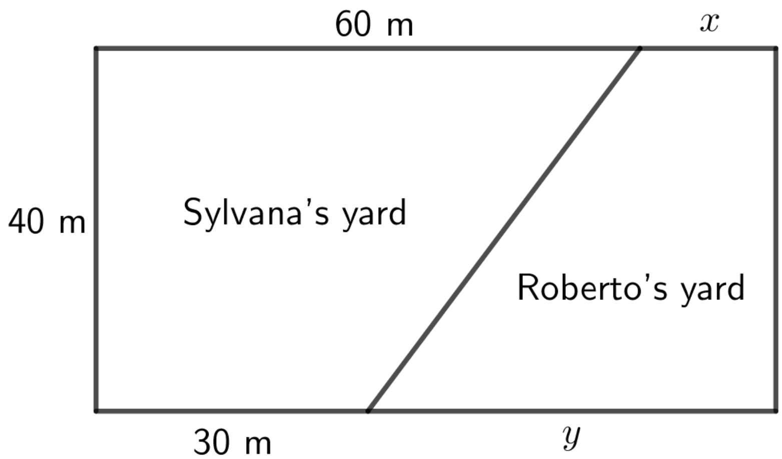



(c) Following the steps for Shape 3 creates a trapezoid, which is a quadrilateral with one pair of parallel sides. (Note : The shape could also be called a right trapezoid, which is a trapezoid with two right angles.)



# Problem of the Week
## Problem B
## Seeking Parts Unknown...

Sylvana and Roberto divide a 40 m by 75 m rectangular lot to form two yards, as shown in the diagram below.




The area of Roberto's yard is 40% of the total area of the two properties.

(a) What are the values of $x$ and $y$, the missing dimensions of Roberto's yard?

(b) What is the area of each yard?

# Problem of the Week

## Problem B and Solution

## Seeking Parts Unknown...

### Problem

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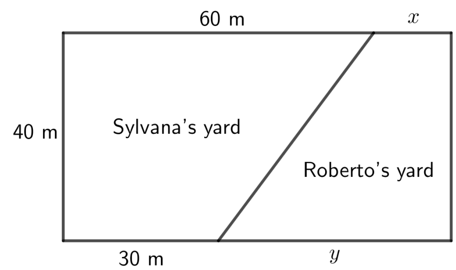



The area of Roberto's yard is 40% of the total area of the two properties.

(a) What are the values of $x$ and $y$, the missing dimensions of Roberto's yard?

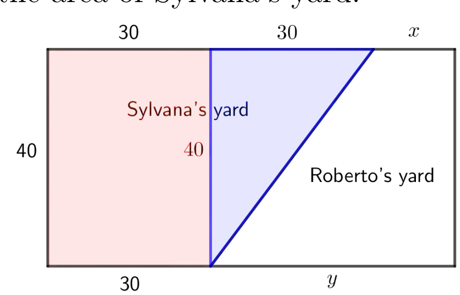
(b) What is the area of each yard?

## Solution

(a) From the two sides of the rectangle of length 75 m, we must have $60\,\text{m} + x = 75$ m and $30\,\text{m} + y = 75$ m. Thus, the missing dimensions of Roberto's yard are $x = 75 - 60 = 15$ m and $y = 75 - 30 = 45$ m.

(b) The total area of the two yards is $40\,\text{m} \times 75\,\text{m} = 3000\ \text{m}^2$. The area of each yard can be found in a variety of ways:

- The area of Roberto's yard is 40% of the total area. Thus, the area of Roberto's yard is 40% of 3000, or $0.4 \times 3000 = 1200\ \text{m}^2$, and the area of Sylvana's yard is $3000 - 1200 = 1800\ \text{m}^2$.
- Alternatively, since the area of Roberto's yard is 40% of the total area, the area of Sylvana's yard must be $100\% - 40\% = 60\%$ of the total area. Thus, the area of Sylvana's yard is $0.6 \times 3000 = 1800\ \text{m}^2$, and the area of Roberto's yard is $3000 - 1800 = 1200\ \text{m}^2$.
- We can find the area of one of the yards, and subtract that from the total area to find the area of the other yard. We will show how to find the area of Sylvana's yard.
  Notice that Sylvana's yard is shaped like a trapezoid. We can calculate the area of Sylvana's yard by dividing the trapezoid into a 40 m by 30 m rectangle (shown in red) and a triangle with a base of 30 m and a height of 40 m (shown in blue).

  The area of the rectangle is $40 \times 30 = 1200\ \text{m}^2$ and the area of the triangle is $\frac{40\times 30}{2} = 600\ \text{m}^2$.
  Thus, the total area of Sylvana's yard is $1200 + 600 = 1800\ \text{m}^2$, and so the total area of Roberto's yard is $3000 - 1800 = 1200\ \text{m}^2$.

# Problem of the Week

## Problem B

## What's in a Measure?

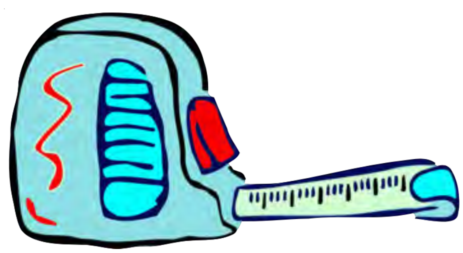
Listed below are measurements for some specific items. However, the unit of measure may not be what you are used to. Convert the measurements to a unit that makes more sense to you.

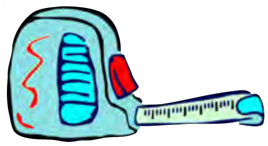
(a) The time between your 10th and 11th birthdays is 31 536 000 seconds.

(b) The distance between Montreal and Toronto is 54 160 000 centimetres.

(c) The length of my toothbrush is 0.00019 kilometres.

(d) I pour 0.25 litres of milk on my cereal in the morning.





# Problem of the Week
## Problem B and Solution
## What's in a Measure?

### Problem

Listed below are measurements for some specific items. However, the unit of measure may not be what you are used to. Convert the measurements to a unit that makes more sense to you.

(a) The time between your 10th and 11th birthdays is 31 536 000 seconds.

(b) The distance between Montreal and Toronto is 54 160 000 centimetres.

(c) The length of my toothbrush is 0.00019 kilometres.

(d) I pour 0.25 litres of milk on my cereal in the morning.

### Solution

(a) The time between your 10th and 11th birthdays is more commonly known as 1 year. (Hopefully, students did not need to do any calculations to determine this!) If you want to do the calculations, divide 31 536 000 by 60 to get 525 600 minutes. Then divide 525 600 by 60 to get 8760 hours. Finally, divide 8760 by 24 to get 365 days, which is equal to 1 year.

(b) A more reasonable unit for the distance between Montreal and Toronto is kilometres. So we will first convert the distance to metres and then to kilometres.

That is, 54 160 000 cm is equal to $54\,160\,000 \div 100 = 541\,600$ m, and $541\,600 \div 1000 = 541.6$ km.

Therefore, the distance between Montreal and Toronto is 541.6 km.

(c) A more reasonable unit for the length of my toothbrush would be centimetres. So we will first convert the length to metres and then to centimetres.

That is, 0.00019 km is equal to $0.00019 \times 1000 = 0.19$ m, and $0.19 \times 100 = 19$ cm.

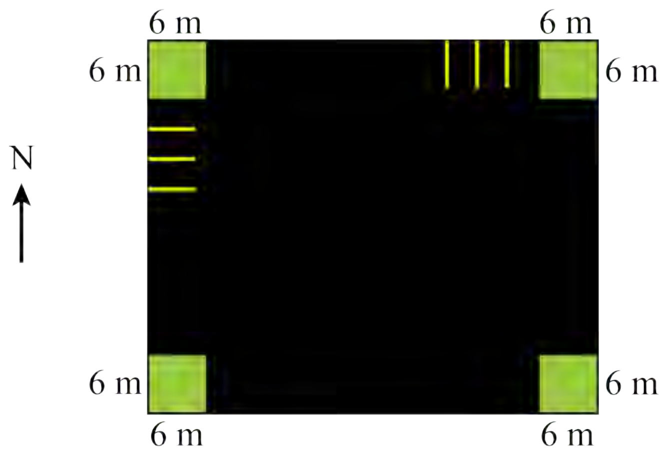
Therefore, the length of the toothbrush is 19 cm.

(d) A more reasonable unit for the amount of milk is millilitres. Therefore, the amount of milk is $0.25\,\text{L} \times 1000 = 250\,\text{mL}$.

NOTE: The units chosen may vary, since different students may find different units reasonable.

# Problem of the Week
## Problem B
## Parking by Design

KalMart has a paved, rectangular parking lot with a 6 m by 6 m curbed garden in each corner. There are parking spots along the north and west sides of the parking lot. Some of the parking spots on the north and west sides are shown in the diagram.




Each parking spot is 2.5 m wide, and the lines separating the parking spots are 7.5 cm thick.

(a) There are 25 parking spots along the north side of the parking lot. What is the length, in metres, of the north side of the parking lot, including the gardens?

(b) There are 20 parking spots along the west side of the parking lot. What is the length, in metres, of the west side of the parking lot, including the gardens?

(c) What is the total area, in square metres, of the paved portion of the parking lot, excluding the gardens?

# Problem of the Week
## Problem B and Solution
## Parking by Design

### Problem

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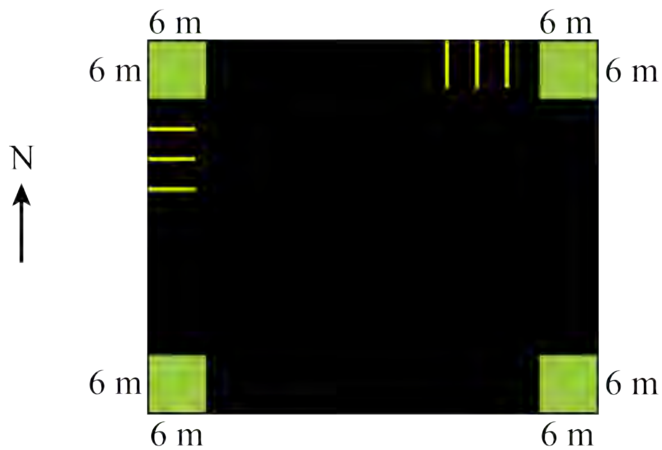


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(c) What is the total area, in square metres, of the paved portion of the parking lot, excluding the gardens?

## Solution

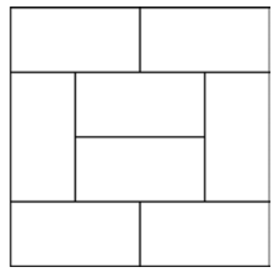
(a) There are 25 parking spots on the north side, plus 24 lines between them, since there are no lines at the corners next to the gardens. Since each parking spot is 2.5 m wide, the parking spots occupy a total of $25 \times 2.5 = 62.5$ m. Since each line is 7.5 cm = 0.075 m thick, the lines occupy a total of $24 \times 0.075 = 1.8$ m. The corner gardens occupy a total of $2 \times 6 = 12$ m. Thus, the total length of the north side is $62.5 + 1.8 + 12 = 76.3$ m.

(b) Similarly, there are 20 parking spots on the west side, plus 19 lines between them. Since each parking spot is 2.5 m wide, the parking spots occupy a total of $20 \times 2.5 = 50$ m. Since each line is 7.5 cm $= 0.075$ m thick, the lines occupy a total of $19 \times 0.075 = 1.425$ m. The corner gardens occupy a total of $2 \times 6 = 12$ m. Thus, the total length of the west side is $50 + 1.425 + 12 = 63.425$ m.

(c) The total area of the parking lot is $76.3 \times 63.425 = 4839.3275$ m$^2$. Each corner garden has an area of $6 \times 6 = 36$ m$^2$. The total garden area is then $4 \times 36 = 144$ m$^2$. Thus, excluding the four gardens, the area of the paved portion of the lot is $4839.3275 - 144 = 4695.3275$ m$^2$.

# Problem of the Week
## Problem B
## Mystery Dimensions

Eight congruent rectangles are arranged to form a larger rectangle as shown.



(a) If the congruent rectangles each have a length of 6 cm and a width of 3 cm, what is the perimeter of the larger rectangle?

