The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

Problem of the Week
Problems and Solutions
2022 - 2023

Problem A (Grade 3/4)

Themes

(Click on a theme name to jump to that section.)

Number Sense (N)

Geometry & Measurement (G)

Algebra (A)

Data Management (D)

Computational Thinking (C)

The problems in this booklet are organized into themes.
A problem often appears in multiple themes.

Number Sense (N)

TAKE ME TO THE COVER

Problem of the Week
Problem A
Power Savings

An electric clothes dryer uses approximately 5 kilowatts of energy to dry a single load of laundry.

On average, Mary Lois's family washes 4 loads of laundry per week. If they use a clothes line instead of the electric dryer to dry half of their loads of laundry, approximately how many kilowatts of energy will they save in a year?

Themes Geometry & Measurement, Number Sense

Problem of the Week
Problem A and Solution
Power Savings

Problem

An electric clothes dryer uses approximately 5 kilowatt-hours of energy to dry a single load of laundry.

On average, Mary Lois's family washes 4 loads of laundry per week. If they use a clothes line instead of the electric dryer to dry half of their loads of laundry, approximately how many kilowatt-hours of energy will they save in a year?

Solution

Since half of 4 is 2, then on average Mary Lois's family uses the clothes line to dry two loads of laundry per week. This means they would save approximately $2 \times 5 = 10$ kilowatt-hours of energy each week. Since there are approximately 52 weeks in a year, we calculate the total savings as $52 \times 10 = 520$ kilowatt-hours of energy.

Problem of the Week
Problem A
Counting Collections

The students in Riverside Public School love to play a game called "Counting Collections". In this game, the students collect objects, and the student who has collected the most objects wins.

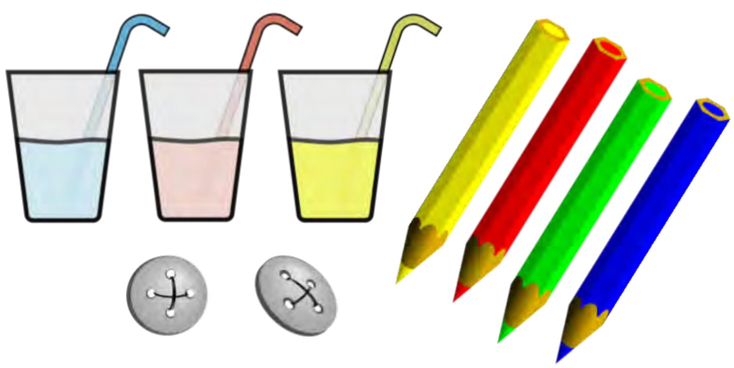
Santosh has collected 262 erasers, 451 buttons and 173 pencils. Alyssa has collected 489 straws and 446 rocks.

To estimate who collected the most objects, they round the total number of each type of object, and then use the rounded numbers to calculate the total for each player. Santosh rounds all the numbers to the nearest 100. Alyssa rounds all the numbers to the nearest 10.

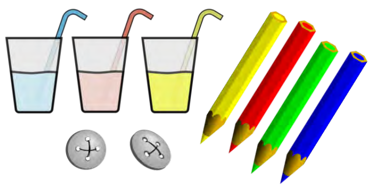
(a) Based on his rounded calculation, who does Santosh think won the game?

(b) Based on her rounded calculation, who does Alyssa think won the game?

(c) Who is correct? Justify your answer.



Theme Number Sense



Problem of the Week
Problem A and Solution
Counting Collections

Problem

The students in Riverside Public School love to play a game called “Counting Collections”. In this game, the students collect objects, and the student who has collected the most objects wins.

Santosh has collected 262 erasers, 451 buttons and 173 pencils. Alyssa has collected 489 straws and 446 rocks.

To estimate who collected the most objects, they round the total number of each type of object, and then use the rounded numbers to calculate the total for each player. Santosh rounds all the numbers to the nearest 100. Alyssa rounds all the numbers to the nearest 10.

(a) Based on his rounded calculation, who does Santosh think won the game?

(b) Based on her rounded calculation, who does Alyssa think won the game?

(c) Who is correct? Justify your answer.

Solution

(a) Rounding to the nearest 100, Santosh collected approximately $300 + 500 + 200 = 1000$ objects and Alyssa collected approximately $500 + 400 = 900$ objects. So Santosh thinks that he is the winner.

(b) Rounding to the nearest 10, Santosh collected approximately $260 + 450 + 170 = 880$ objects and Alyssa collected approximately $490 + 450 = 940$ objects. So Alyssa thinks that she is the winner.

(c) The actual totals are $262 + 451 + 173 = 886$ for Santosh and $489 + 446 = 935$ for Alyssa. So Alyssa collected more objects and is therefore the winner. The estimation when rounding to the nearest 10 is more accurate than when rounding to the nearest 100.

Teacher's Notes

When we use rounding to estimate the results of calculated values it is important to remember that this is an approximation of the actual result. Estimations can be valuable, especially in knowing when an answer is reasonable or unreasonable. However, there is a margin of error when we use rounding.

In this problem, when we see estimated totals that differ by 100 when the numbers were rounded to the nearest 100, we should not assume that we have enough information to make a conclusion about which actual total is greater. Rounding to the nearest 10 gives us a better estimation, but it is still an approximation. However, given that we only have five numbers in our calculations, and the difference between our estimated totals is more than 5×10, we should have more confidence that our conclusion is correct in this case.

Problem of the Week
Problem A
Better Value?

Skyla wants to buy a new outfit. She wants new jeans, a t-shirt, a coat, and a hat. She searches the store flyers and finds the exact same clothing options at three different stores.

- *Noether's Nook* has the jeans for \$59, the t-shirt on sale for \$15, the coat for \$82, and the hat for \$16.
- *Gauss Garb* has the exact same jeans for \$42, the t-shirt for \$27, the coat for \$76, and the hat for \$19.
- *Hypatia's Hidden Gems* has the jeans for \$51, the t-shirt for \$23, the coat for \$94, and the hat on sale for \$14.

Skyla's grandparents want to send her a store gift card for her birthday to cover the exact cost of her outfit. Which store's gift card would you recommend for Skyla's grandparents to purchase?

Problem of the Week
Problem A and Solution
Better Value?

Problem

Skyla wants to buy a new outfit. She wants new jeans, a t-shirt, a coat, and a hat. She searches the store flyers and finds the exact same clothing options at three different stores.

- *Noether's Nook* has the jeans for \$59, the t-shirt on sale for \$15, the coat for \$82, and the hat for \$16.
- *Gauss Garb* has the exact same jeans for \$42, the t-shirt for \$27, the coat for \$76, and the hat for \$19.
- *Hypatia's Hidden Gems* has the jeans for \$51, the t-shirt for \$23, the coat for \$94, and the hat on sale for \$14.

Skyla's grandparents want to send her a store gift card for her birthday to cover the exact cost of her outfit. Which store's gift card would you recommend for Skyla's grandparents to purchase?

Solution

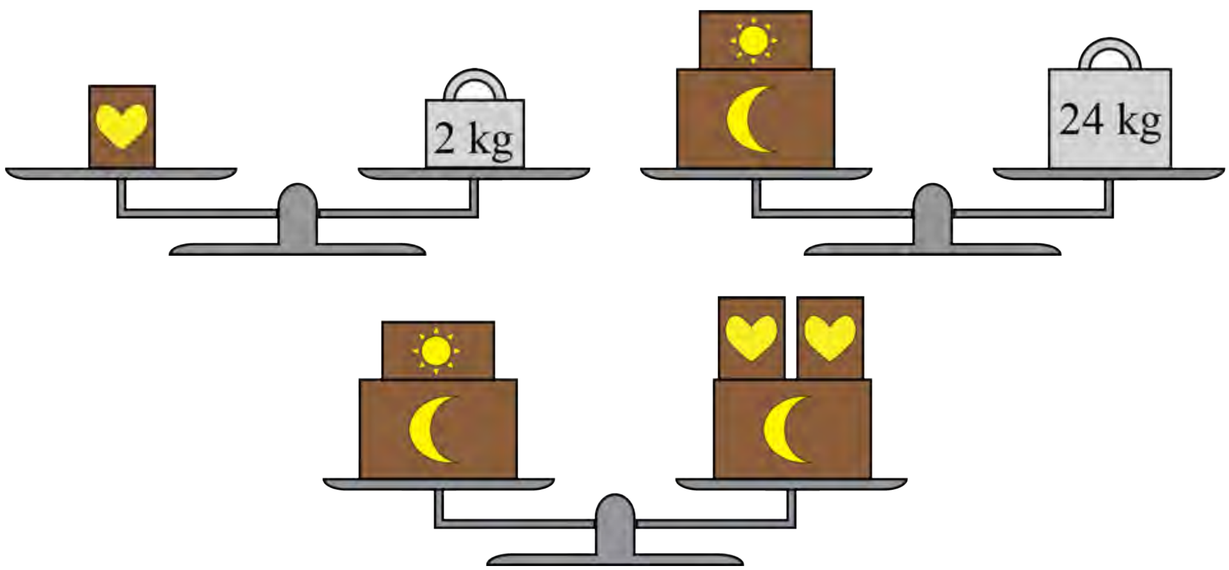
Let's calculate the total cost of the items at each store:

- *Noether's Nook*: $59 + 15 + 82 + 16 = \$172$
- *Gauss Garb*: $42 + 27 + 76 + 19 = \$164$
- *Hypatia's Hidden Gems*: $51 + 23 + 94 + 14 = \$182$

We notice that the least amount they would have to pay is \$164. Therefore, if they want to pay the least amount of money, then the gift card should be purchased from *Gauss Garb*.

Problem of the Week
Problem A
Balancing Act

Bailey is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass, and the cost of sending a box depends on its mass. Bailey has a balance scale and a few standard weights to help with the job. The following diagrams show what Bailey observed when arranging some of the boxes and standard weights on the scales.



Find the mass of each box.

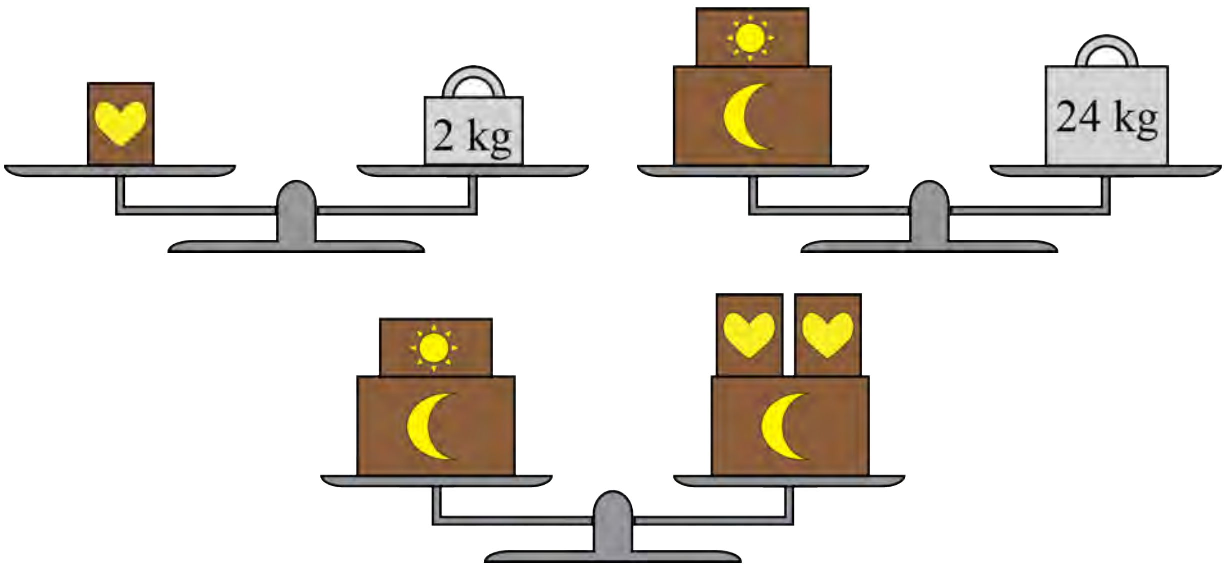
Problem of the Week

Problem A and Solution

Balancing Act

Problem

Bailey is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass, and the cost of sending a box depends on its mass. Bailey has a balance scale and a few standard weights to help with the job. The following diagrams show what Bailey observed when arranging some of the boxes and standard weights on the scales.



Find the mass of each box.

Solution

From the diagrams we notice the following.

- One heart box has a mass of 2 kg.
- One moon box and one sun box have a total mass of 24 kg.
- One moon box and one sun box have the same total mass as one moon box and two heart boxes.

From this, we can conclude that one moon box and two heart boxes have a total mass of 24 kg. Also, two heart boxes have the same mass as one sun box.

Since one heart box has a mass of 2 kg, then two heart boxes have a mass of 4 kg. Therefore, one sun box has a mass of 4 kg.

This means 4 kg + (mass of a moon box) = 24 kg. Since $4 + 20 = 24$, we can determine that one moon box must have a mass of 20 kg.

Teacher's Notes

The idea of a balance scale is a nice analogy for an algebraic equation. We can represent the information in the problem using equations with variables to represent the masses of the different types of boxes. Here is one way to solve the problem algebraically.

Let x represent the mass of a heart box.
Let y represent the mass of a sun box.
Let z represent the mass of a moon box.

From the information in the diagrams, we can write the following equations:

$$x = 2 \tag{1}$$
$$y + z = 24 \tag{2}$$
$$y + z = 2x + z \tag{3}$$

From equation (1), we already know that a heart box has a mass of 2 kg. From equations (2) and (3), we notice that the left sides are the same, so the right sides of the equations must be equal to each other. This means we know:

$$2x + z = 24 \tag{4}$$

Now, substituting $x = 2$ into equation (4), we get

$$2(2) + z = 24$$

Subtracting 4 from each side of this equation, we get

$$z = 20$$

Finally, substituting $z = 20$ into equation (2), we get

$$y + 20 = 24$$

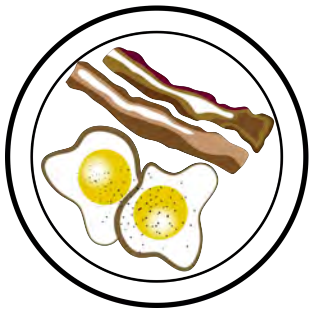
Subtracting 20 from each side of this equation, we get

$$y = 4$$

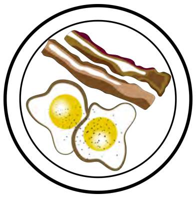
So, a heart box has a mass of 2 kg, a sun box has a mass of 4 kg, and a moon box has a mass of 20 kg.

Problem of the Week
Problem A
Breakfast Supplies

A carton of eggs costs \$3.90 and a package of bacon costs \$7.10. If I went to the store and bought 2 cartons of eggs and 1 package of bacon, how much change would I have left from \$20.00?



Theme Number Sense



Problem of the Week

Problem A and Solution

Breakfast Supplies

Problem

A carton of eggs costs \$3.90 and a package of bacon costs \$7.10. If I went to the store and bought 2 cartons of eggs and 1 package of bacon, how much change would I have left from \$20.00?

Solution

We can start by calculating the total spent on bacon and eggs as $\$3.90 + \$3.90 + \$7.10 = \$14.90$. If we subtract that total from \$20.00, we get $\$20.00 - \$14.90 = \$5.10$. So, we would have \$5.10 left over from our \$20.00.

If we wanted to avoid doing calculations with decimal places, we could change all the amounts to cents. In this case, we multiply all of the dollar amounts by 100. So, the total spent would be $390 + 390 + 710 = 1490$ cents. The amount left over would be $2000 - 1490 = 510$ cents. Then, we can divide the number of cents by 100 to get back to a dollar amount of \$5.10.

Problem of the Week

Problem A

One Room Schoolhouse

The terms *mean, median*, and *mode* are defined at the bottom of the page.

