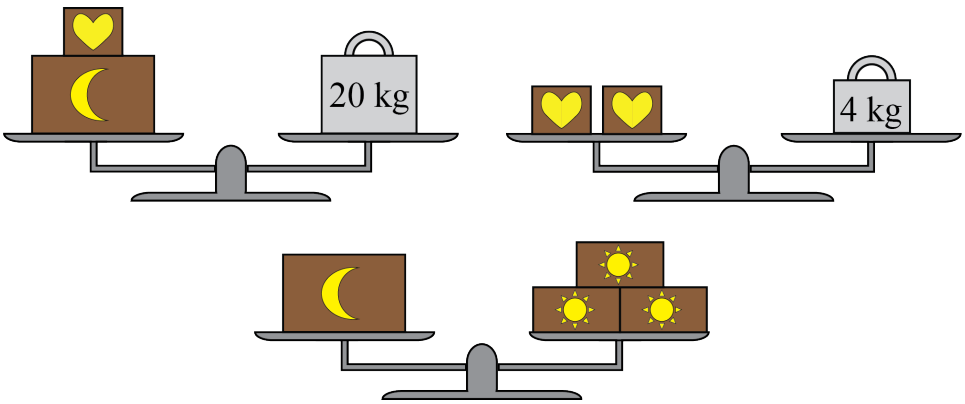
Problem of the Week
Problem A and Solution
Balancing Act

Problem

Robbie is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass.

The following diagrams show what Robbie observed when arranging some of the boxes and standard weights on a scale.



Given that each scale is balanced, determine the mass of each box.

Solution

From the diagrams we notice the following.

- One moon box and one heart box have a total mass of 20 kg.
- Two heart boxes have a mass of 4 kg.
- One moon box has the same total mass as three sun boxes.

Since two heart boxes have a mass of 4 kg, then the mass of one heart box is $\frac{1}{2}$ of 4 kg. Therefore, one heart box has a mass of 2 kg.

Since one heart box and one moon box have a total mass of 20 kg, and one heart box has a mass of 2 kg, then one moon box has a mass of $20 - 2 = 18$ kg.

Since 3 sun boxes have the same total mass as one moon box, then 1 sun box must be $\frac{1}{3}$ the mass of one moon box. Since $\frac{1}{3}$ of 18 is 6, one sun box must have a mass of 6 kg.

Therefore, one heart box has a mass of 2 kg, one moon box has a mass of 18 kg, and one sun box has a mass of 6 kg.

Teacher's Notes

The idea of a balance scale is a nice analogy for an algebraic equation. We can represent the information in the problem using equations with variables to represent the masses of the different types of boxes. Here is one way to solve the problem algebraically.

Let x represent the mass of a heart box, in kg.
Let y represent the mass of a sun box, in kg.
Let z represent the mass of a moon box, in kg.

From the information in the diagrams, we can write the following equations:

$$x + z = 20 \quad (1)$$
$$2x = 4 \quad (2)$$
$$z = 3y \quad (3)$$

We can divide both sides of equation (2) by 2 to get

$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

Now, substituting $x = 2$ into equation (1), we get

$$2 + z = 20$$

Subtracting 2 from each side of this equation, we get

$$z = 18$$

Finally, substituting $z = 18$ into equation (3), we get

$$18 = 3y$$

Dividing both sides of this equation by 3, we get

$$\frac{18}{3} = \frac{3y}{3}$$
$$6 = y$$

Therefore, one heart box has a mass of 2 kg, one moon box has a mass of 18 kg, and one sun box has a mass of 6 kg.