(b) Suppose that the congruent rectangles each have a longer side of length $L$ cm and a shorter side of length 4 cm. Suppose also that the perimeter of the larger rectangle is 64 cm.

(i) What is the value of $L$?

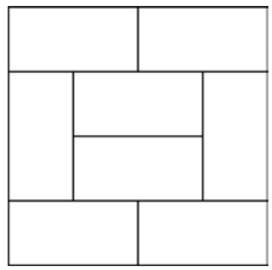
(ii) What is the area of one of the eight congruent rectangles?

**Extension:** Can you solve part (b) without knowing that the length of the shorter side of each rectangle is 4 cm? If so, how?

# Problem of the Week
## Problem B and Solution
## Mystery Dimensions

### Problem

Eight congruent rectangles are arranged to form a larger rectangle as shown.



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(b) Suppose that the congruent rectangles each have a longer side of length $L$ cm and a shorter side of length 4 cm. Suppose also that the perimeter of the larger rectangle is 64 cm.

(i) What is the value of $L$?

(ii) What is the area of one of the eight congruent rectangles?

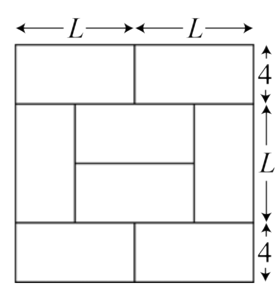
**Extension:** Can you solve part (b) without knowing that the length of the shorter side of each rectangle is 4 cm? If so, how?

## Solution

(a) Since each rectangle has a length of 6 cm and a width of 3 cm, the larger rectangle must have sides of lengths $6 + 6 = 12$ cm and $3 + 6 + 3 = 12$ cm. Thus, the perimeter of the larger rectangle is $12 + 12 + 12 + 12 = 48$ cm.

(b)

(i) Since each rectangle has a longer side of length $L$ cm and shorter side of length 4 cm, we can label our diagram to find the dimensions of the larger rectangle. Using this, we determine that the lengths of the sides of the larger rectangle are $L + L = 2L$ and $4 + L + 4 = L + 8$. Since we know the perimeter of the larger rectangle is 64 cm, we can write the following equation.



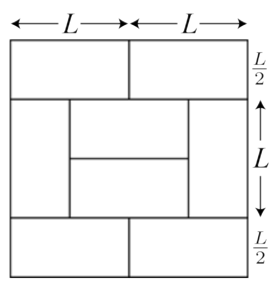
$$\begin{aligned} 2L + 2L + (L+8) + (L+8) &= 64 \\ 6L + 16 &= 64 \\ 6L &= 64 - 16 \\ 6L &= 48 \end{aligned}$$

Since $6 \times 8 = 48$, it follows that $L = 8$ cm.

(ii) The area of a rectangle is equal to its length times its width. Thus, the area of each congruent rectangle is $8 \times 4 = 32$ cm$^2$.

**Extension Solution:**

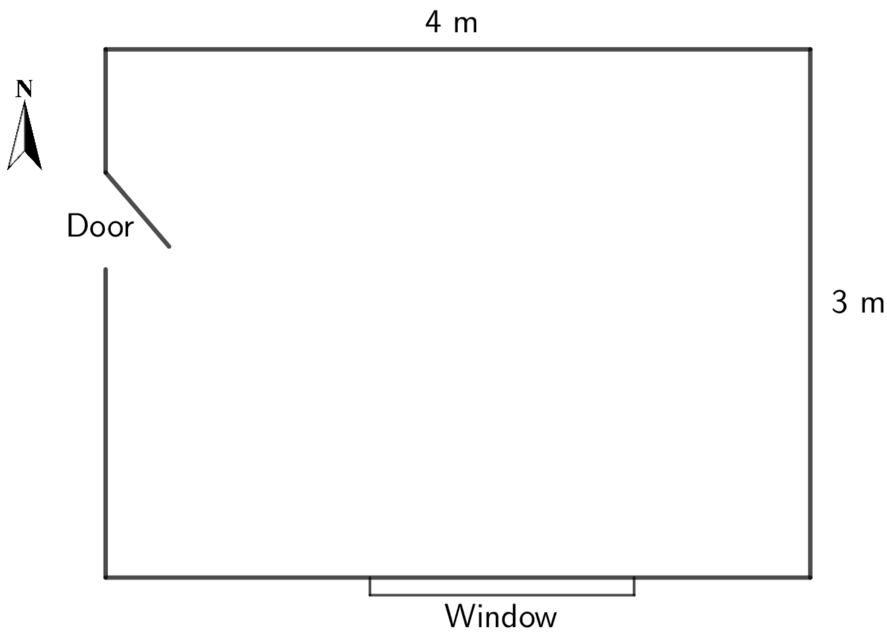
If we ignore the two rectangles on the top and the two rectangles on the bottom, we can see that two rectangles placed on top of each other horizontally have a height of $L$. Therefore, the shorter side of each rectangle equals half its longer side, or $\frac{L}{2}$. We can label our diagram to find the dimensions of the larger rectangle.




Using this, we determine that the larger rectangle has sides of length $L + L = 2L$ and $\frac{L}{2} + L + \frac{L}{2} = 2L$. So the larger rectangle is actually a square with side length $2L$. Since we know its perimeter is 64 cm, it follows that $2L + 2L + 2L + 2L = 64$, or $8L = 64$. Since $8 \times 8 = 64$, it follows that $L = 8$ cm. So, we can solve this problem without knowing the width of each rectangle.

# Problem of the Week
## Problem B
## Redecoration Station

Nimrat wants to redecorate her bedroom. The floor plan for her bedroom is shown below.




The walls in Nimrat's bedroom are 2.5 m high.

(a) Nimrat wants nice, plush, wall-to-wall carpet in her bedroom. How many square metres of carpet will she need to buy? If the carpet she buys costs \$20 per square metre, how much will her carpet cost in total?

(b) Wallpaper costs \$8 per square metre. How much wallpaper will she need to cover the north and east walls? How much will it cost for the wallpaper for those two walls?

(c) She decides to paint the south and west walls, and the cost for paint to do so is \$75. If her total budget is \$500 for carpet, wallpaper, and paint, how much over or under her budget is she?

**Themes** Geometry & Measurement, Number Sense

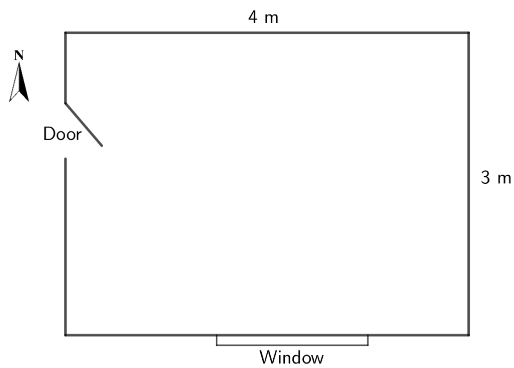
# Problem of the Week

## Problem B and Solution

## Redecoration Station

### Problem

Nimrat wants to redecorate her bedroom. The floor plan for her bedroom is shown below.

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(a) Nimrat wants nice, plush, wall-to-wall carpet in her bedroom. How many square metres of carpet will she need to buy? If the carpet she buys costs \$20 per square metre, how much will her carpet cost in total?

(b) Wallpaper costs \$8 per square metre. How much wallpaper will she need to cover the north and east walls? How much will it cost for the wallpaper for those two walls?

(c) She decides to paint the south and west walls, and the cost for paint to do so is \$75. If her total budget is \$500 for carpet, wallpaper, and paint, how much over or under her budget is she?

## Solution

(a) Since the total floor area of Nimrat's bedroom is $4 \times 3 = 12\,\text{m}^2$, and the carpet costs \$20 per square metre, the cost of her wall-to-wall carpet will be $\$20 \times 12 = \$240$.

(b) Since the north wall is 4 m long and 2.5 m high, its area is $4 \times 2.5 = 10\,\text{m}^2$. Since the east wall is 3 m long and 2.5 m high, its area is $3 \times 2.5 = 7.5\,\text{m}^2$. Thus, the total area to be wallpapered is $10 + 7.5 = 17.5\,\text{m}^2$. Therefore, the cost of the wallpaper at \$8 per square metre will be $17.5 \times \$8 = \$140$.

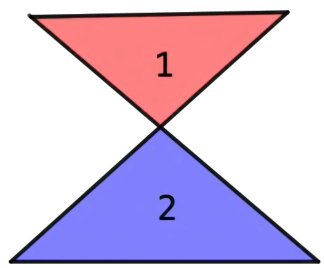
(c) The total cost of wallpaper, carpet, and paint will be $\$240 + \$140 + \$75 = \$455$. Since her total budget is \$500, she will be $\$500 - \$455 = \$45$ under her budget.

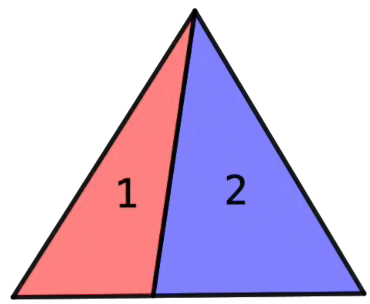
# Problem of the Week

## Problem B

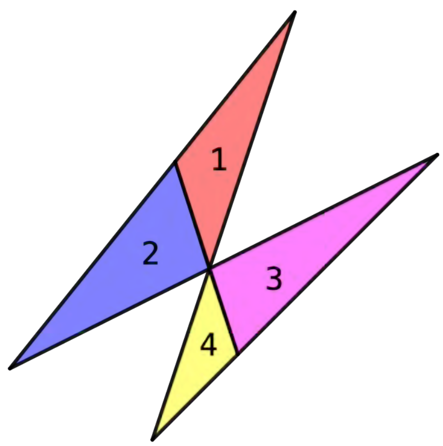
## "Try"angles

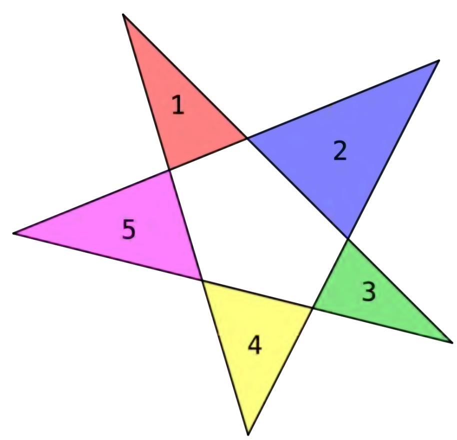
Using four straight lines, it is only possible to construct up to two non-overlapping triangles. Here are some examples:




Using five straight lines, it is only possible to construct up to five non-overlapping triangles. Here are some examples:




Notice that the first diagram has four non-overlapping triangles and the second diagram has five non-overlapping triangles. Notice also that the diagram with five non-overlapping triangles also has a pentagon which is not counted.

(a) How many non-overlapping triangles can you make using six straight lines?

(b) How many non-overlapping triangles can you make using seven straight lines?

Trade ideas with a classmate.

THEME GEOMETRY & MEASUREMENT

# Problem of the Week
## Problem B and Solution
## "Try"angles

### Problem

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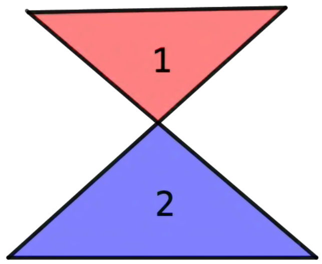


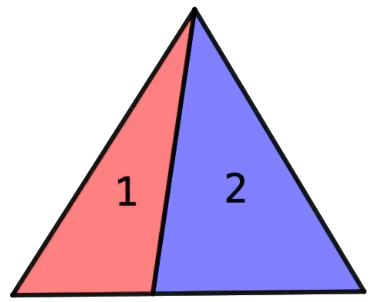


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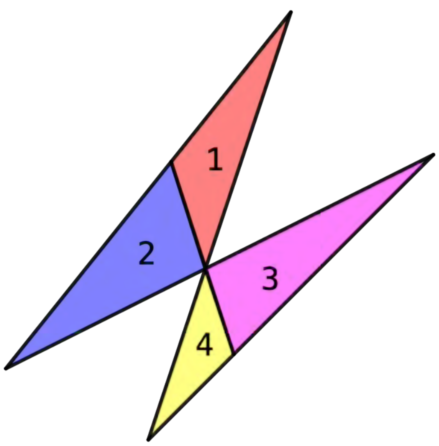


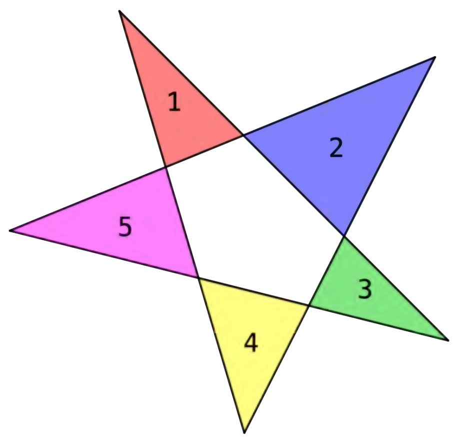


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(a) How many non-overlapping triangles can you make using six straight lines?