The following list of numbers represents the ages of students in the one-room schoolhouse in Muggleland:

$$7,\ 9,\ 15,\ 9,\ 14,\ 13,\ 14,\ 7,\ 6,\ 10,\ 12,\ 8,\ 15,\ 14,\ 12,$$
$$8,\ 7,\ 12,\ 12,\ 16,\ 14,\ 12,\ 7,\ 9,\ 10,\ 8,\ 12,\ 14,\ 11,\ 13$$

(a) Are the mode and the median of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.

(b) Are the median and the mean of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.

Mode refers to the most frequently occurring number in a data set. If there is a tie, then we assign more than one number as the modes of the data set.

Median refers to the middle number in a data set after the numbers have been arranged in order. If a data set has an even number of values, then there are two "middle numbers". In this case we calculate the sum of the two numbers and divide by 2 to get the median of the data set.

Mean refers to the result of calculating the sum of the numbers in the data set and then dividing the sum by the number of values in the data set. This is what is commonly called the *average*.

Themes Data Management, Number Sense

Problem of the Week
Problem A and Solution
One Room Schoolhouse

Problem

The following list of numbers represents the ages of students in the one-room schoolhouse in Muggleland:

$$7, 9, 15, 9, 14, 13, 14, 7, 6, 10, 12, 8, 15, 14, 12,$$
$$8, 7, 12, 12, 16, 14, 12, 7, 9, 10, 8, 12, 14, 11, 13$$

(a) Are the mode and the median of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.

(b) Are the median and the mean of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.

Mode refers to the most frequently occurring number in a data set. If there is a tie, then we assign more than one number as the modes of the data set.

Median refers to the middle number in a data set after the numbers have been arranged in order. If a data set has an even number of values, then there are two "middle numbers". In this case we calculate the sum of the two numbers and divide by 2 to get the median of the data set.

Mean refers to the result of calculating the sum of the numbers in the data set and then dividing the sum by the number of values in the data set. This is what is commonly called the *average*.

Solution

(a) We can start by filling in a tally chart to count how many times each age appears in our data set.

Age	6	7	8	9	10	11	12	13	14	15	16
Tally	$\vert$	$\vert\vert\vert\vert$	$\vert\vert\vert$	$\vert\vert\vert$	$\vert\vert$	$\vert$	$\cancel{\vert\vert\vert\vert}\,\vert$	$\vert\vert$	$\cancel{\vert\vert\vert\vert}$	$\vert\vert$	$\vert$

The most frequently occurring value is 12, so the mode is 12.

We can use the tally chart to list the values in order:

$$6, 7, 7, 7, 7, 8, 8, 8, 9, 9, 9, 10, 10, 11, 12,$$
$$12, 12, 12, 12, 12, 13, 13, 14, 14, 14, 14, 14, 15, 15, 16$$

Since there is an even number of ages in this data set, we need to calculate the median using the two middle numbers. In this case, we calculate the sum of the 15th and 16th numbers to get $12 + 12 = 24$ and then divide by 2 to get $24 \div 2 = 12$.

So, the median of this set of numbers is 12.

For this data set, the mode and the median are the same. However we would not expect that to always be the case. For example, in the data set

$$1, 1, 1, 1, 2, 3, 4, 5, 6$$

the mode is 1, but the median is 2.

(b) To determine the mean, we could add up the 30 numbers in the data set to find the sum, but another way to calculate this total would be to use our tally chart again. For each age, we calculate the product of the age and the tally:

Age	6	7	8	9	10	11	12	13	14	15	16
Tally	1	4	3	3	2	1	6	2	5	2	1
Product	6	28	24	27	20	11	72	26	70	30	16

Now, instead of adding 30 numbers together, we only need to find the sum of the 11 products to determine the total to be

$$6 + 28 + 24 + 27 + 20 + 11 + 72 + 26 + 70 + 30 + 16 = 330$$

Now, to find the mean we can use skip counting or divide by the number of ages, which is 30, to get $330 \div 30 = 11$.

So, the mean is 11.

For this data set, the mean and the median are different. However, we would not expect that to always be the case. For example, in the data set

$$5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5$$

the mean is 5 and the median is also 5.

Problem of the Week

Problem A

What Number Am I?

I am a 3-digit number.

All of my digits are even numbers greater than 1.

My hundreds digit is greater than my tens digit.

My tens digit minus my ones digit is twice as much as my hundreds digit minus my tens digit.

My tens digit is a multiple of 3.

What number am I?

Themes Computational Thinking, Number Sense

Problem of the Week
Problem A and Solution
What Number Am I?

Problem

I am a 3-digit number.

All of my digits are even numbers greater than 1.

My hundreds digit is greater than my tens digit.

My tens digit minus my ones digit is twice as much as my hundreds digit minus my tens digit.

My tens digit is a multiple of 3.

What number am I?

Solution

First, we note that the even digits that are greater than 1 are 2, 4, 6, and 8. Thus, the only possible digits in the number are 2, 4, 6, and 8.

Since the tens digit is also a multiple of 3, the tens digit must be 6. This is the only number in the list above that is a multiple of 3.

Since the hundreds digit is greater than the tens digit, the hundreds digit must be 8. This is the only number in the list above that is greater than 6.

The difference between the hundreds digit and the tens digit is $8 - 6 = 2$. Twice that difference is $2 \times 2 = 4$. Now, we need a number such that when we subtract it from 6, we get 4. Since $6 - 2 = 4$, the ones digit must be 2.

Therefore, the number is 862.

Problem of the Week

Problem A

Snow Days

Tapeesa monitored the amount of snowfall each day, Monday through Sunday, for four weeks. Here is what she recorded:

- The first week it snowed 5 mm each day.
- The second week snow only fell on two days: 8 cm of snow fell on Tuesday and 2 cm of snow fell on Friday.
- It did not snow the third week at all.
- A total of half a metre of snow fell during the last week.

What was the total amount of snowfall over the four weeks?

Themes Geometry & Measurement, Number Sense

Problem of the Week
Problem A and Solution
Snow Days

Problem

Tapeesa monitored the amount of snowfall each day, Monday through Sunday, for four weeks. Here is what she recorded:

- The first week it snowed 5 mm each day.
- The second week snow only fell on two days: 8 cm of snow fell on Tuesday and 2 cm of snow fell on Friday.
- It did not snow the third week at all.
- A total of half a metre of snow fell during the last week.

What was the total amount of snowfall over the four weeks?

Solution

To find the total, we should have all the measurements with the same unit. Let's consider all of the snowfall amounts in centimetres.

The first week it snowed $7 \times 5 = 35$ mm in total. This is equal to 3.5 cm.

The second week it snowed a total of $8 + 2 = 10$ cm.

Since 0.5 m is equal to 50 cm, that is how much it snowed during the fourth week.

Therefore, the total snowfall for the four weeks was $3.5 + 10 + 50 = 63.5$ cm.

Alternatively, we could calculate the amounts in millimetres.

In the second week, it snowed a total of $10 \times 8 = 80$ mm and $10 \times 2 = 20$ mm, for a total of $80 + 20 = 100$ mm. Since 1 m is equal to 1000 mm, then in the last week it snowed half of 1000 mm, or 500 mm. Therefore, the total amount of snowfall for the four weeks was $35 + 100 + 500 = 635$ mm, which is equal to 63.5 cm.

Problem of the Week

Problem A

The Pencil Case

When trying to solve a mystery involving some missing pencils, you discover the following riddle about the number of pencils that were in the box originally.

> There are fewer than forty pencils in the box.
>
> If you remove four pencils at a time, eventually there will be 2 pencils left.
>
> If you remove three pencils at a time, eventually there will be 0 pencils left.
>
> If you remove five pencils at a time, eventually there will be 0 pencils left.

How many pencils were in the box originally?

Problem of the Week
Problem A and Solution
The Pencil Case

Problem

When trying to solve a mystery involving some missing pencils, you discover the following riddle about the number of pencils that were in the box originally.

> There are fewer than forty pencils in the box.
>
> If you remove four pencils at a time, eventually there will be 2 pencils left.
>
> If you remove three pencils at a time, eventually there will be 0 pencils left.
>
> If you remove five pencils at a time, eventually there will be 0 pencils left.

How many pencils were in the box originally?

Solution

Since there would be 2 pencils left if we remove 4 pencils at a time, it cannot be the case that there are 0 pencils in the box.

From this point, one way to solve this problem is to start with the last clue. From this information, we know that the number of pencils in the box is a multiple of 5. The multiples of 5 that are greater than 0 and less than 40 are:

$$5,\ 10,\ 15,\ 20,\ 25,\ 30,\ 35$$

From the second last clue, we know that the number of pencils in the box is a multiple of 3. The multiples of 3 that are greater than 0 and less than 40 are:

$$3,\ 6,\ 9,\ 12,\ 15,\ 18,\ 21,\ 24,\ 27,\ 30,\ 33,\ 36,\ 39$$

The only two numbers that are in both lists are 15 and 30. Now we can count down by 4 from each to see which one ends up at 2.

Starting at 15 we get this sequence: $15, 11, 7, 3$

Starting at 30 we get this sequence: $30, 26, 22, 18, 14, 10, 6, 2$

We see that there must have been 30 pencils in the box to start, since this is a multiple of 5 and a multiple of 3 and will have 2 pencils left in the box if we remove 4 at a time.

Teacher's Notes

The solution provided to this problem essentially uses trial and error to find the answer. We should notice that if we were not given a maximum number of pencils in the box, then there are theoretically an infinite number of answers to this problem. For example if there were 90 pencils in the box, the statements about removing pencils would all be true as well.

If you were to ask a mathematician this question they might solve the problem without using trial and error. They could use *congruence notation* to represent the information and then solve the problem algebraically.

The statement "If you remove four pencils at a time, eventually there will be 2 pencils left" could be restated as "If you divide the number of pencils in the box by 4, the remainder is 2". If we know congruence notation, we can restate this information as

$$p \equiv 2 \mod 4$$

assuming that p is the number of pencils in the box. When we read this statement we say that "p is *congruent* to 2, *modulo* 4".

The other information from the problem could be restated as

$$p \equiv 0 \mod 3$$

and

$$p \equiv 0 \mod 5$$

Then we can solve this problem algebraically by converting these congruences to linear equations.

Problem of the Week

Problem A

Saving Up

Fionnlay earns \$8 each week by walking the neighbour's dog. Starting in the second week, Fionnlay earns an extra \$3 by helping maintain the community garden every other week. He is saving all of his earnings for a bicycle, which costs \$157. He will also need a helmet which costs \$24, a bike horn which costs \$16, and a bike light which costs \$9. All prices include taxes. Fionnlay keeps track of the money he has earned.

(a) How much money will Fionnlay have saved after 8 weeks?

(b) How much money will Fionnlay have saved after 15 weeks?

(c) How long will it take him to save enough money to buy the bike and its accessories?

Themes Algebra, Number Sense

Problem of the Week
Problem A and Solution
Saving Up

Problem

Fionnlay earns \$8 each week by walking the neighbour's dog. Starting in the second week, Fionnlay earns an extra \$3 by helping maintain the community garden every other week. He is saving all of his earnings for a bicycle, which costs \$157. He will also need a helmet which costs \$24, a bike horn which costs \$16, and a bike light which costs \$9. All prices include taxes. Fionnlay keeps track of the money he has earned.

(a) How much money will Fionnlay have saved after 8 weeks?

(b) How much money will Fionnlay have saved after 15 weeks?

(c) How long will it take him to save enough money to buy the bike and its accessories?

Solution

One way to solve this problem is to create a table that records the total that Fionnlay has saved each week. In the odd-numbered weeks, the total will increase by \$8. In the even-numbered weeks, the total will increase by $\$8 + \$3 = \$11$.

We will also need to know how much money Fionnlay needs to save by calculating the total cost of the bike and its accessories. We can do this by adding $\$157 + \$24 + \$16 + \$9 = \$206$.

Using the table, we can now answer the questions.

Week	Increase (in \$)	Total Savings (in \$)
1	8	8
2	11	19
3	8	27
4	11	38
5	8	46
6	11	57
7	8	65
8	11	76
9	8	84
10	11	95
11	8	103
12	11	114
13	8	122
14	11	133
15	8	141
16	11	152
17	8	160
18	11	171
19	8	179
20	11	190
21	8	198
22	11	209

(a) Fionnlay will have saved \$76 after 8 weeks.

(b) Fionnlay will have saved \$141 after 15 weeks.

(c) Fionnlay will need 22 weeks to save enough money for the bike and its accessories.

Problem of the Week
Problem A
Old Faithful

Old Faithful is a geyser in Yellowstone National Park. It is so named because it was believed that it erupted every 60 to 90 minutes all day long. Assuming that Old Faithful erupts at 12 midnight and then erupts every 60 to 90 minutes after the last eruption, answer the following questions.

(a) After the first eruption at 12 midnight, what is the minimum number of eruptions you could see until up to and including 12 midnight the next night?

(b) After the first eruption at 12 midnight, what is the maximum number of eruptions you could see until up to and including 12 midnight the next night?

Themes Algebra, Number Sense

Problem of the Week
Problem A and Solution
Old Faithful

Problem

Old Faithful is a geyser in Yellowstone National Park. It is so named because it was believed that it erupted every 60 to 90 minutes all day long. Assuming that Old Faithful erupts at 12 midnight and then erupts every 60 to 90 minutes after the last eruption, answer the following questions.

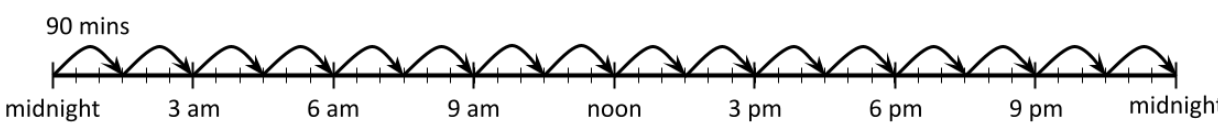
(a) After the first eruption at 12 midnight, what is the minimum number of eruptions you could see until up to and including 12 midnight the next night?

(b) After the first eruption at 12 midnight, what is the maximum number of eruptions you could see until up to and including 12 midnight the next night?

Solution

(a) One way to solve this problem is to make a timeline.

We would see fewer eruptions if the time between eruptions is longer. The longest gap is 90 minutes. Therefore, the fewest number of eruptions will occur if the time between each eruption is 90 minutes.



From this timeline we can count the ending point of each of the arrows in the diagram and see that, after the first eruption at 12 midnight, there would be 16 eruptions if they happened every 90 minutes. Note that the last eruption would be at exactly midnight on the next night.

Alternatively, we might notice that 90 minutes is equal to $1\frac{1}{2}$ hours, and 180 minutes (or two geyser eruption intervals) is equal to 3 hours. Then we can make a table keeping track of how many eruptions take place over time.

Eruptions	2	4	6	8	10	12	14	16
Hours Elapsed	3	6	9	12	15	18	21	24

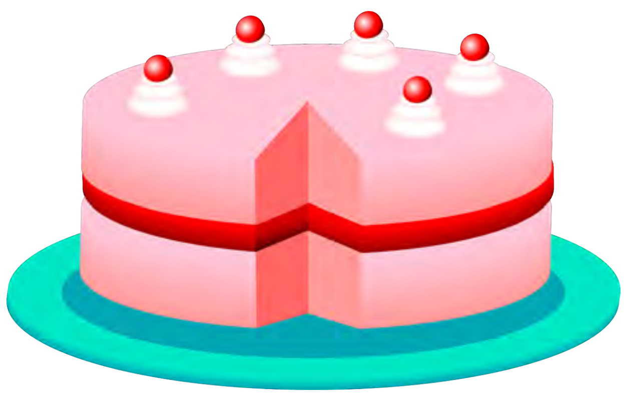
(b) We would see more eruptions if the time between eruptions is shorter. The shortest amount of time between eruptions is 60 minutes. Since 60 minutes is equal to 1 hour, and there are 24 hours from 12 midnight until 12 midnight the next night, then the maximum number of eruptions we could see is 24 eruptions.

Problem of the Week
Problem A
Colossal Cake

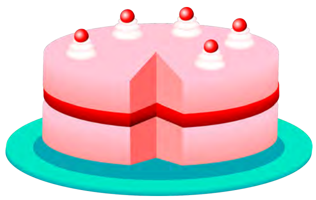
Jose's 12th birthday is coming up and his family is having all of their friends and neighbours over for the party. They ordered a colossal cake from the bakery for the celebration. The mass of the cake, including the icing and the tray, is 5 kg. The tray has a mass of 400 grams.

(a) What is the mass of the cake and the icing without the tray?

(b) If the icing has a mass of 1350 grams, what is the mass of the cake without the icing or the tray?



Themes Geometry & Measurement, Number Sense



Problem of the Week

Problem A and Solution

Colossal Cake

Problem

Jose's 12th birthday is coming up and his family is having all of their friends and neighbours over for the party. They ordered a colossal cake from the bakery for the celebration. The mass of the cake, including the icing and the tray, is 5 kg. The tray has a mass of 400 grams.

(a) What is the mass of the cake and the icing without the tray?

(b) If the icing has a mass of 1350 grams, what is the mass of the cake without the icing or the tray?

Solution

To solve this problem, it would be easier if all of the amounts were described with the same units. Since the mass of the tray and the icing are in grams, we could convert the total mass to grams as well. Since 1 kg is equal to 1000 grams, the total mass is $5 \times 1000 = 5000$ grams.

(a) The mass of the cake and the icing without the tray is $5000 - 400 = 4600$ grams.

(b) The mass of the cake without the icing or the tray is $4600 - 1350 = 3250$ grams.

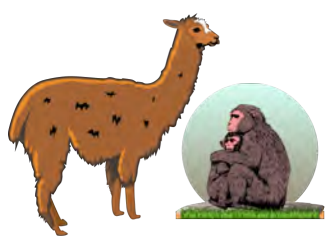
Problem of the Week

Problem A

Zoo Field Trip

The following list gives the fees at the POTW Children's Zoo.

Yesterday the POTW Children's Zoo had 24 visitors. One quarter of the visitors went to pet the llamas, one half of the visitors used the zip line, and 5 visitors had the zoo lunch. How much money did the POTW Children's Zoo make yesterday?



Problem of the Week
Problem A and Solution
Zoo Field Trip

Problem
The following list gives the fees at the POTW Children's Zoo.

Yesterday the POTW Children's Zoo had 24 visitors. One quarter of the visitors went to pet the llamas, one half of the visitors used the zip line, and 5 visitors had the zoo lunch. How much money did the POTW Children's Zoo make yesterday?

Solution
We can determine the amount of money that the POTW Children's Zoo made yesterday by adding the total amounts for each type of fee.

Entry fees would be $24 \times \$8 = \$192$.

Since one quarter of 24 is $24 \div 4 = 6$, then the extra fees for petting the llamas would total $6 \times \$5 = \$30$.

Since one half of 24 is $24 \div 2 = 12$, then the extra fees for the zip line would total $12 \times \$4 = \$48$.

The money made for the 5 lunches would total $5 \times \$7 = \$35$.

Altogether, the POTW Children's Zoo made $\$192 + \$30 + \$48 + \$35 = \$305$ yesterday.

Problem of the Week
Problem A
A Hop, Skip, and a Jump!

Alexis and her friends Mikai, Sophia, and Casper enter a team competition that involves hopping, skipping, biking, and rollerblading for a total of 3 km. Each team member picks an activity and must take turns completing a section of the course by doing their activity.

Alexis will hop for 100 m, then Mikai will bike the next 150 m, followed by Sophia who will rollerblade for the next 200 m, and finally Casper will skip the next 50 m. They will repeat their activity until the team completes the 3 km race.

(a) How many times does each member have to do their activity to complete the race?

(b) What fraction of the race does each team member complete? Put the fractions in order from least to greatest.

Problem of the Week

Problem A and Solution

A Hop, Skip, and a Jump!

Problem

Alexis and her friends Mikai, Sophia, and Casper enter a team competition that involves hopping, skipping, biking, and rollerblading for a total of 3 km. Each team member picks an activity and must take turns completing a section of the course by doing their activity.

Alexis will hop for 100 m, then Mikai will bike the next 150 m, followed by Sophia who will rollerblade for the next 200 m, and finally Casper will skip the next 50 m. They will repeat their activity until the team completes the 3 km race.

(a) How many times does each member have to do their activity to complete the race?

(b) What fraction of the race does each team member complete? Put the fractions in order from least to greatest.

Solution

(a) The total distance travelled after each participant has completed the activity once is $100 + 150 + 200 + 50 = 500$ m. We know that 3 km is equal to $3 \times 1000 = 3000$ m. We can use skip counting by 500 to see how many times they have to complete the cycle of four activities to travel 3000 m: $500,\ 1000,\ 1500,\ 2000,\ 2500,\ 3000$. This means that each team member would need to do their activity 6 times to complete the race.

(b) One way to determine the fractional amounts is to focus on the distance of one cycle of the race, which is 500 m. Then we can take each distance the individuals travelled as the numerator of the fraction and 500 as the denominator of the fraction.

The fractions are: hopping $\frac{100}{500}$, biking $\frac{150}{500}$, rollerblading $\frac{200}{500}$, and skipping $\frac{50}{500}$.

Alternatively, we can determine the fractional amounts using the entire 3000 m. Then we would have found that the fractions are: hopping $\frac{600}{3000}$, biking $\frac{900}{3000}$, rollerblading $\frac{1200}{3000}$, and skipping $\frac{300}{3000}$.

We could also reduce each of these fractions by dividing the numerator and denominator by the same number to get the following fractions: hopping: $\frac{600}{3000} = \frac{100}{500} = \frac{1}{5}$, biking: $\frac{900}{3000} = \frac{150}{500} = \frac{3}{10}$, rollerblading: $\frac{1200}{3000} = \frac{200}{500} = \frac{2}{5}$, and skipping: $\frac{300}{3000} = \frac{50}{500} = \frac{1}{10}$.

We can compare the fractions in many different ways to determine the relative sizes. It is easy to compare fractions with the same denominator, which we have with the fractions with 500 or 3000 as the denominators.

Arranging these in order from least to greatest we get:

skipping	hopping	biking	rollerblading
$\frac{300}{3000} = \frac{50}{500} = \frac{1}{10}$,	$\frac{600}{3000} = \frac{100}{500} = \frac{1}{5}$,	$\frac{900}{3000} = \frac{150}{500} = \frac{3}{10}$,	$\frac{1200}{3000} = \frac{200}{500} = \frac{2}{5}$.

Problem of the Week

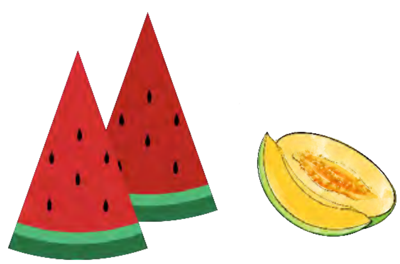
Problem A

How Many Plants?

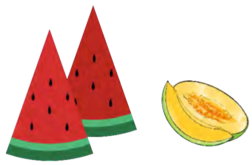
Last year, POTW Farms grew watermelons. They used two different sizes of garden plots: single garden plots and double garden plots. In a single plot they grew one watermelon plant, and in a double plot they grew two watermelon plants. This year, they are planning to grow cantaloupes instead of watermelons. In each single plot they will grow five cantaloupe plants and in each double plot they will grow ten cantaloupe plants. The farm has 14 single plots and 26 double plots.

(a) How many watermelon plants did the farm grow last year?

(b) How many cantaloupe plants are they expecting to grow this year?



Themes Algebra, Number Sense



Problem of the Week

Problem A and Solution

How Many Plants?

Problem

Last year, POTW Farms grew watermelons. They used two different sizes of garden plots: single garden plots and double garden plots. In a single plot they grew one watermelon plant, and in a double plot they grew two watermelon plants. This year, they are planning to grow cantaloupes instead of watermelons. In each single plot they will grow five cantaloupe plants and in each double plot they will grow ten cantaloupe plants. The farm has 14 single plots and 26 double plots.

(a) How many watermelon plants did the farm grow last year?

(b) How many cantaloupe plants are they expecting to grow this year?

Solution

(a) Since each double plot contained two watermelon plants, there were $2 \times 26 = 52$ watermelon plants in the double plots. This means there were a total of $14 + 52 = 66$ watermelon plants.

(b) Since they expect to grow 5 cantaloupe plants in each single plot, they expect to grow $5 \times 14 = 70$ cantaloupe plants in the single plots. Since they expect to grow 10 cantaloupe plants in each double plot, they expect to grow $10 \times 26 = 260$ cantaloupe plants in the double plots. This means POTW Farms is expecting to grow $70 + 260 = 330$ cantaloupe plants.

Problem of the Week

Problem A

Family Facts

Myra has a school project to describe her family. As part of this, she wants to find out the ages of all the people in her family.

Myra knows that the following facts are true today.

Myra is 9 years old.

Myra's sister Vivienne is 3 years older than her.

Myra was 2 years old when her baby brother Jacob was born and today is Jacob's birthday.

Myra's mother Aven is 36 years old and her father Pierce is 5 years older than her mother.

Myra's grandmother Teagan was 55 years old when Jacob was born.

Myra's grandfather John was 49 when Vivienne was born and they have the same birthday.

What are the ages of everyone in Myra's family today?

Problem of the Week

Problem A and Solution

Family Facts

Problem

Myra has a school project to describe her family. As part of this, she wants to find out the ages of all the people in her family.

Myra knows that the following facts are true today.

Myra is 9 years old.

Myra's sister Vivienne is 3 years older than her.

Myra was 2 years old when her baby brother Jacob was born and today is Jacob's birthday.

Myra's mother Aven is 36 years old and her father Pierce is 5 years older than her mother.

Myra's grandmother Teagan was 55 years old when Jacob was born.

Myra's grandfather John was 49 when Vivienne was born and they have the same birthday.

What are the ages of everyone in Myra's family today?

Solution

Since Myra is 9 years old and Vivienne is 3 years older, then Vivienne is $9 + 3 = 12$ years old today.

Since Myra was 2 years old when Jacob was born and today is Jacob's birthday, then Jacob is $9 - 2 = 7$ years old today.

Since Aven is 36 years old and Pierce is 5 years older, then Pierce is $36 + 5 = 41$ years old today.

Since Teagan was 55 years old when Jacob was born and we know that was 7 years ago, then Teagan is $55 + 7 = 62$ years old today.

Since John was 49 years old when Vivienne was born on his birthday and Vivienne is 12 years old today, then John is $49 + 12 = 61$ years old today.

In summary, the ages of everyone in Myra's family today are as follows.

- Myra's brother Jacob is 7 years old.
- Myra is 9 years old.
- Myra's sister Vivienne is 12 years old.
- Myra's mother Aven is 36 years old.
- Myra's father Pierce is 41 years old.
- Myra's grandfather John is 61 years old.
- Myra's grandmother Teagan is 62 years old.

Teacher's Notes

In math word problems we often see some extraneous information that is used to make the text more interesting. This means that we need to determine what facts are necessary to solve the problem. In this problem there are some details that may seem unnecessary, but they are actually critically important to calculating the definitive answer.

There would be more than one possible answer to this problem if we did not know that it was Jacob's birthday today. Suppose we did not know that today was Jacob's birthday. Knowing that Myra was 2 years old when Jacob was born is not be enough information to know how old Jacob is on other days of the year. For example, if her birthday was one day after Jacob's and today was Myra's birthday, then Myra's age today would actually be 3 years more than Jacob's age.

When we are solving problems, we want them to be *deterministic*. This means there is a single, correct solution and we can be confident that we have the right answer. Unfortunately, real life problems do not always fall into this category.

Problem of the Week

Problem A

Fun With Fudge

At the end of the division unit, Mr. Chocolate baked two special batches of fudge for his school. One batch was double chocolate chunk and the other was maple. He cut each batch into equally sized pieces. There were 150 pieces of double chocolate chunk and 284 pieces of maple.

Each class in the school could select one type of fudge. Four classes selected the double chocolate chunk, and eight classes selected the maple. Mr. Chocolate divided up the batches of each type of fudge equally to distribute to the classes.

Did each class get the same amount of fudge?
Were there any pieces of fudge left over?
Justify your answers.

Problem of the Week
Problem A and Solution
Fun With Fudge

Problem

At the end of the division unit, Mr. Chocolate baked two special batches of fudge for his school. One batch was double chocolate chunk and the other was maple. He cut each batch into equally sized pieces. There were 150 pieces of double chocolate chunk and 284 pieces of maple.

Each class in the school could select one type of fudge. Four classes selected the double chocolate chunk, and eight classes selected the maple. Mr. Chocolate divided up the batches of each type of fudge equally to distribute to the classes.

Did each class get the same amount of fudge?
Were there any pieces of fudge left over?
Justify your answers.

Solution

Since there were 4 classes that selected double chocolate chunk, we can divide the number of pieces by 4. This is $150 \div 4 = 37$ with a remainder of 2. So, each of these classes gets 37 pieces of double chocolate chunk fudge and there are 2 pieces left over.

Since there were 8 classes that selected maple, we can divide the number of pieces by 8. This is $284 \div 8 = 35$ with a remainder of 4. So, each of these classes gets 35 pieces of maple fudge and there are 4 pieces left over.

So, the classes that chose double chocolate chunk fudge got more pieces than the classes that chose maple fudge.

Problem of the Week
Problem A
Ski Time

Graham and Olga are going skiing. A gondola takes them from the bottom of a mountain to the top, and then they ski back down. It takes them 20 minutes to ride the gondola to the top of the mountain, and 15 minutes to ski back down.

They begin their first gondola ride up the mountain at 9:30 a.m. and plan to meet their parents at the bottom of the mountain at 12:30 p.m. for lunch. What is the maximum number of ski runs they can do before meeting their parents for lunch, assuming they don't have to wait in line for the gondola?

Themes Algebra, Number Sense

Problem of the Week

Problem A and Solution

Ski Time

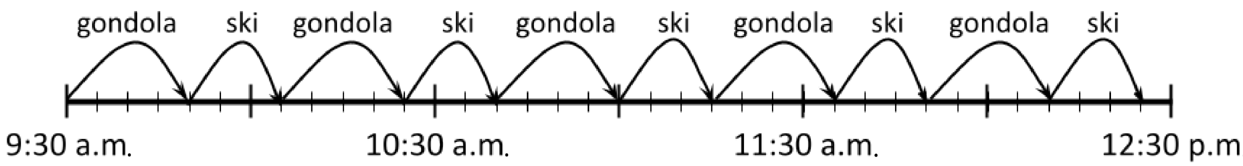
Problem

Graham and Olga are going skiing. A gondola takes them from the bottom of a mountain to the top, and then they ski back down. It takes them 20 minutes to ride the gondola to the top of the mountain, and 15 minutes to ski back down.

They begin their first gondola ride up the mountain at 9:30 a.m. and plan to meet their parents at the bottom of the mountain at 12:30 p.m. for lunch. What is the maximum number of ski runs they can do before meeting their parents for lunch, assuming they don't have to wait in line for the gondola?

Solution

One way to solve this problem would be to use a timeline divided into 5 minute intervals to keep track of how long Graham and Olga have been skiing.



The timeline shows that the maximum number of ski runs that Graham and Olga can do between 9:30 a.m. and 12:30 p.m. is five.

Another way we could solve this problem is to calculate the number of minutes between the start time and lunch time. There are 3 hours between 9:30 a.m. and 12:30 p.m. Since each hour is 60 minutes, this is a total of $60 \times 3 = 180$ minutes. The total time it takes to ride the gondola and then ski back down the mountain once is $20 + 15 = 35$ minutes. Now we could use skip counting to figure out how many ski runs Graham and Olga can do within 180 minutes.

$$35,\ 70,\ 105,\ 140,\ 175,\ 210$$

Therefore, the maximum number of ski runs that Graham and Olga can do within 180 minutes is five.