(b) How many non-overlapping triangles can you make using seven straight lines?

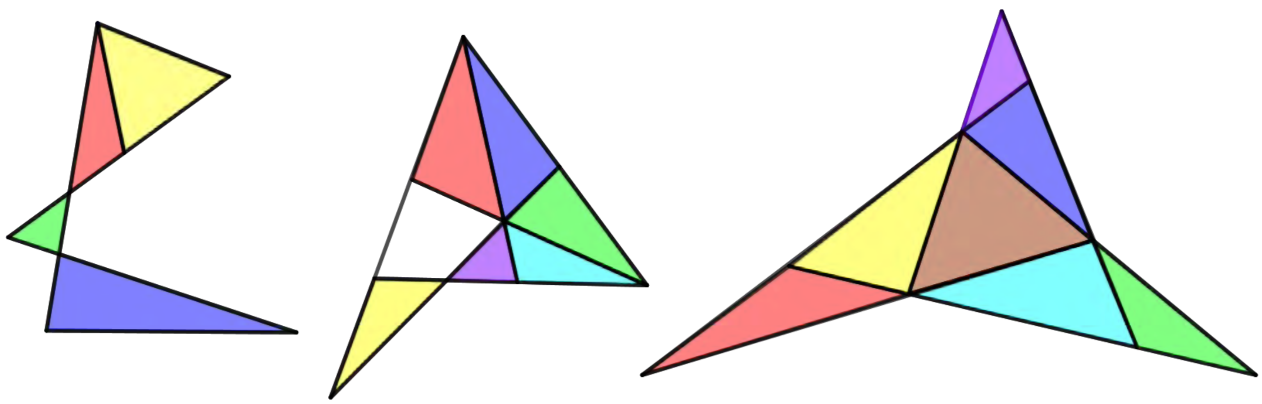
Trade ideas with a classmate.

**Solution**

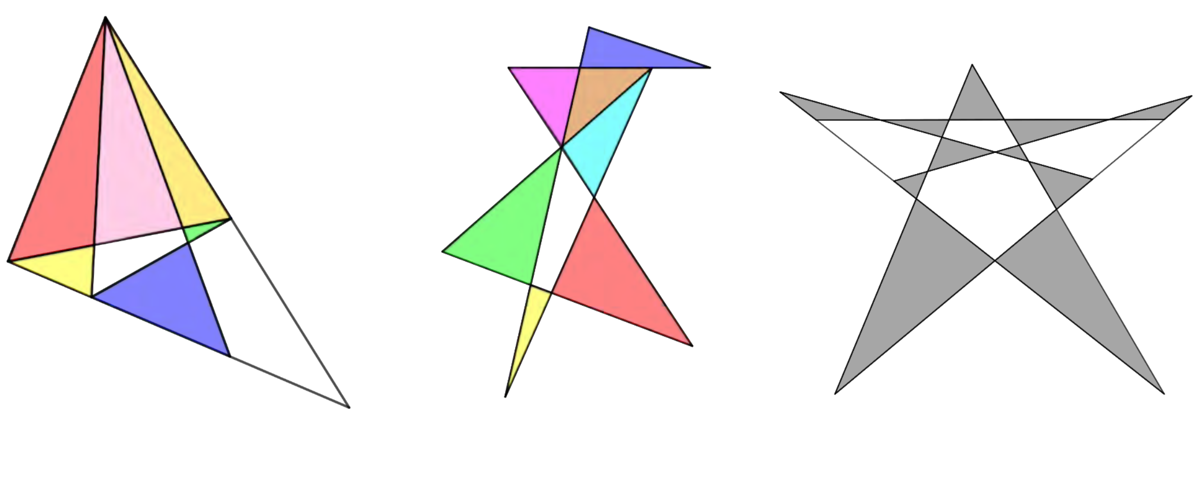
This geometry problem of finding non-overlapping triangles with sides lying on a specified number of straight lines is known as the *Kobon triangle problem.* Note: the Kobon triangle problem is not fully solved!

Here are some sample solutions. Students will likely find many others.

(a) It is known that seven triangles is the maximum possible number of non-overlapping triangles that can be formed using six lines. Here are some solutions for six lines, showing four, six, and seven non-overlapping triangles.



(b) It is known that eleven triangles is the maximum possible number of non-overlapping triangles that can be formed using seven lines. Here are some solutions for seven lines, showing six, seven, and eleven non-overlapping triangles.



# Problem of the Week

## Problem B

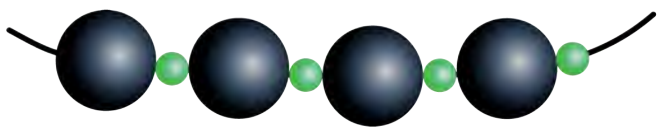
## A String of Beads

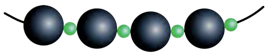
Aurora is making a beaded necklace using black and green beads. The black beads are all 1.2 cm wide and the green beads are all 4 mm wide. Aurora will make her necklace by alternating the black and green beads.

(a) If Aurora wants her necklace to be 80 cm long, how many beads will she need in total?

(b) If the black beads cost \$0.10 each and the green beads cost \$0.03 each, how much will it cost for Aurora to buy all the beads she needs for her necklace?

(c) Would it cost more or less for Aurora to buy the beads if instead of alternating the black and green beads, she put two green beads after each black bead? Explain.





# Problem of the Week
## Problem B and Solution
## A String of Beads

### Problem

Aurora is making a beaded necklace using black and green beads. The black beads are all 1.2 cm wide and the green beads are all 4 mm wide. Aurora will make her necklace by alternating the black and green beads.

(a) If Aurora wants her necklace to be 80 cm long, how many beads will she need in total?

(b) If the black beads cost \$0.10 each and the green beads cost \$0.03 each, how much will it cost for Aurora to buy all the beads she needs for her necklace?

(c) Would it cost more or less for Aurora to buy the beads if instead of alternating the black and green beads, she put two green beads after each black bead? Explain.

## Solution

(a) First we need to write the widths of the beads with the same unit of measurement. If we choose centimetres, then the green beads are $4 \div 10 = 0.4$ cm wide. Since Aurora is alternating black and green beads, the necklace will be made up of pairs of black and green beads. Each pair of black and green beads is $1.2 + 0.4 = 1.6$ cm wide. We need to determine how many pairs of black and green beads will fit on the necklace. Since $80 \div 1.6 = 50$, there will be 50 pairs of black and green beads on the necklace. So Aurora will need 50 black beads and 50 green beads, which is a total of 100 beads.

(b) Aurora needs 50 black beads. Since the black beads cost \$0.10 each, it will cost $50 \times \$0.10 = \$5$ to buy them all. Aurora needs 50 green beads. Since the green beads cost \$0.03 each, it will cost $50 \times \$0.03 = \$1.50$ to buy them all. Therefore, in total, it will cost $\$5 + \$1.50 = \$6.50$ to buy all the beads for the necklace.

(c) If Aurora puts two green beads after each black bead, then the necklace will be made up of groups of one black bead and two green beads. Each of these groups is $1.2 + 0.4 + 0.4 = 2$ cm wide. Since the necklace is 80 cm long, and $80 \div 2 = 40$, it follows that 40 of these groups will fit on the necklace. So the necklace will have 40 black beads and $40 \times 2 = 80$ green beads. Since the black beads cost \$0.10 each, it will cost $40 \times \$0.10 = \$4$ to buy them all. Since the green beads cost \$0.03 each, it will cost $80 \times \$0.03 = \$2.40$ to buy them all. Therefore, in total, it will cost $\$4 + \$2.40 = \$6.40$ to buy all the beads for the necklace. Since $\$6.40 < \$6.50$, it will be cheaper to buy the beads if Aurora puts two green beads after each black bead.

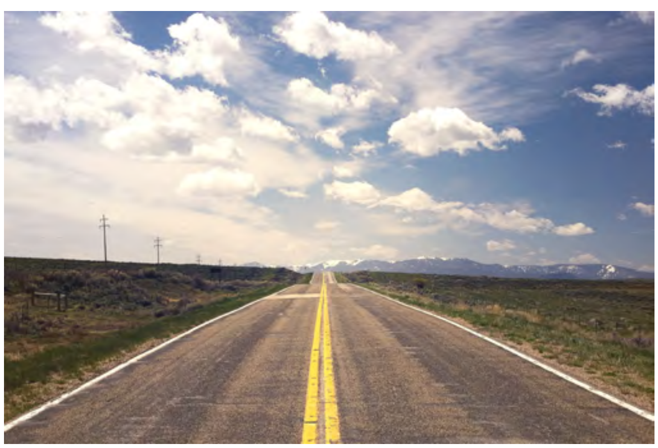
Alternatively, we could have justified this without doing all the calculations. Notice that the width of three green beads is $3 \times 0.4 = 1.2$ cm, which is the width of one black bead. However, the cost of three green beads is $3 \times \$0.03 = \$0.09$, but the cost of one black bead is \$0.10. So three green beads take up the same space as one black bead, but are \$0.01 cheaper to buy. If Aurora puts two green beads after each black bead instead of alternating the black and green beads, then she will end up using more green beads and fewer black beads in her necklace. Every time she replaces one black bead with three green beads she will save \$0.01, so it will be cheaper to buy the beads if Aurora puts two green beads after each black bead.

# Problem of the Week
## Problem B
## On the Road Again!

This problem looks at the epic journeys of two young men whose fortitude knew no bounds.

(a) In 1980, Terry Fox set out to run across Canada in order to raise money for cancer research, in what is called the Marathon of Hope. He planned to run the entire length of the Trans-Canada Highway, which is 7821 km. Terry Fox ran an average of 42 km every day, but had to stop after 143 days and 5373 km. If he had been able to complete his journey and had continued at the same pace, how many days would it have taken him to complete the remainder of his run across Canada?

(b) In 1985, Rick Hansen, the Man in Motion, wheeled around the world in his wheelchair in order to help people understand the importance of a world without barriers for people with disabilities. Starting on March 21, 1985, and finishing on May 22, 1987, he went through 34 countries and travelled a total of 40 075 km. On average, how many kilometres did he travel on each day of his world tour?



# Problem of the Week
## Problem B and Solution
## On the Road Again!

**Problem**

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## Solution

(a) The Trans-Canada Highway is 7821 km long and Terry Fox completed 5373 km. Thus, $7821 - 5373 = 2448$ km remain. If Terry Fox travelled at 42 km per day, then since $2448 \div 42 \approx 58.286$, it would have taken him 59 days to complete his run across Canada.

(b) To calculate the number of days between March 21, 1985 and May 22, 1987, we will first calculate the number of days between March 21, 1985 and March 20, 1987, inclusive, and then calculate the number of days between March 21, 1987 and May 22, 1987, inclusive.

- Since neither 1986 nor 1987 were leap years, the number of days between March 21, 1985 and March 20, 1987 is $2 \times 365 = 730$.
- To calculate the number of days between March 21, 1987 and May 22, 1987, we will look at the number of days in each month. There are 11 days from March 21 to March 31. April has 30 days, and there are 22 days from May 1 to May 22. This is a total of $11 + 30 + 22 = 63$ days.

Thus, in total, Rick Hansen travelled for $730 + 63 = 793$ days. Since he travelled 40 075 km in total, this means he travelled on average $40\,075 \div 793 \approx 50.5$ km per day.