Problem of the Week

Problem A

Baking Cakes

Todd has a great recipe for a cake. Here is a list of the ingredients and amounts needed to bake one cake:

1 cup white sugar
$\frac{1}{2}$ cup butter
2 eggs
2 teaspoons vanilla extract
$1\frac{1}{2}$ cups all-purpose flour
2 teaspoons baking powder
$\frac{1}{2}$ cup milk

Todd would like to bake four cakes for his classmates. He already has the following ingredients and amounts at home:

6 cups of white sugar
3 cups of butter
8 eggs
9 teaspoons of vanilla extract
5 cups of all-purpose flour
8 teaspoons of baking powder
2 cups of milk

Does he have enough of each ingredient to make the four cakes? If not, which ingredient(s) does he not have enough of? How much more does he need?

Theme Number Sense

Problem of the Week
Problem A and Solution
Baking Cakes

Problem

Todd has a great recipe for a cake. Here is a list of the ingredients and amounts needed to bake one cake:

1 cup white sugar
$\frac{1}{2}$ cup butter
2 eggs
2 teaspoons vanilla extract
$1\frac{1}{2}$ cups all-purpose flour
2 teaspoons baking powder
$\frac{1}{2}$ cup milk

Todd would like to bake four cakes for his classmates. He already has the following ingredients and amounts at home:

6 cups of white sugar
3 cups of butter
8 eggs
9 teaspoons of vanilla extract
5 cups of all-purpose flour
8 teaspoons of baking powder
2 cups of milk

Does he have enough of each ingredient to make the four cakes? If not, which ingredient(s) does he not have enough of? How much more does he need?

Solution

First we can determine how much of each ingredient Todd needs for four cakes. For each ingredient, we can either add the amount needed for one cake to itself four times, or multiply it by 4. The calculations are shown in the following table.

Ingredient	**Amount Needed for One Cake**	**Calculations**	**Amount Needed for Four Cakes**
white sugar	1 cup	1×4	4 cups
butter	$\frac{1}{2}$ cup	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$	2 cups
eggs	2 eggs	2×4	8 eggs
vanilla extract	2 teaspoons	2×4	8 teaspoons
all-purpose flour	$1\frac{1}{2}$ cups	$1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2} = 4+2$	6 cups
baking powder	2 teaspoons	2×4	8 teaspoons
milk	$\frac{1}{2}$ cup	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$	2 cups

When comparing the amounts needed to make four cakes with the amounts Todd has already, we can see that he has enough of all the ingredients except all-purpose flour. Todd needs 6 cups of flour to make four cakes, but he has only 5 cups. Therefore, he needs $6 - 5 = 1$ cup more of all-purpose flour to have enough ingredients to make four cakes.

Problem of the Week
Problem A
Counting Coins

The Coyne family keeps a jar on the counter to hold spare change. At the end of each week, everyone in the family puts any coins they have in the jar. At the end of each month they empty the jar and donate the money to a local charity.

At the end of last month the Coyne family counted the coins in their jar and noticed the following totals:

- There were twice as many toonies as quarters.
- There were 3 times as many loonies as toonies.
- There were 5 more dimes than toonies.
- There were 4 fewer nickels than quarters.
- There were 8 quarters in the jar.

(a) How *many coins* were in the jar at the end of last month?

(b) How *much money* was in the jar at the end of last month?

Note: In Canada, a nickel is worth 5 cents, a dime is worth 10 cents, a quarter is worth 25 cents, a loonie is worth 100 cents, and a toonie is worth 200 cents. Also, 100 cents is equal to 1 dollar ($1).

Theme Number Sense

Problem of the Week

Problem A and Solution

Counting Coins

Problem

The Coyne family keeps a jar on the counter to hold spare change. At the end of each week, everyone in the family puts any coins they have in the jar. At the end of each month they empty the jar and donate the money to a local charity.

At the end of last month the Coyne family counted the coins in their jar and noticed the following totals:

- There were twice as many toonies as quarters.
- There were 3 times as many loonies as toonies.
- There were 5 more dimes than toonies.
- There were 4 fewer nickels than quarters.
- There were 8 quarters in the jar.

(a) How *many coins* were in the jar at the end of last month?

(b) How *much money* was in the jar at the end of last month?

Note: In Canada, a nickel is worth 5 cents, a dime is worth 10 cents, a quarter is worth 25 cents, a loonie is worth 100 cents, and a toonie is worth 200 cents. Also, 100 cents is equal to 1 dollar ($1).

Solution

To solve this problem we need to determine how many of each type of coin we have.

Since there were twice as many toonies as quarters and there were 8 quarters, then there were $8 \times 2 = 16$ toonies in the jar.

Since there were 3 times as many loonies as toonies and there were 16 toonies, then there were $16 \times 3 = 48$ loonies in the jar.

Since there were 5 more dimes than toonies and there were 16 toonies, then there were $16 + 5 = 21$ dimes in the jar.

Since there were 4 fewer nickels than quarters and there were 8 quarters, then there were $8 - 4 = 4$ nickels in the jar.

In summary, there were 4 nickels, 21 dimes, 8 quarters, 48 loonies, and 16 toonies in the jar at the end of the month.

(a) In total there were $4 + 21 + 8 + 48 + 16 = 97$ coins in the jar at the end of the month.

(b) To determine the amount of money in the jar, we will first calculate the total value of each type of coin by multiplying the coin's value by the number of that type of coin in the jar.

Type of Coin	**Value of One Coin** (*in cents*)	**Number of Coins**	**Total Value** (*in cents*)
Nickel	5	4	$5 \times 4 = 20$
Dime	10	21	$10 \times 21 = 210$
Quarter	25	8	$25 \times 8 = 200$
Loonie	100	48	$100 \times 48 = 4800$
Toonie	200	16	$200 \times 16 = 3200$

The total amount of money amount in the jar was:
$20 + 210 + 200 + 4800 + 3200 = 8430$ cents, which is equal to \$84.30.

Geometry & Measurement (G)

TAKE ME TO THE COVER

Problem of the Week
Problem A
Power Savings

An electric clothes dryer uses approximately 5 kilowatts of energy to dry a single load of laundry.

On average, Mary Lois's family washes 4 loads of laundry per week. If they use a clothes line instead of the electric dryer to dry half of their loads of laundry, approximately how many kilowatts of energy will they save in a year?

Themes Geometry & Measurement, Number Sense

Problem of the Week
Problem A and Solution
Power Savings

Problem

An electric clothes dryer uses approximately 5 kilowatt-hours of energy to dry a single load of laundry.

On average, Mary Lois's family washes 4 loads of laundry per week. If they use a clothes line instead of the electric dryer to dry half of their loads of laundry, approximately how many kilowatt-hours of energy will they save in a year?

Solution

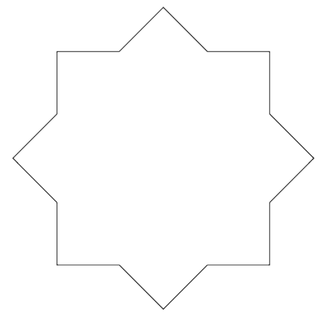
Since half of 4 is 2, then on average Mary Lois's family uses the clothes line to dry two loads of laundry per week. This means they would save approximately $2 \times 5 = 10$ kilowatt-hours of energy each week. Since there are approximately 52 weeks in a year, we calculate the total savings as $52 \times 10 = 520$ kilowatt-hours of energy.

Problem of the Week

Problem A

Star of Lakshmi

The Star of Lakshmi can be found in many places around the world. You can construct this shape by starting with two congruent squares on top of each other. (Two squares are congruent if they have the same side length.) Then imagine you have a pin in the centre of the square and you rotate the top square by 45° (half of a quarter turn). The outline of the result would look like the shape shown on the right.



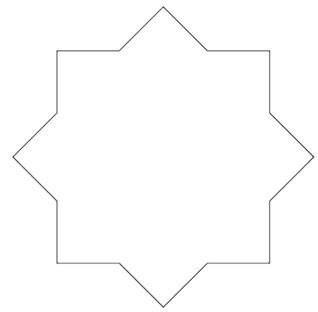
Fill in the table below with details about this shape.

Number of sides	
Number of vertices	
Number of interior right angles	
Number of interior acute angles (angles that are less than 90°)	
Number of lines of symmetry	

Problem of the Week
Problem A and Solution
Star of Lakshmi

Problem

The Star of Lakshmi can be found in many places around the world. You can construct this shape by starting with two congruent squares on top of each other. (Two squares are congruent if they have the same side length.) Then imagine you have a pin in the centre of the square and you rotate the top square by 45° (half of a quarter turn). The outline of the result would look like the shape shown on the right.



Fill in the table below with details about this shape.

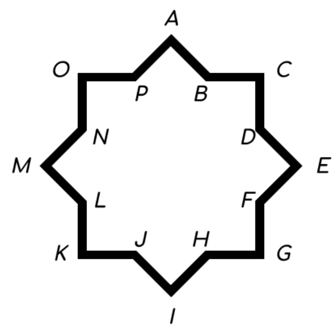
Solution

Number of sides	16
Number of vertices	16
Number of interior right angles	8
Number of interior acute angles (angles that are less than 90°)	0
Number of lines of symmetry	8

The solution continues on page 2.

Here are some notes about the answers.

A diagram of the Lakshmi star with its 16 vertices labelled A through P is below.

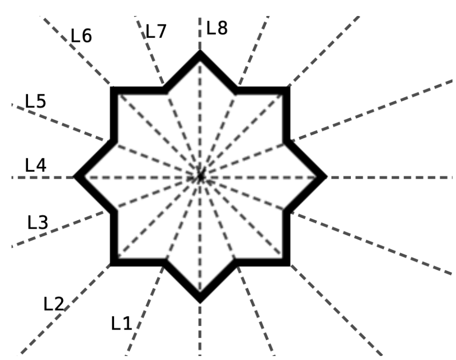


The interior angles at vertices A, C, E, G, I, K, M, and O are all right angles, since they were originally parts of one of the squares used to construct the star.

Also, the interior angles at vertices B, D, F, H, J, L, N, and P are all greater than 90°. In fact these angles are all greater than 180° (a straight line). Note: angles that are greater than 180° are called *reflex angles*.

Since we have now considered all 16 angles in the star, there are no interior acute angles.

Since the shape was formed by two squares, we can draw lines of symmetry between 8 pairs of opposite vertices as shown in the diagram below. We label these lines of symmetry as L1, L2, L3, L4, L5, L6, L7, and L8.



Problem of the Week

Problem A

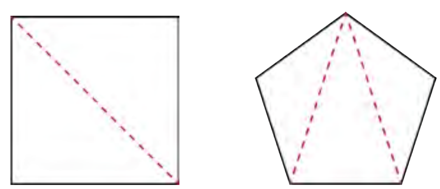
Tracking Triangles

A regular polygon is a closed shape where all the side lengths are the same. Pauline draws lines inside regular polygons according to the following rules.

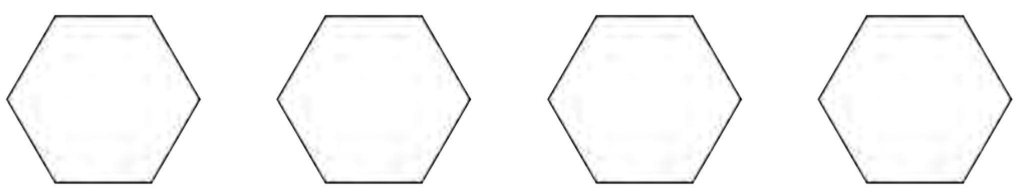
1. The lines must connect two vertices that are *not* beside each other.

2. The lines must be straight and cannot cross.

Pauline continues to draw lines until she cannot draw any more. At this point, the inside of her polygon will be made up entirely of triangles. For example, after drawing lines in a square she creates 2 triangles, and after drawing lines in a regular pentagon she creates 3 triangles, as shown.



(a) Notice that if Pauline had drawn lines between different pairs of vertices in the square and the regular pentagon, the resulting diagrams would have been rotations or reflections of the diagrams above, but would otherwise have been the same. Is it possible for Pauline to draw lines in a regular hexagon and create more than one diagram, which cannot be obtained from the others by a rotation or reflection? Use the hexagons below to test it out.



(b) Look at the number of triangles Pauline creates in a square, pentagon, and hexagon. Use this to predict the number of triangles Pauline creates in an octagon, and then check to see if you are correct.

Not printing this page? Try our interactive worksheet.

THEME GEOMETRY & MEASUREMENT

Problem of the Week
Problem A and Solution
Tracking Triangles

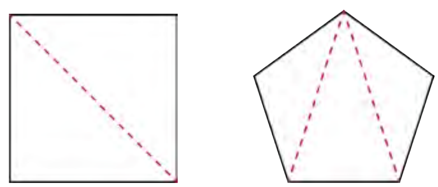
Problem

A regular polygon is a closed shape where all the side lengths are the same. Pauline draws lines inside regular polygons according to the following rules.

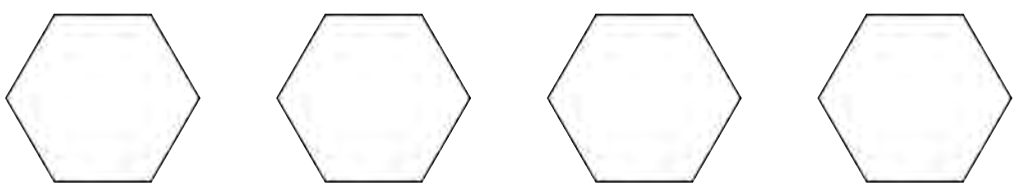
1. The lines must connect two vertices that are *not* beside each other.

2. The lines must be straight and cannot cross.

Pauline continues to draw lines until she cannot draw any more. At this point, the inside of her polygon will be made up entirely of triangles. For example, after drawing lines in a square she creates 2 triangles, and after drawing lines in a regular pentagon she creates 3 triangles, as shown.



(a) Notice that if Pauline had drawn lines between different pairs of vertices in the square and the regular pentagon, the resulting diagrams would have been rotations or reflections of the diagrams above, but would otherwise have been the same. Is it possible for Pauline to draw lines in a regular hexagon and create more than one diagram, which cannot be obtained from the others by a rotation or reflection? Use the hexagons below to test it out.

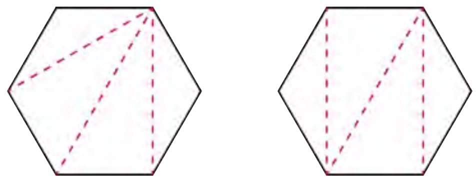


(b) Look at the number of triangles Pauline creates in a square, pentagon, and hexagon. Use this to predict the number of triangles Pauline creates in an octagon, and then check to see if you are correct.

Not printing this page? Try our interactive worksheet.

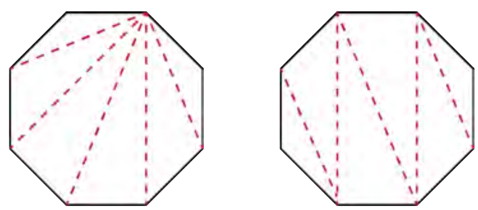
Solution

(a) There are three different diagrams that can be created by drawing lines in a regular hexagon. They are shown below.



All other possible diagrams are reflections or rotations of one of these three.

(b) Pauline creates 2 triangles in a square, 3 triangles in a pentagon, and 4 triangles in a hexagon. It appears as though the number of triangles is always 2 less than the number of sides in the polygon. Then we can predict that Pauline can create $8 - 2 = 6$ triangles in an octagon. In fact this is true. Two examples are shown.



Teacher's Notes

It takes some advanced mathematics to actually prove that in any regular polygon the **maximum** number of triangles we can draw according to the rules is always 2 less than the number of sides of the polygon. However, we can informally convince ourselves that it is possible to draw **at least** this many triangles. In general, one way we can draw the triangles inside the polygon is to pick one vertex as the end point of all the lines and draw a line to each of the other vertices in the polygon that are not adjacent to that vertex. If we examine that pattern, we see we always end up with the number of triangles being 2 less than the number of sides in the polygon.

Remembering this pattern can help us determine another feature of a polygon. It is known that the sum of the interior angles a triangle is always $180°$. You can test this by drawing many different triangles and measuring the interior angles within them. This can also be proven true for all triangles without having to measure specific angles, but again we need some more rules of geometry to do so. However, knowing this we can figure out the sum of the interior angles of any polygon by using the pattern from this problem. For example, since we can draw 4 triangles inside a hexagon, then the sum of the interior angles of a hexagon must be $4 \times 180° = 720°$. Similarly, the sum of the interior angles of a *icosagon* (a 20-sided polygon) is $18 \times 180° = 3240°$. In general, the sum of the interior angles of a polygon with n sides is $(n - 2) \times 180°$.

Problem of the Week
Problem A
Snow Days

Tapeesa monitored the amount of snowfall each day, Monday through Sunday, for four weeks. Here is what she recorded:

- The first week it snowed 5 mm each day.
- The second week snow only fell on two days: 8 cm of snow fell on Tuesday and 2 cm of snow fell on Friday.
- It did not snow the third week at all.
- A total of half a metre of snow fell during the last week.

What was the total amount of snowfall over the four weeks?

Themes Geometry & Measurement, Number Sense

Problem of the Week
Problem A and Solution
Snow Days

Problem

Tapeesa monitored the amount of snowfall each day, Monday through Sunday, for four weeks. Here is what she recorded:

- The first week it snowed 5 mm each day.
- The second week snow only fell on two days: 8 cm of snow fell on Tuesday and 2 cm of snow fell on Friday.
- It did not snow the third week at all.
- A total of half a metre of snow fell during the last week.

What was the total amount of snowfall over the four weeks?

Solution

To find the total, we should have all the measurements with the same unit. Let's consider all of the snowfall amounts in centimetres.

The first week it snowed $7 \times 5 = 35$ mm in total. This is equal to 3.5 cm.

The second week it snowed a total of $8 + 2 = 10$ cm.

Since 0.5 m is equal to 50 cm, that is how much it snowed during the fourth week.

Therefore, the total snowfall for the four weeks was $3.5 + 10 + 50 = 63.5$ cm.

Alternatively, we could calculate the amounts in millimetres.

In the second week, it snowed a total of $10 \times 8 = 80$ mm and $10 \times 2 = 20$ mm, for a total of $80 + 20 = 100$ mm. Since 1 m is equal to 1000 mm, then in the last week it snowed half of 1000 mm, or 500 mm. Therefore, the total amount of snowfall for the four weeks was $35 + 100 + 500 = 635$ mm, which is equal to 63.5 cm.

Problem of the Week

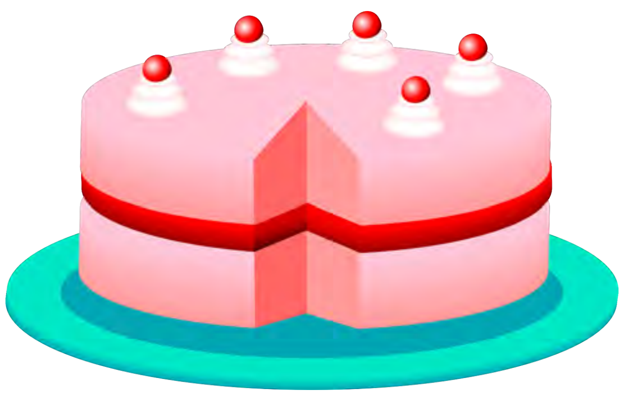
Problem A

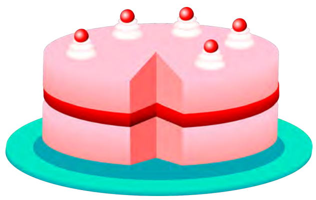
Colossal Cake

Jose's 12th birthday is coming up and his family is having all of their friends and neighbours over for the party. They ordered a colossal cake from the bakery for the celebration. The mass of the cake, including the icing and the tray, is 5 kg. The tray has a mass of 400 grams.

(a) What is the mass of the cake and the icing without the tray?

(b) If the icing has a mass of 1350 grams, what is the mass of the cake without the icing or the tray?





Problem of the Week
Problem A and Solution
Colossal Cake

Problem

Jose's 12th birthday is coming up and his family is having all of their friends and neighbours over for the party. They ordered a colossal cake from the bakery for the celebration. The mass of the cake, including the icing and the tray, is 5 kg. The tray has a mass of 400 grams.

(a) What is the mass of the cake and the icing without the tray?

(b) If the icing has a mass of 1350 grams, what is the mass of the cake without the icing or the tray?

Solution

To solve this problem, it would be easier if all of the amounts were described with the same units. Since the mass of the tray and the icing are in grams, we could convert the total mass to grams as well. Since 1 kg is equal to 1000 grams, the total mass is $5 \times 1000 = 5000$ grams.

(a) The mass of the cake and the icing without the tray is $5000 - 400 = 4600$ grams.

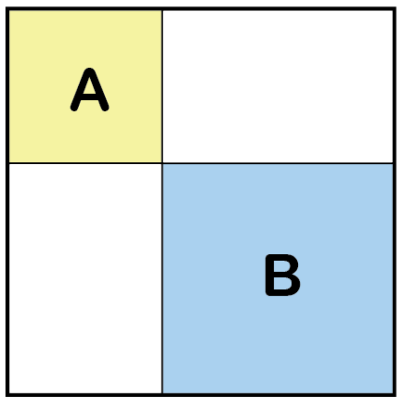
(b) The mass of the cake without the icing or the tray is $4600 - 1350 = 3250$ grams.

Problem of the Week

Problem A

What's My Perimeter?

A large square is divided into four rectangles by drawing two straight lines inside the square; one line parallel to the bottom of the square, and one line parallel to the side of the square. Two of the rectangles created are also squares. These squares are labelled A and B in the diagram.



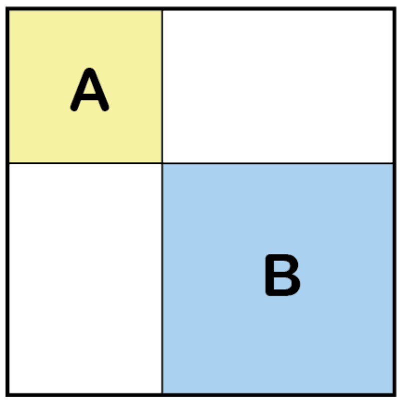
Square A has an area of $16\,\text{cm}^2$ and square B has an area of $36\,\text{cm}^2$.

What is the perimeter of the original square?

Problem of the Week
Problem A and Solution
What's My Perimeter?

Problem

A large square is divided into four rectangles by drawing two straight lines inside the square; one line parallel to the bottom of the square, and one line parallel to the side of the square. Two of the rectangles created are also squares. These squares are labelled A and B in the diagram.

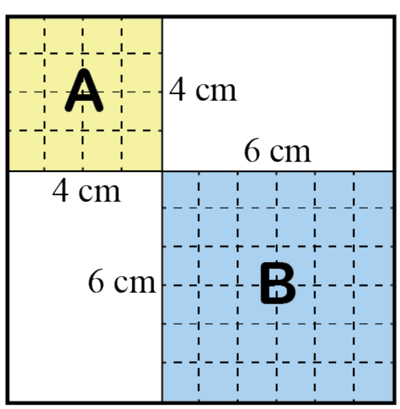


Square A has an area of $16\,\text{cm}^2$ and square B has an area of $36\,\text{cm}^2$.

What is the perimeter of the original square?

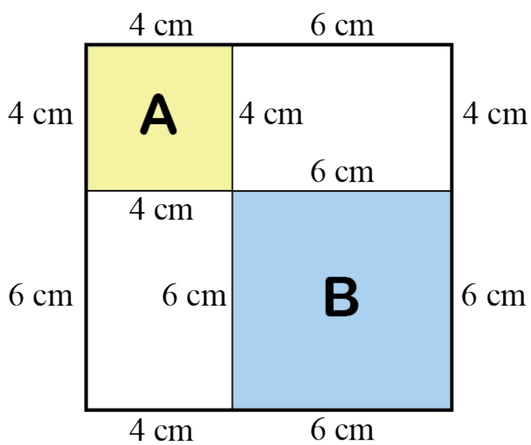
Solution

To solve this problem we need to determine the side lengths of squares A and B. One way we can determine the side lengths is to build each square using $1\,\text{cm} \times 1\,\text{cm}$ tiles. To build square A we would need 16 tiles. The only way to arrange 16 tiles into a square is to make a square with 4 rows and 4 columns. We call this a 4×4 square. That means the side length of square A is $4\,\text{cm}$. Similarly, to build square B we would need 36 tiles. The only way to arrange 36 tiles into a square is to make a 6×6 square. That means the side length of square B is $6\,\text{cm}$.



Alternatively, we know that the area of a square is equal to the side length of the square multiplied by itself. To find the side length of square A, we look for a number that when multiplied by itself gives a result of 16. Since $4 \times 4 = 16$, it follows that the side length of square A is $4\,\text{cm}$. To find the side length of square B, we look for a number that when multiplied by itself gives a result of 36. Since $6 \times 6 = 36$, it follows that the side length of square B is $6\,\text{cm}$.

Since the shapes inside the square are all rectangles, the opposite sides of each rectangle have the same lengths.



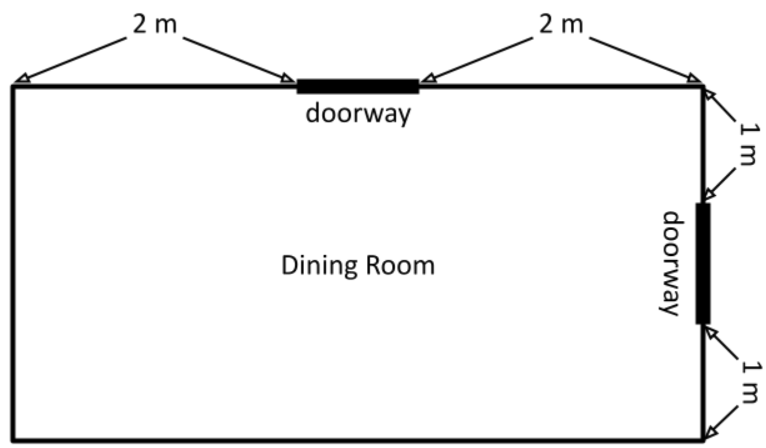
Therefore, the side length of the original square is $6 + 4 = 10\,\text{cm}$. This means that the perimeter of the original square is $10 + 10 + 10 + 10 = 40\,\text{cm}$.

Problem of the Week

Problem A

Painting Plans

Judith and James want to repaint their dining room. They have a floor plan of the dining room that shows the dimensions of the rectangular room as well as the location of the doorways.



The walls are each 3 metres high and the doorways are each 2 metres high and 1 metre wide. They do not want to paint the doorways.

(a) For each wall, draw a diagram showing the area that needs to be painted. Show the dimensions of the walls and the doorways on each diagram.

(b) Calculate the total area that will be painted.

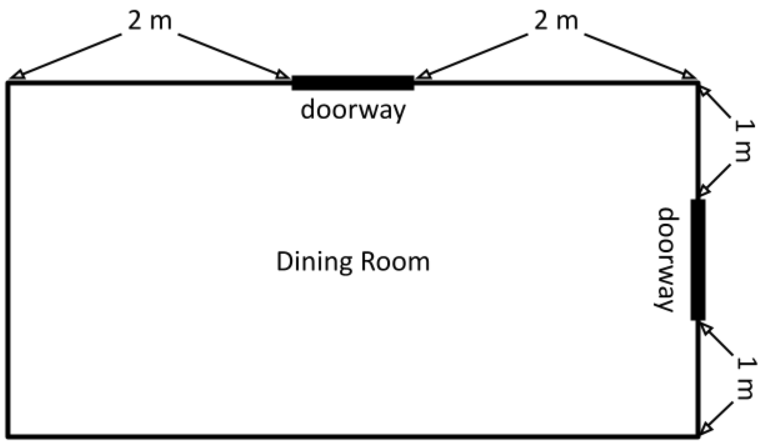
Problem of the Week

Problem A and Solution

Painting Plans

Problem

Judith and James want to repaint their dining room. They have a floor plan of the dining room that shows the dimensions of the rectangular room as well as the location of the doorways.



The walls are each 3 metres high and the doorways are each 2 metres high and 1 metre wide. They do not want to paint the doorways.

(a) For each wall, draw a diagram showing the area that needs to be painted. Show the dimensions of the walls and the doorways on each diagram.

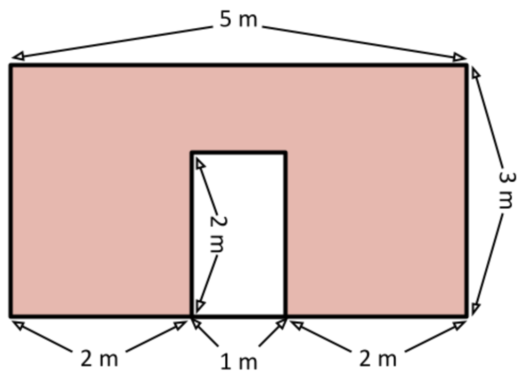
(b) Calculate the total area that will be painted.

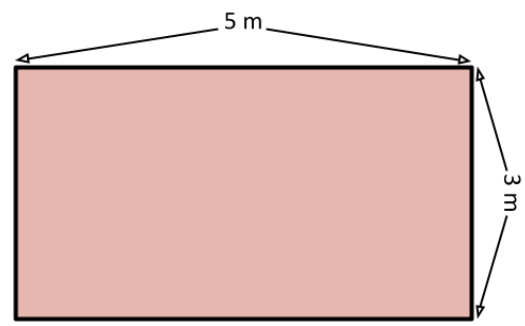
Solution

(a) Since the dining room is rectangular, the lengths of the opposite walls are the same. We can create a diagram of the walls based on the information from the floor plan and the known dimensions of the doorways and wall height.

From the floor plan, we know that on the longer wall with a doorway, the distance from one edge of the doorway to the edge of the wall is 2 m. We also know the width of the doorway is 1 m. Thus, the width of the entire wall is $2 + 1 + 2 = 5$ m. The longer wall without a doorway has the same width and height.

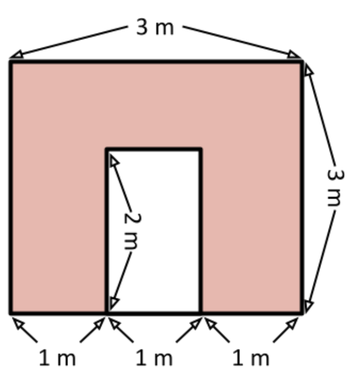
Here is a diagram of the longer walls of the dining room:

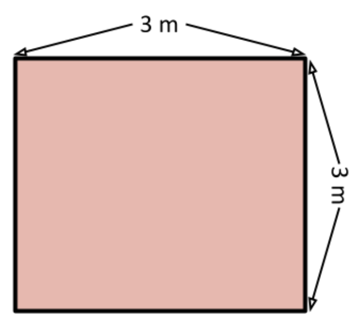




From the floor plan, we know that on the shorter wall with a doorway, the distance from one edge of the doorway to the edge of the wall is 1 m. We also know the width of the doorway is 1 m. Thus, the width of the entire wall is $1 + 1 + 1 = 3$ m. The shorter wall without a doorway has the same width and height.

Here is a diagram of the shorter walls of the dining room:





(b) The area of the longer wall without a doorway is $5 \times 3 = 15$ m^2. The area of the shorter wall without a doorway is $3 \times 3 = 9$ m^2.

For the walls that include the doorways, we could divide them into rectangular pieces to determine the areas requiring paint on those walls. This means we would need to determine the dimensions of those rectangular pieces. Another way to do this is to calculate the area of the wall ignoring the doorway, and then subtract the area of the doorway to determine how much paint is required. The area of any of each doorway is $2 \times 1 = 2$ m^2.