# Problem of the Week
## Problem B
## Fields of Flowers

Sadie has garden beds that are 11 m by 14 m. She wants to grow giant sunflowers in one of her garden beds and dwarf sunflowers in another garden bed.

(a) Sadie spaces the giant sunflower seeds 50 cm apart, in rows that are 50 cm apart, leaving a 100 cm border on all sides of the garden bed. How many giant sunflower seeds can she plant in one garden bed?

(b) In another garden bed, Sadie plants dwarf sunflower seeds. She spaces the seeds 25 cm apart, in rows that are 25 cm apart, leaving a 100 cm border on all sides of the garden bed. How many dwarf sunflowers can she plant in this garden bed?

(c) All of Sadie's sunflowers have germinated and matured, but then in a cold early frost one evening, she loses 20% of the giant sunflowers and 10% of the dwarf sunflowers. If Sadie sells all the surviving sunflowers at \$5.00 each for the giants and \$3.00 each for the dwarfs, which crop will provide the greater income?

# Problem of the Week
## Problem B and Solution
## Fields of Flowers

### Problem

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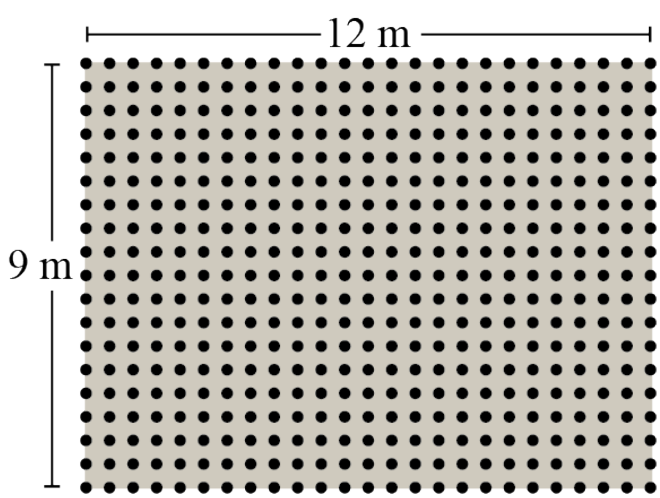
## Solution

(a) Since 100 cm is equal to 1 m, the border around Sadie's garden bed is 1 m on each side. That means the planting area inside the garden is 9 m by 12 m. If she plants rows of seeds starting right on the edge of the planting area, and plants them 50 cm (or $\frac{1}{2}$ m) apart, then in each row she can plant 2 seeds per metre, plus 1 more seed at the end of the row. So, along the 9 m width she can plant $9 \times 2 + 1 = 18 + 1 = 19$ seeds. Along the 12 m length she can plant $12 \times 2 + 1 = 24 + 1 = 25$ seeds. Thus, Sadie can plant a total of $19 \times 25 = 475$ giant sunflower seeds in one garden bed, as shown.



(b) As in part (a), we can conclude that the planting area inside this garden bed is also 9 m by 12 m. If Sadie plants rows of seeds starting right on the edge of the planting area, and plants them 25 cm (or $\frac{1}{4}$ m) apart, then in each row she can plant 4 seeds per metre, plus 1 more seed at the end of the row. So, along the 9 m width she can plant $9 \times 4 + 1 = 36 + 1 = 37$ seeds. Along the 12 m length she can plant $12 \times 4 + 1 = 48 + 1 = 49$ seeds. Thus, Sadie can plant a total of $37 \times 49 = 1813$ dwarf sunflower seeds in this garden bed.

(c) After the loss of 20% of the giant sunflowers, Sadie will have 80% of 475, or $0.8 \times 475 = 380$ flowers left. These giant sunflowers will provide an income of $380 \times \$5.00 = \$1900$.

After the loss of 10% of the dwarf sunflowers, Sadie will have 90% of 1813, or $0.9 \times 1813 \approx 1632$ flowers left. These dwarf sunflowers will provide an income of $1632 \times \$3.00 = \$4896$. Thus, the income from the dwarf sunflowers is more than double that of the giant sunflowers.

# Algebra (A)

TAKE ME TO THE COVER

# Problem of the Week
## Problem B
## Joey Prepares for Winter

Joey the chipmunk will soon be hibernating, so he's gathering acorns, his food supply for the long winter months.

Joey has four acorns remaining from the previous day, and has gathered acorns over the last few hours as shown in the following table.

| **Hour** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Total Number of Acorns** | 4 | 20 | 36 | 52 | 68 | 84 |

(a) Is the total number of acorns a linear growing pattern? Verify your answer by creating a graph.

(b) Suppose Joey continues collecting acorns at this same rate.

(i) How many acorns would Joey have collected by the end of Hour 12?

(ii) How many hours would it take him to collect at least 330 acorns?

(iii) Write an algebraic expression to represent the total number of acorns Joey would have after collecting for $n$ hours.

**Themes** Algebra, Number Sense

# Problem of the Week

## Problem B and Solution

## Joey Prepares for Winter

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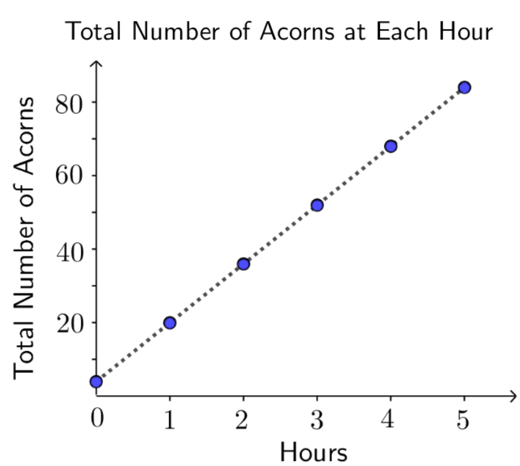
(iii) Write an algebraic expression to represent the total number of acorns Joey would have after collecting for $n$ hours.

## Solution

(a) Looking at the data, we see that the number of acorns increases by the same amount each hour; Joey is collecting acorns at a rate of 16 per hour. So we expect that the pattern of the total number of acorns is a growing linear pattern. This is verified by the following graph.



(b) (i) Hour 12 is 7 more hours after Hour 5. Since Joey will collect 16 acorns in each of those hours, he will have $7 \times 16 = 112$ more acorns, giving a total of $84 + 112 = 196$ acorns by the end of Hour 12.

(ii) To collect at least 330 acorns in total, Joey needs $330 - 196 = 134$ more acorns than he has after 12 hours. After 8 more hours, he would have $8 \times 16 = 128$ more acorns. After 9 more hours, he would have $9 \times 16 = 144$ more acorns. Therefore, he will need to collect acorns for 9 more hours to get to at least 330 acorns.
Thus, he will need a total of $12 + 9 = 21$ hours to collect at least 330 acorns.

Alternatively: Joey initially has 4 acorns, so to get to 330 acorns, he needs to collect 326 more acorns. Since he collects 16 acorns per hour, this would take him $326 \div 16 = 20\frac{3}{8}$ hours. This means he will have 330 acorns during the $21^{\text{st}}$ hour. That is, he will need to collect for 21 hours to get at least 330 acorns.

(iii) After $n$ hours of collecting 16 acorns each hour, Joey would have $16 \times n$ acorns. Given that he starts with four leftover acorns, Joey would have a total of $(16 \times n) + 4$ acorns.

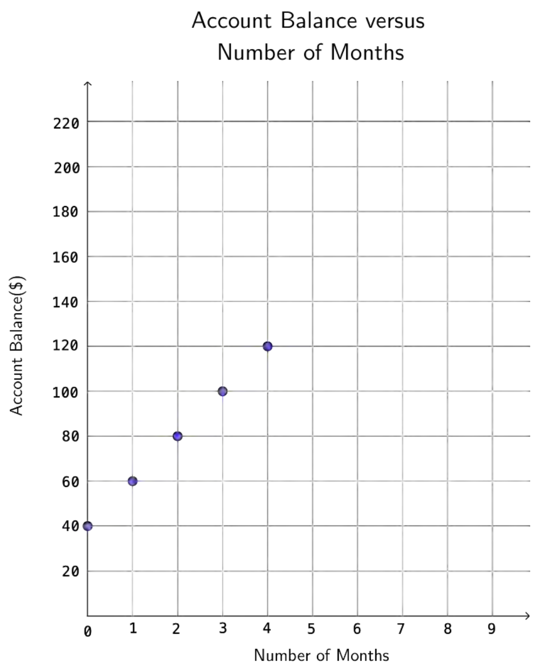
# Problem of the Week

## Problem B

## Banking on Amir

Amir's aunt wants to help him develop an education fund so that he can go to drumming school. He started a bank account with \$40, and each month thereafter a \$20 deposit is to be made.

The graph below shows how the bank account grows over time (with no interest).




(a) Create a table of values, listing the five ordered pairs of coordinates from the graph, as indicated by the dots.

(b) What is the pattern rule for the monthly account balance? Use your rule to add as many points to the graph as possible.

(c) How much will Amir have in his account after 6 months? Show on the graph how you got your answer.

(d) After how many months will Amir have \$220 in his account? Show on the graph how you got your answer.

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THEMES ALGEBRA, DATA MANAGEMENT

# Problem of the Week
## Problem B and Solution
## Banking on Amir

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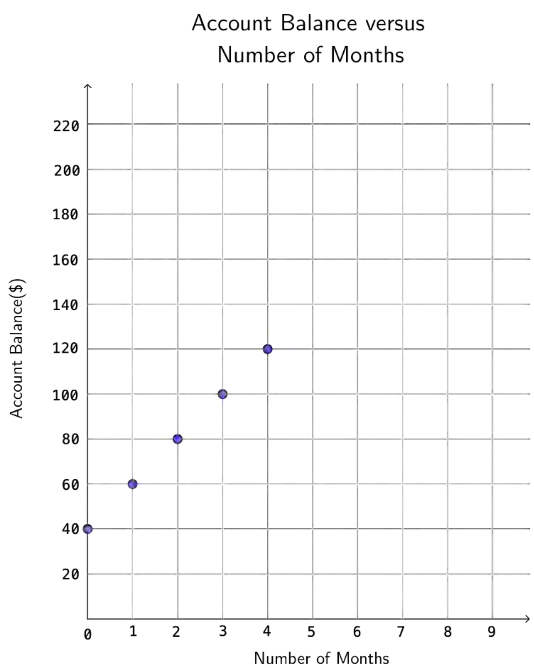


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(d) After how many months will Amir have \$220 in his account? Show on the graph how you got your answer.

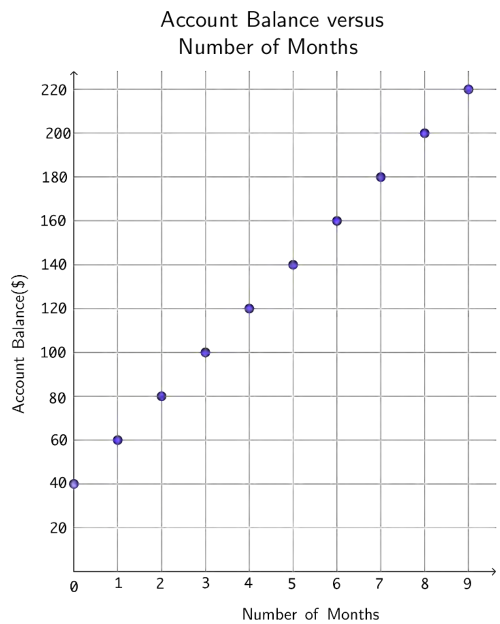
**Not printing this page?** You can use our interactive worksheet.

## Solution

(a) A table of values, listing the five ordered pairs of coordinates from the graph, is below.

| Number of Months | Account Balance ($) |
|---|---|
| 0 | 40 |
| 1 | 60 |
| 2 | 80 |
| 3 | 100 |
| 4 | 120 |

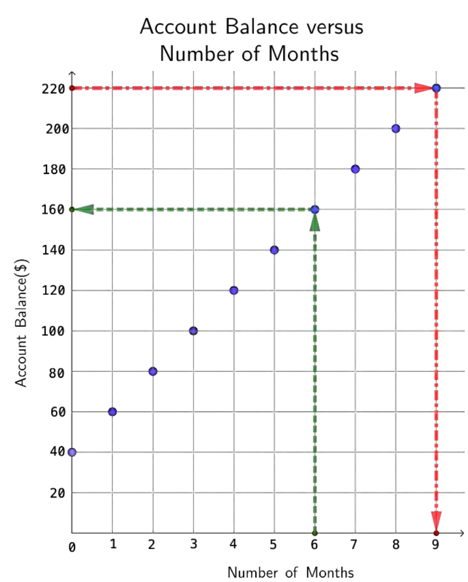
(b) The pattern rule for the monthly account balance is "Start at 40 and add 20 each month". Using this pattern rule, we can complete the graph up to 9 months, as shown below.




(c),(d)

For part (c), we start at $(6, 0)$ on the $x$-axis and move up to the (blue) dot, then left to the $y$-axis, which indicates \$160, as shown by the green arrows (dashed lines) on the graph. Thus, Amir will have \$160 in his account after 6 months.

For part (d), we start at $(220, 0)$ on the $y$-axis, and move to the right, reaching the (blue) dot, and then down to the $x$-axis, which indicates 9 months, as shown by the red arrows (dashed-dotted lines) on the graph. Thus, after 9 months, Amir will have \$220 in his account.

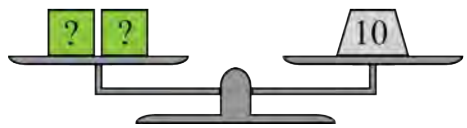


# Problem of the Week

## Problem B

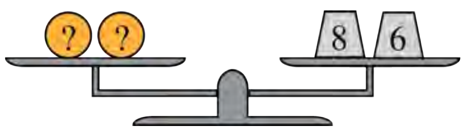
## A Balancing Act

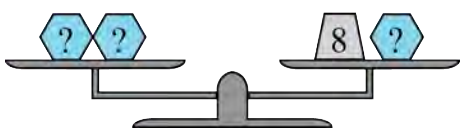
If a scale is balanced, then the total mass on each side of the scale is the same. Consider the following balanced scale, where the number on an object represents its mass, in grams, and two identical objects with question marks on them have the same unknown mass.

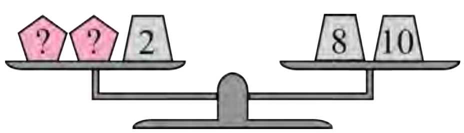



Since the right side has a mass of $10\,\text{g}$, it follows that the two squares must also have a total mass of $10\,\text{g}$. Since the square objects are identical, they must each have a mass of $10 \div 2 = 5\,\text{g}$.

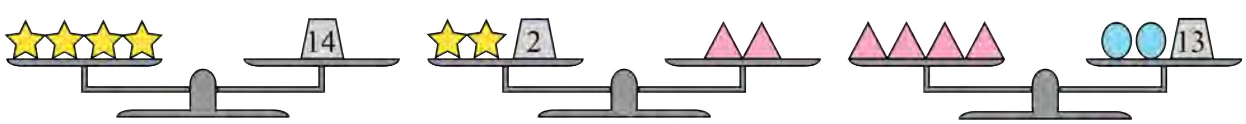
(a) Find the mass of the indicated shape for each of the three balanced scales.

(i) 


(ii) 


(iii) 

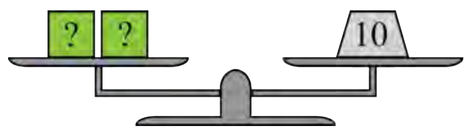

(b) Using the same idea as in part (a), determine the mass of each symbol in the balanced scales shown. Note that here, the information from the previous scale is used in solving the next one.



# Problem of the Week
## Problem B and Solution
## A Balancing Act

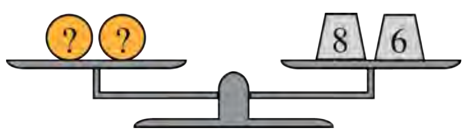
### Problem

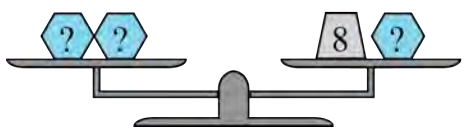
If a scale is balanced, then the total mass on each side of the scale is the same. Consider the following balanced scale, where the number on an object represents its mass, in grams, and two identical objects with question marks on them have the same unknown mass.

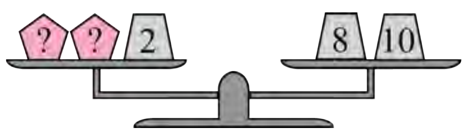



Since the right side has a mass of 10 g, it follows that the two squares must also have a total mass of 10 g. Since the square objects are identical, they must each have a mass of $10 \div 2 = 5$ g.

(a) Find the mass of the indicated shape for each of the three balanced scales.

(i) 


(ii) 


(iii) 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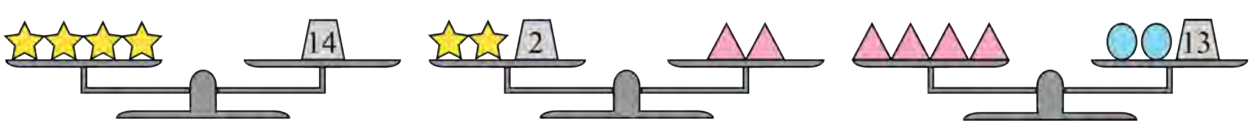


**Solution**

(a) (i) Since 2 circles have a mass of $8 + 6 = 14\,\text{g}$, it follows that one circle has a mass of $14 \div 2 = 7\,\text{g}$.

(ii) If we remove one hexagon from each side of the scale, the scale will remain balanced because the hexagons have equal mass. Then we see that one hexagon has a mass of $8\,\text{g}$.

(iii) If we remove $2\,\text{g}$ from each side of the scale, then it follows that 2 pentagons have a mass of $8 + 10 - 2 = 16\,\text{g}$. Then one pentagon has a mass of $16 \div 2 = 8\,\text{g}$.

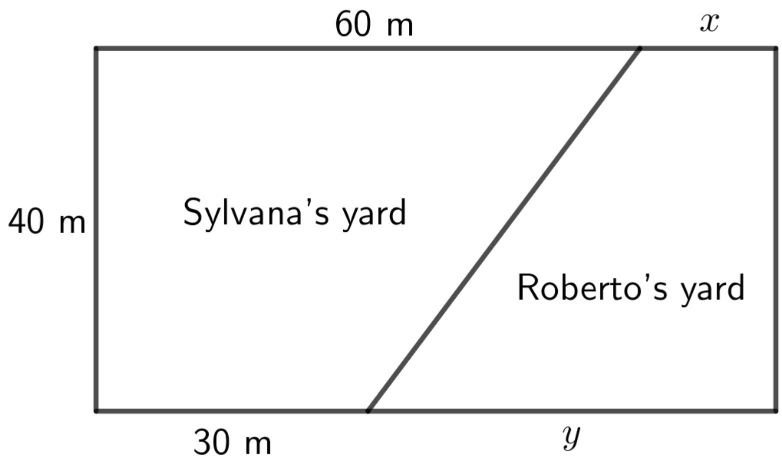
(b) From the first scale, we see that 4 stars have a mass of $14\,\text{g}$, so it follows that one star has a mass of $14 \div 4 = 7 \div 2 = 3\frac{1}{2}\,\text{g}$.

In the second scale there are two stars and an object with a mass of $2\,\text{g}$ on the left side. These have a total mass of $3\frac{1}{2} + 3\frac{1}{2} + 2 = 9\,\text{g}$. Then, two triangles have a mass of $9\,\text{g}$, so one triangle has a mass of $9 \div 2 = 4\frac{1}{2}\,\text{g}$.

In the third scale there are four triangles on the left side. These have a total mass of $4 \times 4\frac{1}{2} = 18\,\text{g}$. If we subtract $13\,\text{g}$ from each side of this scale, then each side will have a mass of $18 - 13 = 5\,\text{g}$. Thus, 2 ovals have a mass of $5\,\text{g}$, so one oval has a mass of $5 \div 2 = 2\frac{1}{2}\,\text{g}$.

# Problem of the Week
## Problem B
## Seeking Parts Unknown...

Sylvana and Roberto divide a 40 m by 75 m rectangular lot to form two yards, as shown in the diagram below.




The area of Roberto's yard is 40% of the total area of the two properties.

(a) What are the values of $x$ and $y$, the missing dimensions of Roberto's yard?

(b) What is the area of each yard?

# Problem of the Week

## Problem B and Solution

## Seeking Parts Unknown...

### Problem

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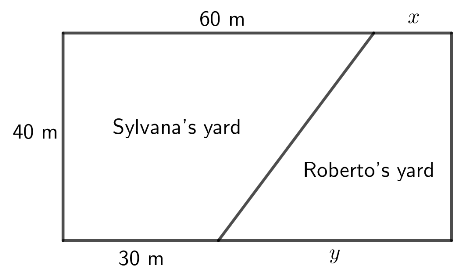



The area of Roberto's yard is 40% of the total area of the two properties.

(a) What are the values of $x$ and $y$, the missing dimensions of Roberto's yard?

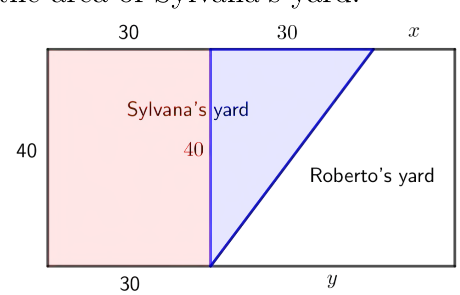
(b) What is the area of each yard?

## Solution

(a) From the two sides of the rectangle of length 75 m, we must have $60\,\text{m} + x = 75$ m and $30\,\text{m} + y = 75$ m. Thus, the missing dimensions of Roberto's yard are $x = 75 - 60 = 15$ m and $y = 75 - 30 = 45$ m.

(b) The total area of the two yards is $40\,\text{m} \times 75\,\text{m} = 3000$ m$^2$. The area of each yard can be found in a variety of ways:

- The area of Roberto's yard is 40% of the total area. Thus, the area of Roberto's yard is 40% of 3000, or $0.4 \times 3000 = 1200$ m$^2$, and the area of Sylvana's yard is $3000 - 1200 = 1800$ m$^2$.
- Alternatively, since the area of Roberto's yard is 40% of the total area, the area of Sylvana's yard must be $100\% - 40\% = 60\%$ of the total area. Thus, the area of Sylvana's yard is $0.6 \times 3000 = 1800$ m$^2$, and the area of Roberto's yard is $3000 - 1800 = 1200$ m$^2$.
- We can find the area of one of the yards, and subtract that from the total area to find the area of the other yard. We will show how to find the area of Sylvana's yard.
  Notice that Sylvana's yard is shaped like a trapezoid. We can calculate the area of Sylvana's yard by dividing the trapezoid into a 40 m by 30 m rectangle (shown in red) and a triangle with a base of 30 m and a height of 40 m (shown in blue).

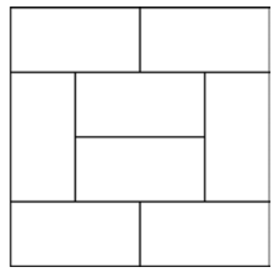
  The area of the rectangle is $40 \times 30 = 1200$ m$^2$ and the area of the triangle is $\frac{40\times 30}{2} = 600$ m$^2$.
  Thus, the total area of Sylvana's yard is $1200 + 600 = 1800$ m$^2$, and so the total area of Roberto's yard is $3000 - 1800 = 1200$ m$^2$.

# Problem of the Week

## Problem B

## Mystery Dimensions

Eight congruent rectangles are arranged to form a larger rectangle as shown.



(a) If the congruent rectangles each have a length of 6 cm and a width of 3 cm, what is the perimeter of the larger rectangle?

(b) Suppose that the congruent rectangles each have a longer side of length $L$ cm and a shorter side of length 4 cm. Suppose also that the perimeter of the larger rectangle is 64 cm.

(i) What is the value of $L$?

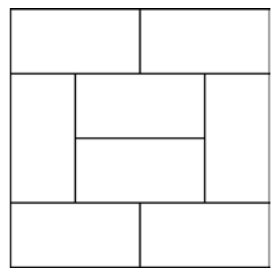
(ii) What is the area of one of the eight congruent rectangles?