So the area to be painted on the longer wall with the doorway is $15 - 2 = 13$ m^2, and the area to be painted on the shorter wall with the doorway is $9 - 2 = 7$ m^2.

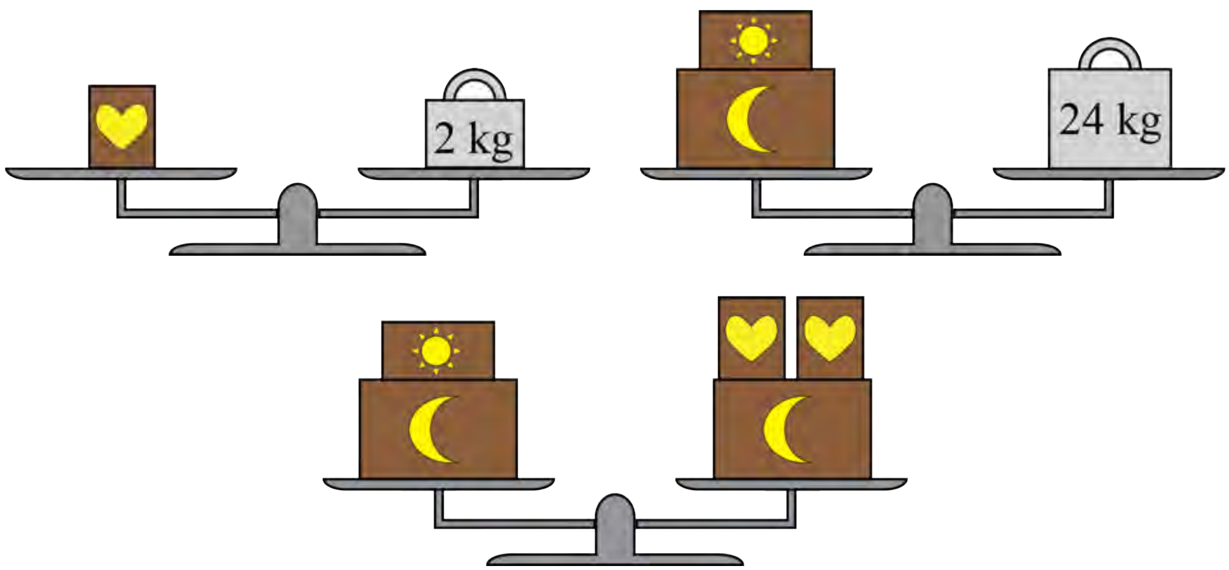
Therefore, the total area that will be painted is $15 + 9 + 13 + 7 = 44$ m^2.

Algebra (A)

TAKE ME TO THE COVER

Problem of the Week
Problem A
Balancing Act

Bailey is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass, and the cost of sending a box depends on its mass. Bailey has a balance scale and a few standard weights to help with the job. The following diagrams show what Bailey observed when arranging some of the boxes and standard weights on the scales.



Find the mass of each box.

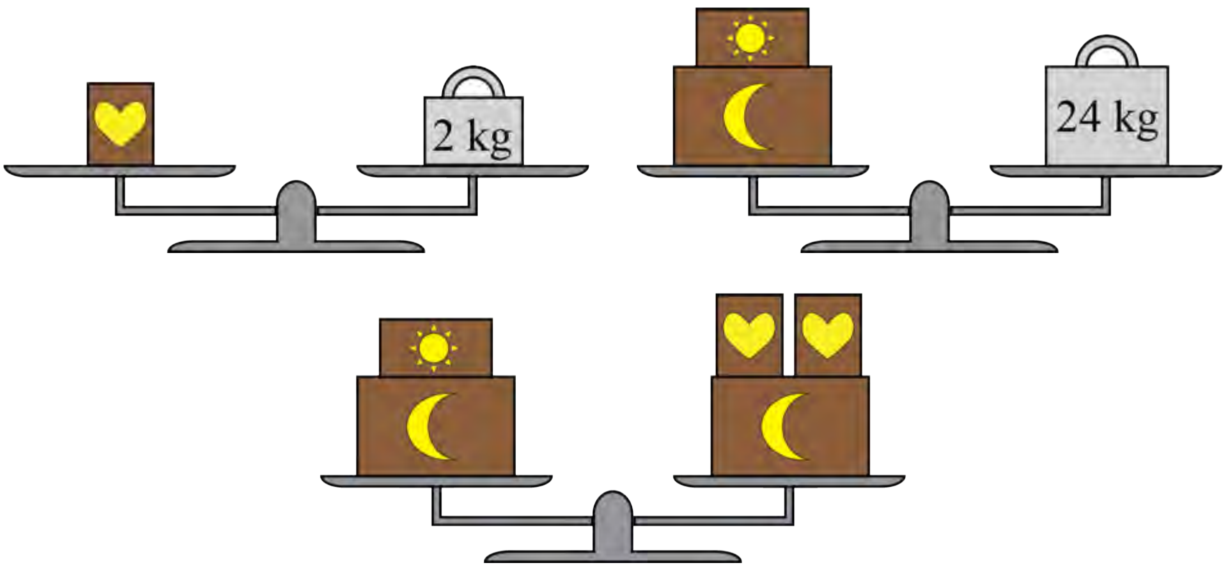
Problem of the Week

Problem A and Solution

Balancing Act

Problem

Bailey is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass, and the cost of sending a box depends on its mass. Bailey has a balance scale and a few standard weights to help with the job. The following diagrams show what Bailey observed when arranging some of the boxes and standard weights on the scales.



Find the mass of each box.

Solution

From the diagrams we notice the following.

- One heart box has a mass of 2 kg.
- One moon box and one sun box have a total mass of 24 kg.
- One moon box and one sun box have the same total mass as one moon box and two heart boxes.

From this, we can conclude that one moon box and two heart boxes have a total mass of 24 kg. Also, two heart boxes have the same mass as one sun box.

Since one heart box has a mass of 2 kg, then two heart boxes have a mass of 4 kg. Therefore, one sun box has a mass of 4 kg.

This means 4 kg + (mass of a moon box) = 24 kg. Since $4 + 20 = 24$, we can determine that one moon box must have a mass of 20 kg.

Teacher's Notes

The idea of a balance scale is a nice analogy for an algebraic equation. We can represent the information in the problem using equations with variables to represent the masses of the different types of boxes. Here is one way to solve the problem algebraically.

Let x represent the mass of a heart box.
Let y represent the mass of a sun box.
Let z represent the mass of a moon box.

From the information in the diagrams, we can write the following equations:

$$x = 2 \tag{1}$$
$$y + z = 24 \tag{2}$$
$$y + z = 2x + z \tag{3}$$

From equation (1), we already know that a heart box has a mass of 2 kg. From equations (2) and (3), we notice that the left sides are the same, so the right sides of the equations must be equal to each other. This means we know:

$$2x + z = 24 \tag{4}$$

Now, substituting $x = 2$ into equation (4), we get

$$2(2) + z = 24$$

Subtracting 4 from each side of this equation, we get

$$z = 20$$

Finally, substituting $z = 20$ into equation (2), we get

$$y + 20 = 24$$

Subtracting 20 from each side of this equation, we get

$$y = 4$$

So, a heart box has a mass of 2 kg, a sun box has a mass of 4 kg, and a moon box has a mass of 20 kg.

Problem of the Week
Problem A
Pumpkin Patch

The mass of a standard carving pumpkin is approximately 12 kg. Lavina plans to sell the pumpkins she has grown at the farmer's market. The table she has to display the pumpkins can support 224 kg. If the mass of each of her pumpkins is 12 kg, what is the largest number of pumpkins that Lavina can put on her table?

Problem of the Week
Problem A and Solution
Pumpkin Patch

Problem

The mass of a standard carving pumpkin is approximately 12 kg. Lavina plans to sell the pumpkins she has grown at the farmer's market. The table she has to display the pumpkins can support 224 kg. If the mass of each of her pumpkins is 12 kg, what is the largest number of pumpkins that Lavina can put on her table?

Solution

We can make a table to calculate the total mass of various quantities of pumpkins.

Number of Pumpkins	1	2	3	4	5	6	7	8	9	10
Total Mass (in kg)	12	24	36	48	60	72	84	96	108	120

Number of Pumpkins	11	12	13	14	15	16	17	18	19
Total Mass (in kg)	132	144	156	168	180	192	204	216	228

The total mass of 19 pumpkins, which is 228 kg, exceeds the capacity of the table. So the largest number of pumpkins Lavina can fit safely on her table is 18.

Having to make a table counting from 1 to 19 takes quite a bit of work. Alternatively, we could try to reduce the work by narrowing the search area. We can use easier numbers such as multiples of 10 to find a narrower range to check. We see that $10 \times 12 = 120$ and $20 \times 12 = 240$. From this we know that the answer must be between 10 and 20 pumpkins. So instead of starting our table with 1 pumpkin, we could start it with 10 pumpkins.

We might also notice that the number we are looking for (224 kg) is much closer to 240 than 120. So rather than counting up, we could count down from 240 in a table.

Number of Pumpkins	20	19	18
Total Mass (in kg)	240	228	216

Again, from this result we can conclude that the largest number of pumpkins she can put on her table is 18.

Teacher's Notes

In this problem we said that the pumpkins all had the same mass of 12 kg. However, in reality, we would not expect each of the pumpkins to have exactly the same mass. A more realistic statement would be that the mass of each pumpkin is approximately 12 kg, but that makes the problem a bit trickier. If we had rounded to the nearest kilogram, that means the mass of each pumpkin could be greater than or equal to 11.5 kg and less than 12.5 kg.

Let's assume that all the pumpkins have a mass of no more than 11.6 kg, which is still approximately 12 kg. In this case we see that $11.6 \times 19 = 220.4$ kg, so we could fit 19 pumpkins on the table.

Let's assume that all the pumpkins have a mass of no less than 12.45 kg, which is still approximately 12 kg. In this case we see that $12.45 \times 18 = 224.1$ kg, which is more than the capacity of the table.

Sometimes we need to recognize a margin of error to reflect that our physical world does not always fit into our nice and neat world of mathematical problems.

Problem of the Week
Problem A
The Pencil Case

When trying to solve a mystery involving some missing pencils, you discover the following riddle about the number of pencils that were in the box originally.

There are fewer than forty pencils in the box.

If you remove four pencils at a time, eventually there will be 2 pencils left.

If you remove three pencils at a time, eventually there will be 0 pencils left.

If you remove five pencils at a time, eventually there will be 0 pencils left.

How many pencils were in the box originally?

Themes Algebra, Number Sense

Problem of the Week
Problem A and Solution
The Pencil Case

Problem

When trying to solve a mystery involving some missing pencils, you discover the following riddle about the number of pencils that were in the box originally.

> There are fewer than forty pencils in the box.
>
> If you remove four pencils at a time, eventually there will be 2 pencils left.
>
> If you remove three pencils at a time, eventually there will be 0 pencils left.
>
> If you remove five pencils at a time, eventually there will be 0 pencils left.

How many pencils were in the box originally?

Solution

Since there would be 2 pencils left if we remove 4 pencils at a time, it cannot be the case that there are 0 pencils in the box.

From this point, one way to solve this problem is to start with the last clue. From this information, we know that the number of pencils in the box is a multiple of 5. The multiples of 5 that are greater than 0 and less than 40 are:

$$5,\ 10,\ 15,\ 20,\ 25,\ 30,\ 35$$

From the second last clue, we know that the number of pencils in the box is a multiple of 3. The multiples of 3 that are greater than 0 and less than 40 are:

$$3,\ 6,\ 9,\ 12,\ 15,\ 18,\ 21,\ 24,\ 27,\ 30,\ 33,\ 36,\ 39$$

The only two numbers that are in both lists are 15 and 30. Now we can count down by 4 from each to see which one ends up at 2.

Starting at 15 we get this sequence: $15, 11, 7, 3$

Starting at 30 we get this sequence: $30, 26, 22, 18, 14, 10, 6, 2$

We see that there must have been 30 pencils in the box to start, since this is a multiple of 5 and a multiple of 3 and will have 2 pencils left in the box if we remove 4 at a time.

Teacher's Notes

The solution provided to this problem essentially uses trial and error to find the answer. We should notice that if we were not given a maximum number of pencils in the box, then there are theoretically an infinite number of answers to this problem. For example if there were 90 pencils in the box, the statements about removing pencils would all be true as well.

If you were to ask a mathematician this question they might solve the problem without using trial and error. They could use *congruence notation* to represent the information and then solve the problem algebraically.

The statement "If you remove four pencils at a time, eventually there will be 2 pencils left" could be restated as "If you divide the number of pencils in the box by 4, the remainder is 2". If we know congruence notation, we can restate this information as

$$p \equiv 2 \mod 4$$

assuming that p is the number of pencils in the box. When we read this statement we say that "p is *congruent* to 2, *modulo* 4".

The other information from the problem could be restated as

$$p \equiv 0 \mod 3$$

and

$$p \equiv 0 \mod 5$$

Then we can solve this problem algebraically by converting these congruences to linear equations.

Problem of the Week
Problem A
Saving Up

Fionnlay earns \$8 each week by walking the neighbour's dog. Starting in the second week, Fionnlay earns an extra \$3 by helping maintain the community garden every other week. He is saving all of his earnings for a bicycle, which costs \$157. He will also need a helmet which costs \$24, a bike horn which costs \$16, and a bike light which costs \$9. All prices include taxes. Fionnlay keeps track of the money he has earned.

(a) How much money will Fionnlay have saved after 8 weeks?

(b) How much money will Fionnlay have saved after 15 weeks?

(c) How long will it take him to save enough money to buy the bike and its accessories?

Themes Algebra, Number Sense

Problem of the Week
Problem A and Solution
Saving Up

Problem

Fionnlay earns \$8 each week by walking the neighbour's dog. Starting in the second week, Fionnlay earns an extra \$3 by helping maintain the community garden every other week. He is saving all of his earnings for a bicycle, which costs \$157. He will also need a helmet which costs \$24, a bike horn which costs \$16, and a bike light which costs \$9. All prices include taxes. Fionnlay keeps track of the money he has earned.

(a) How much money will Fionnlay have saved after 8 weeks?

(b) How much money will Fionnlay have saved after 15 weeks?

(c) How long will it take him to save enough money to buy the bike and its accessories?

Solution

One way to solve this problem is to create a table that records the total that Fionnlay has saved each week. In the odd-numbered weeks, the total will increase by \$8. In the even-numbered weeks, the total will increase by $\$8 + \$3 = \$11$.

We will also need to know how much money Fionnlay needs to save by calculating the total cost of the bike and its accessories. We can do this by adding $\$157 + \$24 + \$16 + \$9 = \$206$.

Using the table, we can now answer the questions.

(a) Fionnlay will have saved \$76 after 8 weeks.

(b) Fionnlay will have saved \$141 after 15 weeks.

(c) Fionnlay will need 22 weeks to save enough money for the bike and its accessories.

Week	Increase (in \$)	Total Savings (in \$)
1	8	8
2	11	19
3	8	27
4	11	38
5	8	46
6	11	57
7	8	65
8	11	76
9	8	84
10	11	95
11	8	103
12	11	114
13	8	122
14	11	133
15	8	141
16	11	152
17	8	160
18	11	171
19	8	179
20	11	190
21	8	198
22	11	209

Problem of the Week

Problem A

Old Faithful

Old Faithful is a geyser in Yellowstone National Park. It is so named because it was believed that it erupted every 60 to 90 minutes all day long. Assuming that Old Faithful erupts at 12 midnight and then erupts every 60 to 90 minutes after the last eruption, answer the following questions.

(a) After the first eruption at 12 midnight, what is the minimum number of eruptions you could see until up to and including 12 midnight the next night?

(b) After the first eruption at 12 midnight, what is the maximum number of eruptions you could see until up to and including 12 midnight the next night?