**Extension:** Can you solve part (b) without knowing that the length of the shorter side of each rectangle is 4 cm? If so, how?

Themes Algebra, Geometry & Measurement

# Problem of the Week
## Problem B and Solution
## Mystery Dimensions

### Problem
Eight congruent rectangles are arranged to form a larger rectangle as shown.



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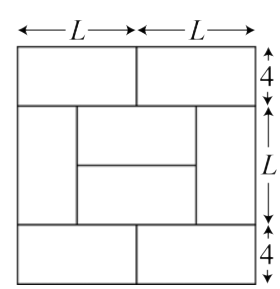
**Extension:** Can you solve part (b) without knowing that the length of the shorter side of each rectangle is 4 cm? If so, how?

## Solution

(a) Since each rectangle has a length of 6 cm and a width of 3 cm, the larger rectangle must have sides of lengths $6 + 6 = 12$ cm and $3 + 6 + 3 = 12$ cm. Thus, the perimeter of the larger rectangle is $12 + 12 + 12 + 12 = 48$ cm.

(b)

(i) Since each rectangle has a longer side of length $L$ cm and shorter side of length 4 cm, we can label our diagram to find the dimensions of the larger rectangle. Using this, we determine that the lengths of the sides of the larger rectangle are $L + L = 2L$ and $4 + L + 4 = L + 8$. Since we know the perimeter of the larger rectangle is 64 cm, we can write the following equation.



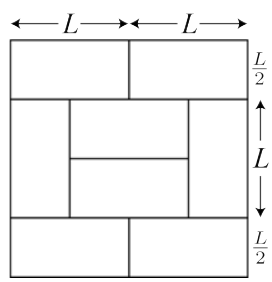
$$\begin{aligned} 2L + 2L + (L+8) + (L+8) &= 64 \\ 6L + 16 &= 64 \\ 6L &= 64 - 16 \\ 6L &= 48 \end{aligned}$$

Since $6 \times 8 = 48$, it follows that $L = 8$ cm.

(ii) The area of a rectangle is equal to its length times its width. Thus, the area of each congruent rectangle is $8 \times 4 = 32$ cm$^2$.

**Extension Solution:**

If we ignore the two rectangles on the top and the two rectangles on the bottom, we can see that two rectangles placed on top of each other horizontally have a height of $L$. Therefore, the shorter side of each rectangle equals half its longer side, or $\frac{L}{2}$. We can label our diagram to find the dimensions of the larger rectangle.




Using this, we determine that the larger rectangle has sides of length $L + L = 2L$ and $\frac{L}{2} + L + \frac{L}{2} = 2L$. So the larger rectangle is actually a square with side length $2L$. Since we know its perimeter is 64 cm, it follows that $2L + 2L + 2L + 2L = 64$, or $8L = 64$. Since $8 \times 8 = 64$, it follows that $L = 8$ cm. So, we can solve this problem without knowing the width of each rectangle.

# Data Management (D)

TAKE ME TO THE COVER

# Problem of the Week

## Problem B

## Water, Water, Everywhere...

Very little of Earth's fresh water is accessible for human consumption, particularly in dry countries, making alternative sources necessary.

(a) The per capita (per person) daily water consumption for nine different countries is given below.

$$155\,\text{L},\ 251\,\text{L},\ 200\,\text{L},\ 147\,\text{L},\ 135\,\text{L},\ 235\,\text{L},\ 373\,\text{L},\ 145\,\text{L},\ 380\,\text{L}$$

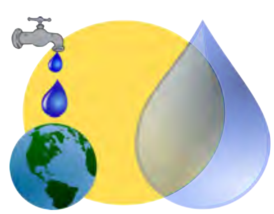
What is the average per capita daily water consumption for these countries? Round your answer to the nearest whole number.

(b) A small city of 110 000 people in an arid (very dry) country obtains its fresh water by desalination of sea water. If the per capita consumption in this city is equal to the average from part (a), how much fresh water must be produced each day by the city's desalination plant?

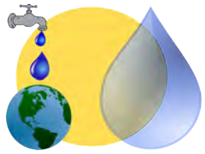
(c) Sea water is 3.5% salt; the remaining 96.5% is fresh water. Thus, if 1000 L of sea water was desalinated, the amount of fresh water produced would be $0.965 \times 1000 = 965$ L. In general, we can use the following equation to show the relationship between the amount of sea water and fresh water in the desalination process.

$$0.965 \times \text{amount of sea water} = \text{amount of fresh water}$$

Use this equation and your answer from part (b) to find the amount of sea water that must be processed by the desalination plant every day in order to fulfill the city's fresh water needs.



THEMES DATA MANAGEMENT, NUMBER SENSE



# Problem of the Week
## Problem B and Solution
## Water, Water, Everywhere...

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Use this equation and your answer from part (b) to find the amount of sea water that must be processed by the desalination plant every day in order to fulfill the city's fresh water needs.

## Solution

(a) Adding the nine countries' daily consumption figures gives 2021 L. Thus, the average daily consumption per capita is $2021 \div 9 = 224.555... \approx 225$ L.

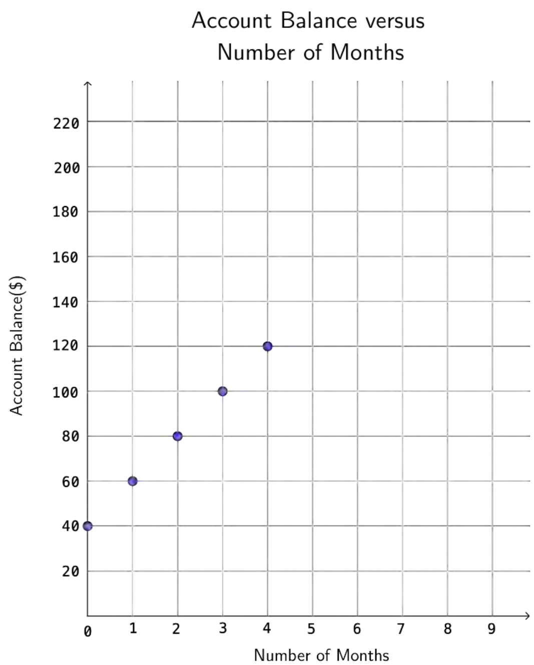
(b) If each of the 110 000 people consumes 225 litres of water per day, then the city's desalination plant must produce $110\,000 \times 225 = 24\,750\,000$ litres of fresh water per day.

(c) Once we substitute our answer from part (b), the equation becomes $0.965 \times \text{amount of sea water} = 24\,750\,000$. We can find the amount of sea water by trial and error, but a more efficient method is to notice that $\text{amount of sea water} = 24\,750\,000 \div 0.965 \approx 25\,647\,668$. Thus the amount of sea water needed each day is approximately 25 647 668 L, or about 25.65 million litres.

# Problem of the Week
## Problem B
## Banking on Amir

Amir's aunt wants to help him develop an education fund so that he can go to drumming school. He started a bank account with \$40, and each month thereafter a \$20 deposit is to be made.

The graph below shows how the bank account grows over time (with no interest).




(a) Create a table of values, listing the five ordered pairs of coordinates from the graph, as indicated by the dots.

(b) What is the pattern rule for the monthly account balance? Use your rule to add as many points to the graph as possible.

(c) How much will Amir have in his account after 6 months? Show on the graph how you got your answer.

(d) After how many months will Amir have \$220 in his account? Show on the graph how you got your answer.

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Themes Algebra, Data Management

# Problem of the Week

## Problem B and Solution

## Banking on Amir

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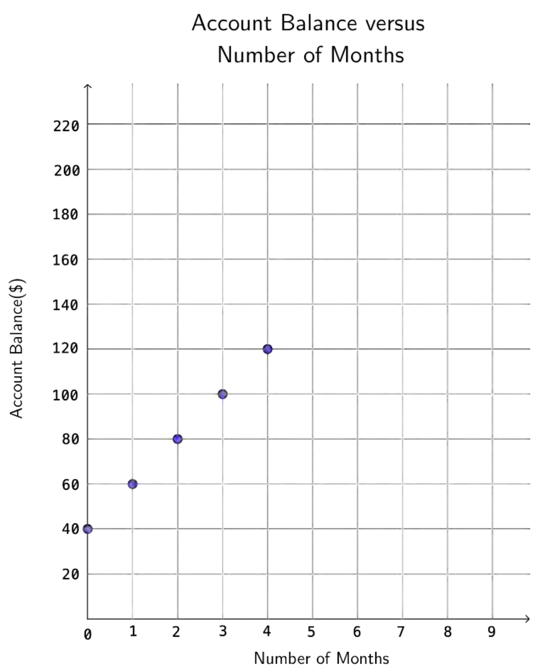


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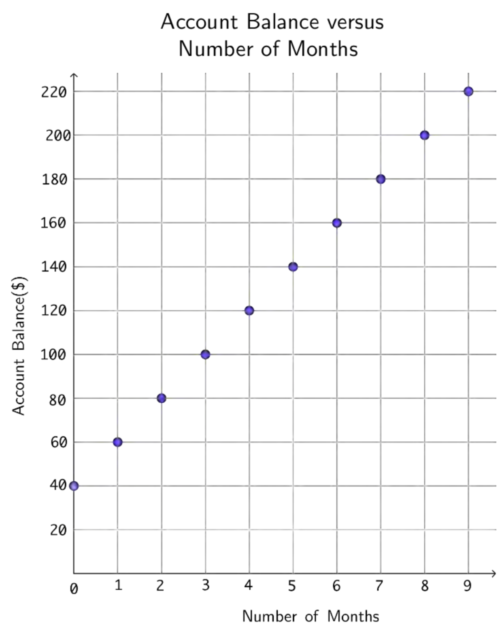
## Solution

(a) A table of values, listing the five ordered pairs of coordinates from the graph, is below.

| Number of Months | Account Balance ($) |
|---|---|
| 0 | 40 |
| 1 | 60 |
| 2 | 80 |
| 3 | 100 |
| 4 | 120 |

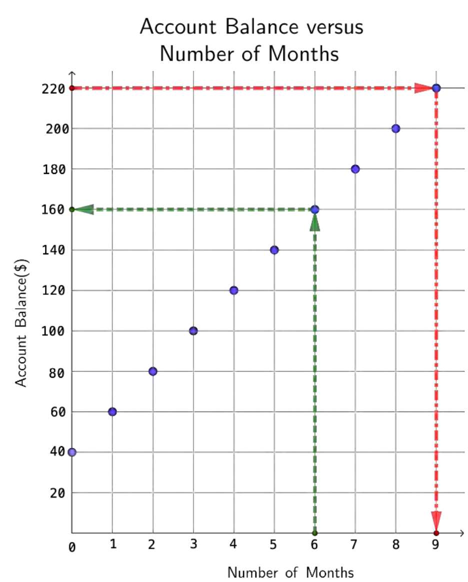
(b) The pattern rule for the monthly account balance is "Start at 40 and add 20 each month". Using this pattern rule, we can complete the graph up to 9 months, as shown below.




(c),(d)

For part (c), we start at $(6, 0)$ on the $x$-axis and move up to the (blue) dot, then left to the $y$-axis, which indicates \$160, as shown by the green arrows (dashed lines) on the graph. Thus, Amir will have \$160 in his account after 6 months.

For part (d), we start at $(220, 0)$ on the $y$-axis, and move to the right, reaching the (blue) dot, and then down to the $x$-axis, which indicates 9 months, as shown by the red arrows (dashed-dotted lines) on the graph. Thus, after 9 months, Amir will have \$220 in his account.



# Problem of the Week

## Problem B

## A Pocketful of Coins

Dakarai has a some Canadian coins in his pocket: one nickel (worth \$0.05), one dime (worth \$0.10), one quarter (worth \$0.25), one loonie (worth \$1.00), and one toonie (worth \$2.00).


(a) Suppose he reaches into his pocket and pulls out one coin at random.

(i) What is the probability that he will pull out
- a nickel?
- a quarter?
- a toonie?

(ii) What is the probability that the total value of the coins remaining in his pocket is
- less than \$1.00?
- greater than \$1.35?
- less than \$2.00?

(b) Suppose Dakarai reaches into his pocket and pulls out two coins at random. Which is greater, the probability that the coins in his hand have a value of \$0.35, or the probability that the coins in his hand have a value of \$3.00?

**Theme** Data Management

# Problem of the Week
## Problem B and Solution
## A Pocketful of Coins

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## Solution

(a) Dakarai is selecting one of the five coins ‘at random’.

(i) Since his selection is ‘at random’, there is an equal chance he will pull out any one of the coins, so the probability for each of these is equal to $\frac{1}{5} = 0.2$, or 20%.

(ii)
- There is no combination of any four of the coins that has a total value less than \$1.00. Therefore, this probability is equal to 0.
- If Dakarai draws the coin of greatest value (the toonie), the total value of the remaining coins will be $\$(1.00 + 0.25 + 0.10 + 0.05) = \$1.40$, which is greater than \$1.35. So the total value of the remaining coins will always be greater than \$1.35, regardless of which coin he chooses. Therefore, this probability is equal to 1, or 100%.
- If Dakarai picks only one coin, the only way the remaining coins could have total value less than \$2.00 is if he pulls out the toonie. Thus, the probability is $\frac{1}{5} = 0.2$, or 20%.

(b) There are exactly two coins with total value \$0.35, namely the dime and the quarter. Similarly, there are exactly two coins with total value \$3.00, namely the loonie and the toonie. Since the coins are drawn ‘at random’, the probabilities of these events must be equal.

NOTE: The actual probability of each event is 0.1 or 10%. This can be illustrated by constructing a tree diagram, or by the following argument. The probability of drawing the dime first is $\frac{1}{5}$. Then there are only four coins in his pocket, so the probability of drawing the quarter next is $\frac{1}{4}$. Thus, the probability of drawing the dime and then the quarter is $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20} = 0.05$, or 5%. Similarly, the probability of drawing the quarter and then the dime is $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20} = 0.05$, or 5%. Thus, the total probability of drawing the dime and quarter is $0.05 + 0.05 = 0.1$, or 10%. A similar analysis can be used to show that the total probability of drawing the loonie and toonie is also 0.1, or 10%.

# Problem of the Week

## Problem B

## Measuring Feet - A Great Feat!

While reading an article in Teen Feat Magazine, Sundip learned that the mean (average) length of an 11-year-old's foot is 22.9 cm. He wondered how many of his 11-year-old friends had "average" feet. Here is the information of the foot lengths, in cm, that he gathered from himself and 11 of his friends:

19.1, 23.3, 21.7, 24.3, 22.1, 22.4, 20.7, 21.9, 22.5, 24.1, 26.4, 24.7

(a) Complete the frequency table below to reveal the number of students with foot lengths within each interval.

| Foot Length | Tally | Frequency | Relative Frequency |
|---|---|---|---|
| 18.0 - 19.9 | | | |
| 20.0 - 21.9 | | | |
| 22.0 - 23.9 | | | |
| 24.0 - 25.9 | | | |
| 26.0 - 27.9 | | | |

(b) What is the mean (average) foot length for Sundip and his friends? How does it compare to the average for 11-year-olds?

(c) Does the information in the table reveal any similarities or differences among the students as to foot length?

(d) Sundip's article also stated that fifty years ago, the average foot length an 11-year-old was about 21.9 cm. How do his friends' sizes compare to those of fifty years ago?

# Problem of the Week

## Problem B and Solution

## Measuring Feet - A Great Feat!

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## Solution

(a) Here is the completed frequency table.

| Foot Length | Tally | Frequency | Relative Frequency |
|---|---|---|---|
| 18.0 - 19.9 | \| | 1 | $\frac{1}{12} \approx 8.3\%$ |
| 20.0 - 21.9 | \|\|\| | 3 | $\frac{3}{12} = 25\%$ |
| 22.0 - 23.9 | \|\|\|\| | 4 | $\frac{4}{12} \approx 33.3\%$ |
| 24.0 - 25.9 | \|\|\| | 3 | $\frac{3}{12} = 25\%$ |
| 26.0 - 27.9 | \| | 1 | $\frac{1}{12} \approx 8.3\%$ |

(b) To find the mean foot length for Sundip and his friends, we add up all the foot lengths and divide by 12. The sum of the foot lengths is 273.2 cm, and so the average is $\frac{273.2}{12} \approx 22.8$ cm. The average for Sundip and his friends is slightly lower than the average of 22.9 cm for 11-year-olds.

(c) The table reveals that the foot lengths of Sundip and his friends are concentrated in their middle range, with only one person at each end of the possible lengths.

(d) The average foot length of Sundip and his friends is about 1 cm longer than the average of 50 years ago.

# Problem of the Week
## Problem B
## Yukon Do It!

The Yukon Quest is one of the most famous endurance sled dog races in the world. The total distance the race covers is 1635 km.

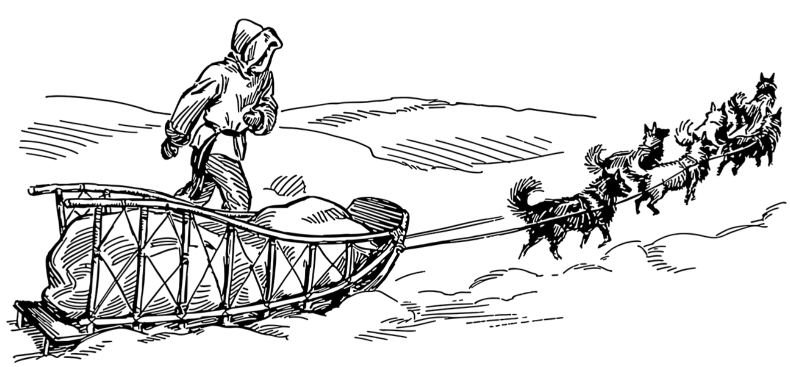
(a) If the average distance travelled per day for each team is 145 km, how many days will it take, on average, for a team to complete the Yukon Quest?

(b) A team of Alaskan huskies travels at 15 km per hour, with an 18 minute rest after every three hours. A team of Siberian huskies runs more quickly at 20 km per hour, but requires a 30 minute rest after every two hours.

On a certain day both teams travel 145 km. Create a broken-line graph of distance versus time for each team for that day.

Suggestion: You may find it helpful to first construct a table for each team, matching the total distance travelled with the elapsed time for each interval of travel and rest.

(c) Suppose that the weight of some additional equipment slows the average speed of the Siberian huskies by 5 km per hour. If the team still travels 145 km in a day, by how many minutes will this increase their travel time for the day?



Themes Data Management, Number Sense

# Problem of the Week
## Problem B and Solution
## Yukon Do It!

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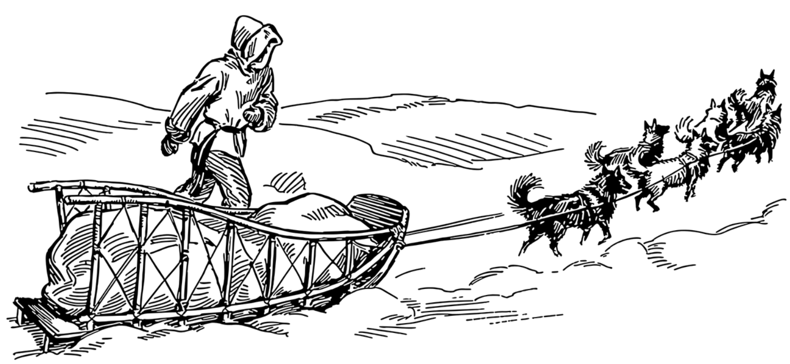
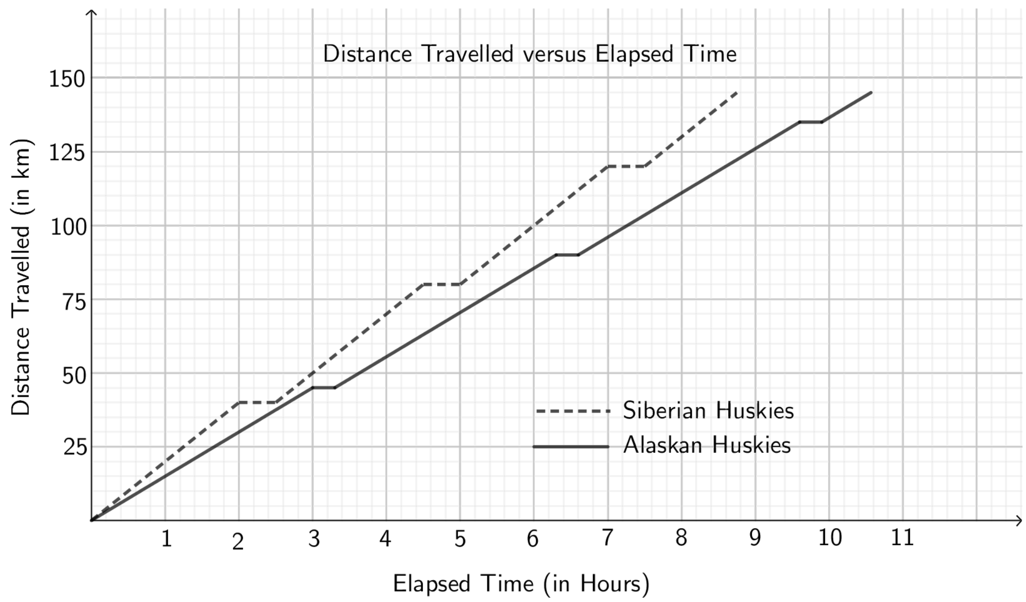


## Solution

(a) Since the average (mean) distance traveled each day is 145 km, and the total distance is 1635 km, the number of days to complete the race is $1635 \div 145 \approx 11.276$ days. Thus, on average, the teams would finish on the 12th day.

(b) A broken-line graph of distance versus time for each team is shown below.




Here is the table for the Alaskan husky team.

| Interval Type | Start Time | End Time | Interval Distance Travelled (km) | Total Distance Travelled (km) |
|---|---|---|---|---|
| Travel | 0 | 3 hrs | 45 | 45 |
| Rest | 3 hrs | 3 hrs 18 min | 0 | 45 |
| Travel | 3 hrs 18 min | 6 hrs 18 min | 45 | 90 |
| Rest | 6 hrs 18 min | 6 hrs 36 min | 0 | 90 |
| Travel | 6 hrs 36 min | 9 hrs 36 min | 45 | 135 |
| Rest | 9 hrs 36 min | 9 hrs 54 min | 0 | 135 |
| Travel | 9 hrs 54 min | 10 hrs 34 min | 10 | 145 |

Here is the table for the Siberian husky team.

| Interval Type | Start Time | End Time | Interval Distance Travelled (km) | Total Distance Travelled (km) |
|---|---|---|---|---|
| Travel | 0 | 2 hrs | 40 | 40 |
| Rest | 2 hrs | 2 hrs 30 min | 0 | 40 |
| Travel | 2 hrs 30 min | 4 hrs 30 min | 40 | 80 |
| Rest | 4 hrs 30 min | 5 hrs | 0 | 80 |
| Travel | 5 hrs | 7 hrs | 40 | 120 |
| Rest | 7 hrs | 7 hrs 30 min | 0 | 120 |
| Travel | 7 hrs 30 min | 8 hrs 45 min | 25 | 145 |

Note that the Alaskan huskies travel 135 km in three segments of 3 hours and 18 minutes each, plus 10 km in a final segment of 40 minutes. This gives a total time of 10 hours and 34 minutes. The Siberian huskies travel 120 km in three segments of 2 hours and 30 minutes each, plus 25 km in a final segment of 1 hour and 15 minutes. This gives a total time of 8 hours and 45 minutes.