Themes Algebra, Number Sense

Problem of the Week
Problem A and Solution
Old Faithful

Problem

Old Faithful is a geyser in Yellowstone National Park. It is so named because it was believed that it erupted every 60 to 90 minutes all day long. Assuming that Old Faithful erupts at 12 midnight and then erupts every 60 to 90 minutes after the last eruption, answer the following questions.

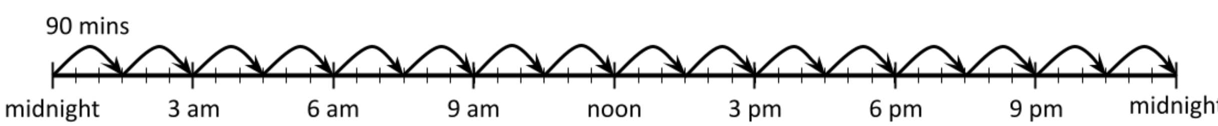
(a) After the first eruption at 12 midnight, what is the minimum number of eruptions you could see until up to and including 12 midnight the next night?

(b) After the first eruption at 12 midnight, what is the maximum number of eruptions you could see until up to and including 12 midnight the next night?

Solution

(a) One way to solve this problem is to make a timeline.

We would see fewer eruptions if the time between eruptions is longer. The longest gap is 90 minutes. Therefore, the fewest number of eruptions will occur if the time between each eruption is 90 minutes.



From this timeline we can count the ending point of each of the arrows in the diagram and see that, after the first eruption at 12 midnight, there would be 16 eruptions if they happened every 90 minutes. Note that the last eruption would be at exactly midnight on the next night.

Alternatively, we might notice that 90 minutes is equal to $1\frac{1}{2}$ hours, and 180 minutes (or two geyser eruption intervals) is equal to 3 hours. Then we can make a table keeping track of how many eruptions take place over time.

Eruptions	2	4	6	8	10	12	14	16
Hours Elapsed	3	6	9	12	15	18	21	24

(b) We would see more eruptions if the time between eruptions is shorter. The shortest amount of time between eruptions is 60 minutes. Since 60 minutes is equal to 1 hour, and there are 24 hours from 12 midnight until 12 midnight the next night, then the maximum number of eruptions we could see is 24 eruptions.

Problem of the Week

Problem A

A Hop, Skip, and a Jump!

Alexis and her friends Mikai, Sophia, and Casper enter a team competition that involves hopping, skipping, biking, and rollerblading for a total of 3 km. Each team member picks an activity and must take turns completing a section of the course by doing their activity.

Alexis will hop for 100 m, then Mikai will bike the next 150 m, followed by Sophia who will rollerblade for the next 200 m, and finally Casper will skip the next 50 m. They will repeat their activity until the team completes the 3 km race.

(a) How many times does each member have to do their activity to complete the race?

(b) What fraction of the race does each team member complete? Put the fractions in order from least to greatest.

Themes Algebra, Number Sense

Problem of the Week
Problem A and Solution
A Hop, Skip, and a Jump!

Problem

Alexis and her friends Mikai, Sophia, and Casper enter a team competition that involves hopping, skipping, biking, and rollerblading for a total of 3 km. Each team member picks an activity and must take turns completing a section of the course by doing their activity.

Alexis will hop for 100 m, then Mikai will bike the next 150 m, followed by Sophia who will rollerblade for the next 200 m, and finally Casper will skip the next 50 m. They will repeat their activity until the team completes the 3 km race.

(a) How many times does each member have to do their activity to complete the race?

(b) What fraction of the race does each team member complete? Put the fractions in order from least to greatest.

Solution

(a) The total distance travelled after each participant has completed the activity once is $100 + 150 + 200 + 50 = 500$ m. We know that 3 km is equal to $3 \times 1000 = 3000$ m. We can use skip counting by 500 to see how many times they have to complete the cycle of four activities to travel 3000 m: $500,\ 1000,\ 1500,\ 2000,\ 2500,\ 3000$. This means that each team member would need to do their activity 6 times to complete the race.

(b) One way to determine the fractional amounts is to focus on the distance of one cycle of the race, which is 500 m. Then we can take each distance the individuals travelled as the numerator of the fraction and 500 as the denominator of the fraction.

The fractions are: hopping $\frac{100}{500}$, biking $\frac{150}{500}$, rollerblading $\frac{200}{500}$, and skipping $\frac{50}{500}$.

Alternatively, we can determine the fractional amounts using the entire 3000 m. Then we would have found that the fractions are: hopping $\frac{600}{3000}$, biking $\frac{900}{3000}$, rollerblading $\frac{1200}{3000}$, and skipping $\frac{300}{3000}$.

We could also reduce each of these fractions by dividing the numerator and denominator by the same number to get the following fractions: hopping: $\frac{600}{3000} = \frac{100}{500} = \frac{1}{5}$, biking: $\frac{900}{3000} = \frac{150}{500} = \frac{3}{10}$, rollerblading: $\frac{1200}{3000} = \frac{200}{500} = \frac{2}{5}$, and skipping: $\frac{300}{3000} = \frac{50}{500} = \frac{1}{10}$.

We can compare the fractions in many different ways to determine the relative sizes. It is easy to compare fractions with the same denominator, which we have with the fractions with 500 or 3000 as the denominators.

Arranging these in order from least to greatest we get:

$$\begin{array}{cccc} \text{skipping} & \text{hopping} & \text{biking} & \text{rollerblading} \\ \frac{300}{3000} = \frac{50}{500} = \frac{1}{10}, & \frac{600}{3000} = \frac{100}{500} = \frac{1}{5}, & \frac{900}{3000} = \frac{150}{500} = \frac{3}{10}, & \frac{1200}{3000} = \frac{200}{500} = \frac{2}{5}. \end{array}$$

Problem of the Week

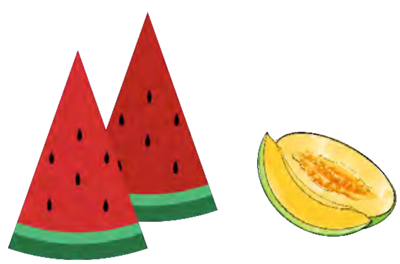
Problem A

How Many Plants?

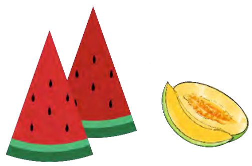
Last year, POTW Farms grew watermelons. They used two different sizes of garden plots: single garden plots and double garden plots. In a single plot they grew one watermelon plant, and in a double plot they grew two watermelon plants. This year, they are planning to grow cantaloupes instead of watermelons. In each single plot they will grow five cantaloupe plants and in each double plot they will grow ten cantaloupe plants. The farm has 14 single plots and 26 double plots.

(a) How many watermelon plants did the farm grow last year?

(b) How many cantaloupe plants are they expecting to grow this year?



Themes Algebra, Number Sense



Problem of the Week
Problem A and Solution
How Many Plants?

Problem

Last year, POTW Farms grew watermelons. They used two different sizes of garden plots: single garden plots and double garden plots. In a single plot they grew one watermelon plant, and in a double plot they grew two watermelon plants. This year, they are planning to grow cantaloupes instead of watermelons. In each single plot they will grow five cantaloupe plants and in each double plot they will grow ten cantaloupe plants. The farm has 14 single plots and 26 double plots.

(a) How many watermelon plants did the farm grow last year?

(b) How many cantaloupe plants are they expecting to grow this year?

Solution

(a) Since each double plot contained two watermelon plants, there were $2 \times 26 = 52$ watermelon plants in the double plots. This means there were a total of $14 + 52 = 66$ watermelon plants.

(b) Since they expect to grow 5 cantaloupe plants in each single plot, they expect to grow $5 \times 14 = 70$ cantaloupe plants in the single plots. Since they expect to grow 10 cantaloupe plants in each double plot, they expect to grow $10 \times 26 = 260$ cantaloupe plants in the double plots. This means POTW Farms is expecting to grow $70 + 260 = 330$ cantaloupe plants.

Problem of the Week

Problem A

Ski Time

Graham and Olga are going skiing. A gondola takes them from the bottom of a mountain to the top, and then they ski back down. It takes them 20 minutes to ride the gondola to the top of the mountain, and 15 minutes to ski back down.

They begin their first gondola ride up the mountain at 9:30 a.m. and plan to meet their parents at the bottom of the mountain at 12:30 p.m. for lunch. What is the maximum number of ski runs they can do before meeting their parents for lunch, assuming they don't have to wait in line for the gondola?

Themes Algebra, Number Sense

Problem of the Week

Problem A and Solution

Ski Time

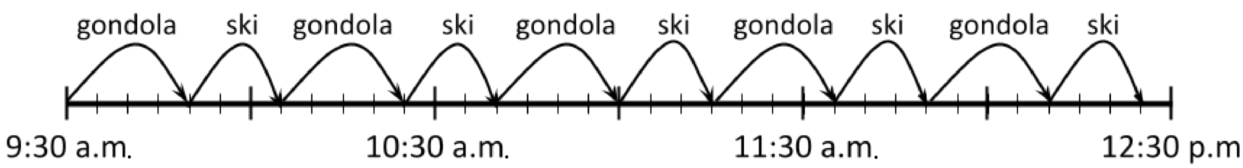
Problem

Graham and Olga are going skiing. A gondola takes them from the bottom of a mountain to the top, and then they ski back down. It takes them 20 minutes to ride the gondola to the top of the mountain, and 15 minutes to ski back down.

They begin their first gondola ride up the mountain at 9:30 a.m. and plan to meet their parents at the bottom of the mountain at 12:30 p.m. for lunch. What is the maximum number of ski runs they can do before meeting their parents for lunch, assuming they don't have to wait in line for the gondola?

Solution

One way to solve this problem would be to use a timeline divided into 5 minute intervals to keep track of how long Graham and Olga have been skiing.



The timeline shows that the maximum number of ski runs that Graham and Olga can do between 9:30 a.m. and 12:30 p.m. is five.

Another way we could solve this problem is to calculate the number of minutes between the start time and lunch time. There are 3 hours between 9:30 a.m. and 12:30 p.m. Since each hour is 60 minutes, this is a total of $60 \times 3 = 180$ minutes. The total time it takes to ride the gondola and then ski back down the mountain once is $20 + 15 = 35$ minutes. Now we could use skip counting to figure out how many ski runs Graham and Olga can do within 180 minutes.

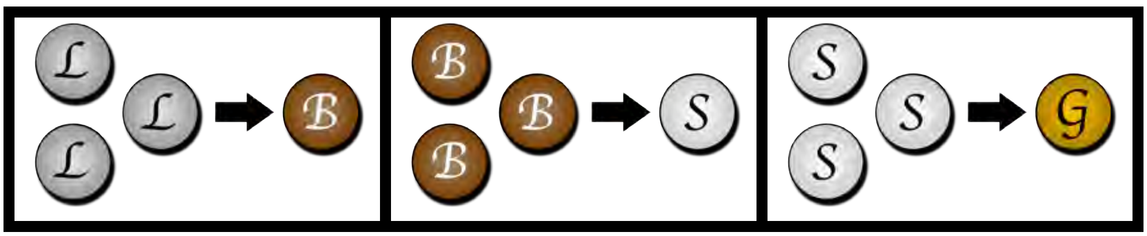
$$35, \ 70, \ 105, \ 140, \ 175, \ 210$$

Therefore, the maximum number of ski runs that Graham and Olga can do within 180 minutes is five.

Problem of the Week
Problem A
Tabulating Tokens

In the Game of Threes, players gather tokens. There are four kinds of tokens: lead, bronze, silver, and gold. Bronze tokens are 10 times more valuable than lead, silver tokens are 10 times more valuable than bronze, and gold tokens are 10 times more valuable than silver.

In the game, whenever you have three of the same kind of token, you can trade them for one of the next most valuable token.



The goal is to get as many of the most valuable tokens as possible. For example it would be better to have 1 gold token than 20 lead tokens, since gold tokens are $10 \times 10 \times 10 = 1000$ times more valuable than lead tokens.

(a) Suppose you start out with 2 lead, 1 bronze, 2 silver, and 1 gold tokens, and on your next turn you gather 1 more of each type. After making the best possible trades, how many lead, bronze, silver, and gold tokens would you have?

(b) Suppose you start out with 43 lead tokens. After making the best possible trades, how many lead, bronze, silver, and gold tokens would you have?

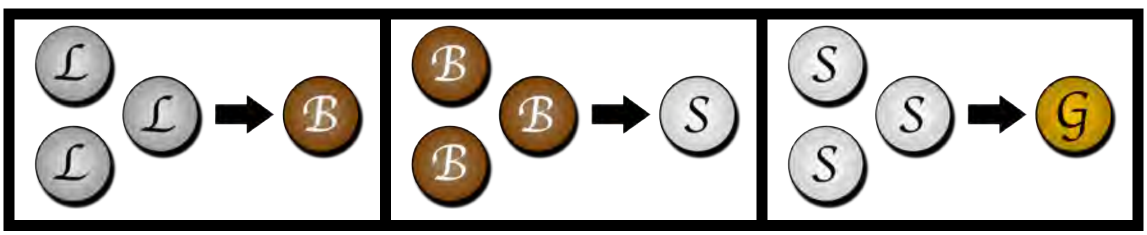
Themes Algebra, Computational Thinking

Problem of the Week
Problem A and Solution
Tabulating Tokens

Problem

In the Game of Threes, players gather tokens. There are four kinds of tokens: lead, bronze, silver, and gold. Bronze tokens are 10 times more valuable than lead, silver tokens are 10 times more valuable than bronze, and gold tokens are 10 times more valuable than silver.

In the game, whenever you have three of the same kind of token, you can trade them for one of the next most valuable token.



The goal is to get as many of the most valuable tokens as possible. For example it would be better to have 1 gold token than 20 lead tokens, since gold tokens are $10 \times 10 \times 10 = 1000$ times more valuable than lead tokens.

(a) Suppose you start out with 2 lead, 1 bronze, 2 silver, and 1 gold tokens, and on your next turn you gather 1 more of each type. After making the best possible trades, how many lead, bronze, silver, and gold tokens would you have?

(b) Suppose you start out with 43 lead tokens. After making the best possible trades, how many lead, bronze, silver, and gold tokens would you have?

Solution

(a) After gathering 1 more coin of each type, we will have 3 lead, 2 bronze, 3 silver, and 2 gold tokens. Now we can start making some trades. The following table shows the results after each trade:

Tokens Before Trade	**Trade**	**Tokens After Trade**
3 lead, 2 bronze, 3 silver, 2 gold	3 lead for 1 bronze	0 lead, 3 bronze, 3 silver, 2 gold
0 lead, 3 bronze, 3 silver, 2 gold	3 bronze for 1 silver	0 lead, 0 bronze, 4 silver, 2 gold
0 lead, 0 bronze, 4 silver, 2 gold	3 silver for 1 gold	0 lead, 0 bronze, 1 silver, 3 gold

This means we end up with 0 lead, 0 bronze, 1 silver, and 3 gold tokens after making the best possible trades. Note that we could have made a trade of 3 silver for 1 gold as our first or second trade and it would not have changed the final token tally.

(b) If we start with 43 lead tokens, we can trade as many groups of three as possible into bronze tokens. We can use skip counting or division to determine that there are 14 groups of 3 lead tokens with 1 lead token left over. Now we can take the 14 groups of 3 lead tokens and trade them for 14 bronze tokens. Next, we can determine that there are 4 groups of 3 bronze tokens with 2 bronze tokens left over. Now we can trade the 4 groups of 3 bronze tokens for 4 silver tokens. Finally, we can trade 3 silver tokens for 1 gold token with 1 silver token left over.

Here is a summary table of the trades:

Tokens Before Trade	**Trade**	**Tokens After Trade**
43 lead, 0 bronze, 0 silver, 0 gold	42 lead for 14 bronze	1 lead, 14 bronze, 0 silver, 0 gold
1 lead, 14 bronze, 0 silver, 0 gold	12 bronze for 4 silver	1 lead, 2 bronze, 4 silver, 0 gold
1 lead, 2 bronze, 4 silver, 0 gold	3 silver for 1 gold	1 lead, 2 bronze, 1 silver, 1 gold

This means we end up with 1 lead, 2 bronze, 1 silver, and 1 gold token.