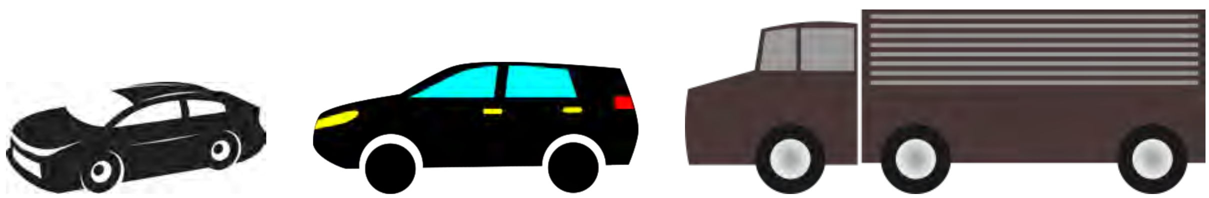
(c) Since the average speed of the Siberian huskies is now only 15 km per hour, they will travel only 30 km in each 2 hour segment. Thus, they will now travel 120 km in four segments of 2 hours and 30 minutes each. That is, they will travel 120 km in 10 hours. They will travel the last 25 km in a final segment of 100 min, or 1 hour and 40 minutes. This gives a total time of 11 hours and 40 minutes. Thus, their time for the day has increased by 2 hours and 55 minutes, or 175 minutes.

# Problem of the Week
## Problem B
## Ingrid's Vehicles

While waiting in a parking lot, Ingrid began counting vehicles. She discovered that there were 22 cars, 16 SUVs, and 3 trucks.

(a) Create a pictograph, a broken line graph, and a bar graph to represent this data.

(b) To best communicate this distribution, which graph do you prefer? Why?



# Problem of the Week

## Problem B and Solution

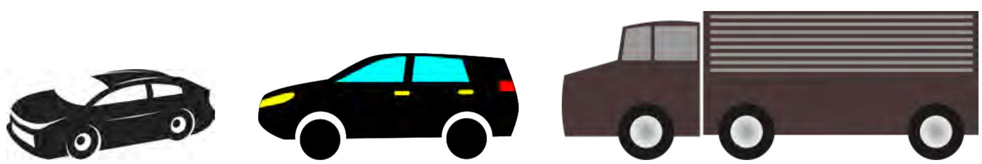
## Ingrid's Vehicles

### Problem

While waiting in a parking lot, Ingrid began counting vehicles. She discovered that there were 22 cars, 16 SUVs, and 3 trucks.

(a) Create a pictograph, a broken line graph, and a bar graph to represent this data.

(b) To best communicate this distribution, which graph do you prefer? Why?



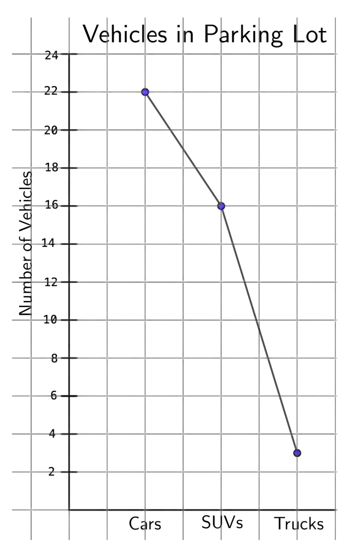
## Solution

(a) A pictograph, a broken line graph, and a bar graph are below.

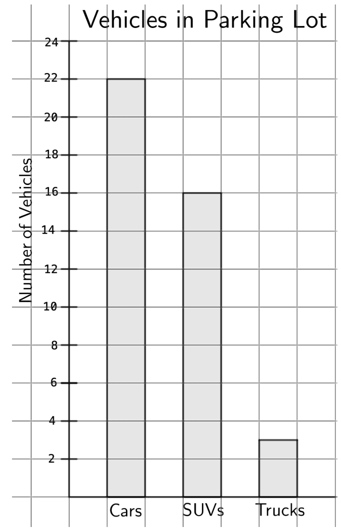
**Pictograph:**


NOTE: The pictograph could be condensed by *unitizing*. For example, each image could be used to represent to two vehicles.

**Broken line graph:**



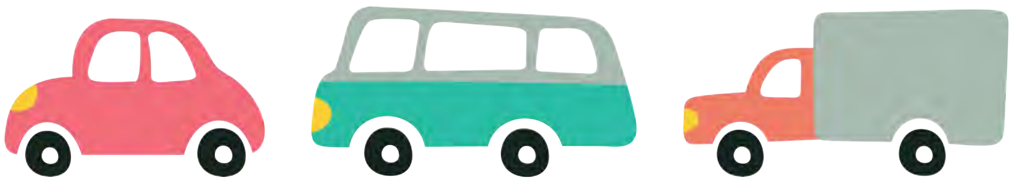
**Bar graph:**




(b) Answers may vary. The bar graph best communicates this distribution as it compares the numbers of vehicles most clearly and concisely.

# Problem of the Week
## Problem B
## Traffic Predictions

Petr was standing at the bus stop during rush hour and started counting the passing vehicles. In the first five minutes he waited, he counted 20 cars, 25 vans and 15 trucks.



(a) Based on Petr's sample data, what is the theoretical probability that the next vehicle will be a truck?

(b) Petr counted vehicles for another five minutes and discovered that the experimental probability of a vehicle being a car was the same for his first and second samples. If Petr counted a total of 84 vehicles in his second sample, how many of those vehicles were cars?

# Problem of the Week
## Problem B and Solution
## Traffic Predictions

### Problem

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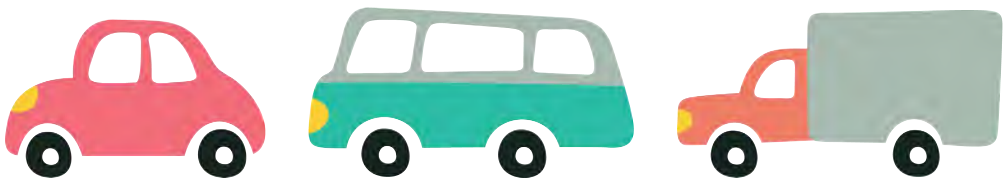


(a) Based on Petr's sample data, what is the theoretical probability that the next vehicle will be a truck?

(b) Petr counted vehicles for another five minutes and discovered that the experimental probability of a vehicle being a car was the same for his first and second samples. If Petr counted a total of 84 vehicles in his second sample, how many of those vehicles were cars?

### Solution

(a) Petr's sample had a total of $20 + 25 + 15 = 60$ vehicles. Since 15 of these were trucks, the theoretical probability that the next vehicle will be a truck is $\frac{15}{60} = \frac{1}{4}$. Notice that the probability is equal to the fraction of trucks in the sample.

(b) Petr's first sample included 20 cars which is $\frac{20}{60} = \frac{1}{3}$ of the vehicles. Thus, the experimental probability of a vehicle in the first sample being a car is $\frac{1}{3}$. If this experimental probability is the same for the second sample, then $\frac{1}{3}$ of the cars in the second sample must have been cars. Since his second sample had a total of 84 vehicles, and $\frac{1}{3}$ of 84 is $\frac{1}{3} \times 84 = 28$, it follows that 28 of the vehicles in the second sample were cars.

Note: You cannot predict the individual numbers of vans or trucks in the second sample, because you don't know the experimental probabilities of a vehicle being a van or a truck for the second sample.

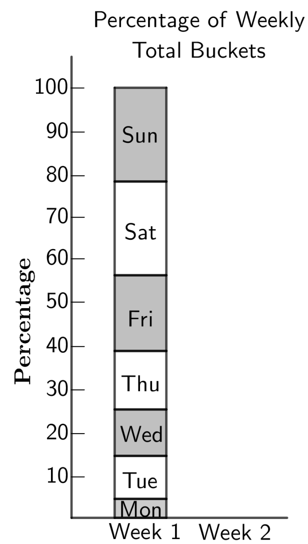
Extension: If Petr had determined that the probability of a vehicle being a car was the same for his first and second samples, would it have been possible for him to have observed 85 vehicles in the second sample?

# Problem of the Week
## Problem B
## Buckets of Golf Balls

Golfers will practice their golf game at a driving range. At a driving range, they hit practice balls by the bucket.

Annie works at a local driving range. Over a period of two weeks, she records the number of buckets of balls that she hands out each day. The table below displays her data.

| Day | Week 1 | Week 2 |
|---|---|---|
| Monday | 11 | 14 |
| Tuesday | 25 | 32 |
| Wednesday | 27 | 34 |
| Thursday | 34 | 37 |
| Friday | 44 | 50 |
| Saturday | 57 | 70 |
| Sunday | 52 | 63 |




(a) A *stacked bar graph* is given for Week 1, showing the percentage of each day's buckets relative to the total (250 buckets) for that week. For example, on Monday Annie gives out 11 buckets, which is $\frac{11}{250} = 4.4\%$ of the total; on Tuesday she gives out 25 buckets, which is $\frac{25}{250} = 10.0\%$ of the total. Verify that the remaining blocks of the graph accurately portray the given data for Week 1 by calculating the remaining daily percentages.

(b) Calculate the daily percentages for Week 2, and create a similar stacked bar graph for Week 2. Round percentages to one decimal place.

(c) By examining the bar graphs, what conclusions could you draw about the number of buckets given out each day?

Themes Data Management, Number Sense

# Problem of the Week

## Problem B and Solution

## Buckets of Golf Balls

### Problem

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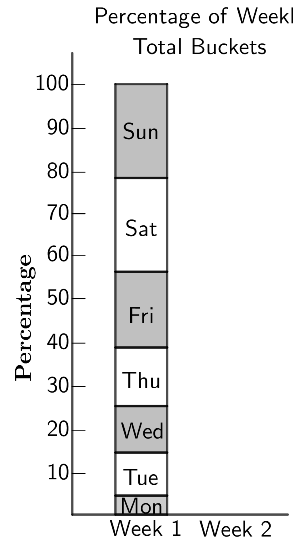


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Verify that the remaining blocks of the graph accurately portray the given data for Week 1 by calculating the remaining daily percentages.

(b) Calculate the daily percentages for Week 2, and create a similar stacked bar graph for Week 2. Round percentages to one decimal place.

(c) By examining the bar graphs, what conclusions could you draw about the number of buckets given out each day?

## Solution

(a) The remaining days' percentages are:
Wednesday: $\frac{27}{250} = 10.8\%$
Thursday: $\frac{34}{250} = 13.6\%$
Friday: $\frac{44}{250} = 17.6\%$
Saturday: $\frac{57}{250} = 22.8\%$
Sunday: $\frac{52}{250} = 20.8\%$

Note: We can find each percentage by rewriting the fraction as an equivalent fraction with a denominator of 100. We will look at the data for Wednesday and show two ways to do this.

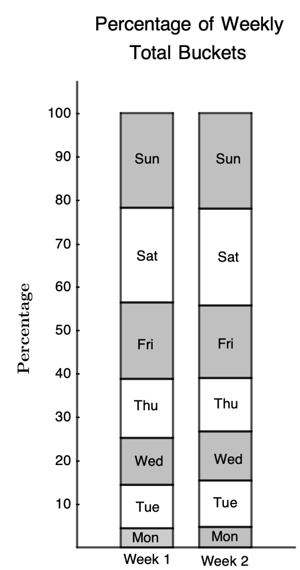
(i) We will get the denominator to be 1000 by multiplying numerator and denominator by 4. Then, we divide each by 10 to get a fraction with a denominator of 100.

$$\frac{27}{250} = \frac{108}{1000} = \frac{10.8}{100} = 10.8\%$$

(ii) Since $250 \div 100 = 2.5$, we can divide both numerator and denominator by 2.5 to get $\frac{10.8}{100} = 10.8\%$.

The heights of the remaining blocks of the graph do portray the given data for Week 1.

(b) During Week 2, Annie handed out a total of 300 buckets. The daily percentages and completed bar graph are below.
Monday: $\frac{14}{300} \approx 4.7\%$
Tuesday: $\frac{32}{300} \approx 10.7\%$
Wednesday: $\frac{34}{300} \approx 11.3\%$
Thursday: $\frac{37}{300} \approx 12.3\%$
Friday: $\frac{50}{300} \approx 16.7\%$
Saturday: $\frac{70}{300} \approx 23.3\%$
Sunday: $\frac{63}{300} = 21.0\%$




(c) The tallest rectangular boxes are for Saturday and Sunday. Therefore, we can say that the most buckets are given out on either Saturday or Sunday. The data in the table shows that it is in fact on Saturday when the most buckets are given out.

The shortest rectangular box is for Monday. Therefore, we can say that the fewest number of buckets are given out on Monday. This is verified by the table.

# Computational Thinking (C)

TAKE ME TO THE COVER