Teacher's Notes

The rules of this game mimic rules that can be used in a *context-free grammar*. Each rule describes a substitution that can take place as you move from a starting point to an end point.

A context-free grammar is used in other disciplines including:

- Linguistics - a way to describe the grammar rules of a natural language
- Biology - a way to describe biological change such as the growth of plants
- Computer Science - a way to describe the rules of a programming language

When you follow the rules from a starting point (such as starting with 43 lead coins) to a successful end point, this is called a *derivation*. This process is also called *parsing*. In Computer Science, parsing is an important step in converting code that people write in a programming language such as Java or Python into a form that a computer can understand and execute.

Data Management (D)

TAKE ME TO THE COVER

Problem of the Week

Problem A

When were you born?

Have you ever thought about whether you were born on an even-numbered or an odd-numbered day of the month? In my family, the four of us were born on odd-numbered days: February 19th, March 15th, September 7th, and December 3rd. Our daughter was born on September 4th, an even-numbered day.

Our best friend just had a baby born in 2021.

(a) Is it more likely that the baby was born on an even-numbered day or an odd-numbered day? Justify your answer.

(b) What is the probability that their baby was born on a day that is a multiple of 5?

Problem of the Week
Problem A and Solution
When were you born?

Problem

Have you ever thought about whether you were born on an even-numbered or an odd-numbered day of the month? In my family, the four of us were born on odd-numbered days: February 19th, March 15th, September 7th, and December 3rd. Our daughter was born on September 4th, an even-numbered day.

Our best friend just had a baby born in 2021.

(a) Is it more likely that the baby was born on an even-numbered day or an odd-numbered day? Justify your answer.

(b) What is the probability that their baby was born on a day that is a multiple of 5?

Solution

We assume that there is an equal probability of being born on any day of the year.

(a) One way to solve this problem is to count the number of even-numbered days in a year. We can make a table that records the number of even-numbered days for each month:

Month	Number of Even-numbered Days
Jan	15
Feb	14
Mar	15
Apr	15
May	15
Jun	15
Jul	15
Aug	15
Sep	15
Oct	15
Nov	15
Dec	15

Adding up the number of even-numbered days in each month, we see that there are 179 even-numbered days in the year. We might make the adding easier by noticing this is equal to $15 + 14 + (10 \times 15) = 29 + 150 = 179$.

Since 2021 is not a leap year, there are 365 days in that year. Since all days must be either even or odd, then the number of odd-numbered days in the year is $365 - 179 = 186$.

Since $186 > 179$, we can see that it is more likely for the baby to be born on an odd-numbered day.

(b) Again we can make a table recording the number of days that are multiples of 5 for each month of the year:

Month	**Number of Days that are Multiples of 5**
Jan	6
Feb	5
Mar	6
Apr	6
May	6
Jun	6
Jul	6
Aug	6
Sep	6
Oct	6
Nov	6
Dec	6

When we add these numbers together we get 71. To calculate the probability that the baby was born on a day that is a multiple of 5, we divide the number of days that are multiples of 5 by the total number of days in a year.

So the probability that the baby was born on a day that is a multiple of 5 is $\frac{71}{365}$.

Teacher's Notes

In the solution, we assumed that there is an equal probability of being born on any day of the year. However, if we look at real data we would conclude that assumption is not correct. Statistics Canada is a great resource for real data. The following link,

https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1310041501

provides recent data listing the number of births in Canada by month.

When we look at the data, it is not surprising to see that there are fewer births during the month of February than during the month of March. We could predict this ourselves since there are only 28 or 29 days in February, but 31 days in March. However, we might be surprised to see that, in this data, there are consistently more births during the month of August than during the month of December. The difference is more than 10%, despite the fact that both months have 31 days.

Although we can make reasonable guesses based on probabilities, there may be factors that we cannot know or that we miss when making decisions. This does not mean we should ignore the statistical methods, it just means we have to realize there are limitations on what we can know for certain.

Problem of the Week
Problem A
One Room Schoolhouse

The terms *mean*, *median*, and *mode* are defined at the bottom of the page.

The following list of numbers represents the ages of students in the one-room schoolhouse in Muggleland:

7, 9, 15, 9, 14, 13, 14, 7, 6, 10, 12, 8, 15, 14, 12,
8, 7, 12, 12, 16, 14, 12, 7, 9, 10, 8, 12, 14, 11, 13

(a) Are the mode and the median of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.

(b) Are the median and the mean of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.

Mode refers to the most frequently occurring number in a data set. If there is a tie, then we assign more than one number as the modes of the data set.

Median refers to the middle number in a data set after the numbers have been arranged in order. If a data set has an even number of values, then there are two "middle numbers". In this case we calculate the sum of the two numbers and divide by 2 to get the median of the data set.

Mean refers to the result of calculating the sum of the numbers in the data set and then dividing the sum by the number of values in the data set. This is what is commonly called the *average*.

Themes Data Management, Number Sense

Problem of the Week
Problem A and Solution
One Room Schoolhouse

Problem

The following list of numbers represents the ages of students in the one-room schoolhouse in Muggleland:

$$7,\ 9,\ 15,\ 9,\ 14,\ 13,\ 14,\ 7,\ 6,\ 10,\ 12,\ 8,\ 15,\ 14,\ 12,$$
$$8,\ 7,\ 12,\ 12,\ 16,\ 14,\ 12,\ 7,\ 9,\ 10,\ 8,\ 12,\ 14,\ 11,\ 13$$

(a) Are the mode and the median of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.

(b) Are the median and the mean of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.

Mode refers to the most frequently occurring number in a data set. If there is a tie, then we assign more than one number as the modes of the data set.

Median refers to the middle number in a data set after the numbers have been arranged in order. If a data set has an even number of values, then there are two "middle numbers". In this case we calculate the sum of the two numbers and divide by 2 to get the median of the data set.

Mean refers to the result of calculating the sum of the numbers in the data set and then dividing the sum by the number of values in the data set. This is what is commonly called the *average*.

Solution

(a) We can start by filling in a tally chart to count how many times each age appears in our data set.

Age	6	7	8	9	10	11	12	13	14	15	16
Tally	𝍷	𝍷𝍷𝍷𝍷	𝍷𝍷𝍷	𝍷𝍷𝍷	𝍷𝍷	𝍷	𝍸𝍷	𝍷𝍷	𝍸	𝍷𝍷	𝍷

The most frequently occurring value is 12, so the mode is 12.

We can use the tally chart to list the values in order:

$$6,\ 7,\ 7,\ 7,\ 7,\ 8,\ 8,\ 8,\ 9,\ 9,\ 9,\ 10,\ 10,\ 11,\ 12,$$
$$12,\ 12,\ 12,\ 12,\ 12,\ 13,\ 13,\ 14,\ 14,\ 14,\ 14,\ 14,\ 15,\ 15,\ 16$$

Since there is an even number of ages in this data set, we need to calculate the median using the two middle numbers. In this case, we calculate the sum of the 15th and 16th numbers to get $12 + 12 = 24$ and then divide by 2 to get $24 \div 2 = 12$.

So, the median of this set of numbers is 12.

For this data set, the mode and the median are the same. However we would not expect that to always be the case. For example, in the data set

$$1, 1, 1, 1, 2, 3, 4, 5, 6$$

the mode is 1, but the median is 2.

(b) To determine the mean, we could add up the 30 numbers in the data set to find the sum, but another way to calculate this total would be to use our tally chart again. For each age, we calculate the product of the age and the tally:

Age	6	7	8	9	10	11	12	13	14	15	16
Tally	1	4	3	3	2	1	6	2	5	2	1
Product	6	28	24	27	20	11	72	26	70	30	16

Now, instead of adding 30 numbers together, we only need to find the sum of the 11 products to determine the total to be

$$6 + 28 + 24 + 27 + 20 + 11 + 72 + 26 + 70 + 30 + 16 = 330$$

Now, to find the mean we can use skip counting or divide by the number of ages, which is 30, to get $330 \div 30 = 11$.

So, the mean is 11.

For this data set, the mean and the median are different. However, we would not expect that to always be the case. For example, in the data set

$$5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5$$

the mean is 5 and the median is also 5.

Problem of the Week

Problem A

Ice Cream Party!

The Grade 3 and Grade 4 classes of Wapella Public School are having an ice cream party. Each student gets one scoop of ice cream. The teachers conducted a survey to find out what ice cream flavours the students wanted.

(a) Fill in the missing vote tallies and frequencies in the following tables.

Grade 3: 78 students polled

Flavour	Vote Tally	Frequency
chocolate	𝍸 𝍸 𝍸 ||	
vanilla	𝍸 𝍸 𝍸 𝍸 |||	
strawberry	𝍸 𝍸 |||	
butterscotch		
mint chip	𝍸 ||||	
bubble gum	𝍸 𝍸 ||	

Grade 4: 82 students polled

Flavour	Vote Tally	Frequency
chocolate	𝍸 𝍸 𝍸 𝍸 |	
vanilla	𝍸 𝍸 𝍸 𝍸	
strawberry		
butterscotch	𝍸 𝍸 𝍸 ||||	
mint chip	|||	
bubble gum	𝍸 𝍸 |||	

(b) How many scoops of strawberry ice cream are needed for the party?

(c) A tub of ice cream contains enough for 6 scoops. How many tubs of each flavour of ice cream will the teachers need to buy?

(d) If each tub of ice cream costs $4, how much will the ice cream for this party cost?

THEME DATA MANAGEMENT

Problem of the Week
Problem A and Solution
Ice Cream Party!

Problem

The Grade 3 and Grade 4 classes of Wapella Public School are having an ice cream party. Each student gets one scoop of ice cream. The teachers conducted a survey to find out what ice cream flavours the students wanted.

(a) Fill in the missing vote tallies and frequencies in the following tables.

Grade 3: 78 students polled

Flavour	Vote Tally	Frequency
chocolate	卌 卌 卌 ||	
vanilla	卌 卌 卌 卌 |||	
strawberry	卌 卌 |||	
butterscotch		
mint chip	卌 ||||	
bubble gum	卌 卌 ||	

Grade 4: 82 students polled

Flavour	Vote Tally	Frequency
chocolate	卌 卌 卌 卌 |	
vanilla	卌 卌 卌 卌	
strawberry		
butterscotch	卌 卌 卌 ||||	
mint chip	|||	
bubble gum	卌 卌 |||	

(b) How many scoops of strawberry ice cream are needed for the party?

(c) A tub of ice cream contains enough for 6 scoops. How many tubs of each flavour of ice cream will the teachers need to buy?

(d) If each tub of ice cream costs $4, how much will the ice cream for this party cost?

Solution

(a) The completed tables are below.

Grade 3: 78 students polled

Flavour	**Vote Tally**	**Frequency**
chocolate	𝍸 𝍸 𝍸 ||	17
vanilla	𝍸 𝍸 𝍸 𝍸 |||	23
strawberry	𝍸 𝍸 |||	13
butterscotch	||||	4
mint chip	𝍸 ||||	9
bubble gum	𝍸 𝍸 ||	12

Grade 4: 82 students polled

Flavour	**Vote Tally**	**Frequency**
chocolate	𝍸 𝍸 𝍸 𝍸 |	21
vanilla	𝍸 𝍸 𝍸 𝍸	20
strawberry	𝍸 |	6
butterscotch	𝍸 𝍸 𝍸 ||||	19
mint chip	|||	3
bubble gum	𝍸 𝍸 |||	13

The known frequencies for the Grade 3 class total $17 + 23 + 13 + 9 + 12 = 74$, which leaves $78 - 74 = 4$ students who selected butterscotch.

The known frequencies for the Grade 4 class total $21 + 20 + 19 + 3 + 13 = 76$, which leaves $82 - 76 = 6$ students who selected strawberry.

(b) Since 13 students in Grade 3 selected strawberry and 6 students in Grade 4 selected strawberry, a total of $13 + 6 = 19$ scoops of strawberry ice cream are needed for the party.

(c) To determine the number of tubs of ice cream to buy, we first need to find the total number of scoops selected for each flavour. We could then use skip counting to determine the number of tubs to buy. We would need to skip to at least the amount of scoops wanted in total and possibly beyond the actual number of scoops needed. For example, we need a total of $17 + 21 = 38$ scoops of chocolate ice cream. Skip counting by 6, we have 6, 12, 18, 24, 36, 42, and we get beyond 38 on the seventh value. So we need 7 tubs of chocolate ice cream. We can go through a similar process for the other flavours.

Alternatively, we could divide the total number of scoops for each flavour by 6, and round up if necessary. We must round up since we need to ensure we have enough scoops to serve everyone that wants a particular flavour.

	chocolate	vanilla	strawberry	butterscotch	mint chip	bubble gum
scoops selected	$17+21 = 38$	$23+20 = 43$	$13 + 6 = 19$	$4 + 19 = 23$	$9 + 3 = 12$	$12+13 = 25$
scoops divided by 6	$38 \div 6 = 6$ remainder 2	$43 \div 6 = 7$ remainder 1	$19 \div 6 = 3$ remainder 1	$23 \div 6 = 3$ remainder 5	$12 \div 6 = 2$ remainder 0	$25 \div 6 = 4$ remainder 1

Rounding up where necessary, this means we need 7 tubs of chocolate, 8 tubs of vanilla, 4 tubs of strawberry, 4 tubs of butterscotch, 2 tubs of mint chip, and 5 tubs of bubble gum.

(d) Since we need a total of $7 + 8 + 4 + 4 + 2 + 5 = 30$ tubs of ice cream, and each tub costs \$4, the ice cream at this party will cost a total of $30 \times 4 = \$120$.

Computational Thinking (C)

TAKE ME TO THE COVER

Problem of the Week

Problem A

What Number Am I?

I am a 3-digit number.

All of my digits are even numbers greater than 1.

My hundreds digit is greater than my tens digit.

My tens digit minus my ones digit is twice as much as my hundreds digit minus my tens digit.

My tens digit is a multiple of 3.

What number am I?

Themes Computational Thinking, Number Sense

Problem of the Week
Problem A and Solution
What Number Am I?

Problem

I am a 3-digit number.

All of my digits are even numbers greater than 1.

My hundreds digit is greater than my tens digit.

My tens digit minus my ones digit is twice as much as my hundreds digit minus my tens digit.

My tens digit is a multiple of 3.

What number am I?

Solution

First, we note that the even digits that are greater than 1 are 2, 4, 6, and 8. Thus, the only possible digits in the number are 2, 4, 6, and 8.

Since the tens digit is also a multiple of 3, the tens digit must be 6. This is the only number in the list above that is a multiple of 3.

Since the hundreds digit is greater than the tens digit, the hundreds digit must be 8. This is the only number in the list above that is greater than 6.

The difference between the hundreds digit and the tens digit is $8 - 6 = 2$. Twice that difference is $2 \times 2 = 4$. Now, we need a number such that when we subtract it from 6, we get 4. Since $6 - 2 = 4$, the ones digit must be 2.

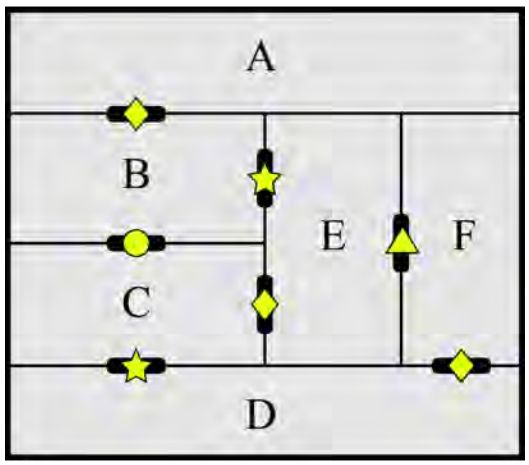
Therefore, the number is 862.

Problem of the Week

Problem A

Finding Treasure

You have been invited to an old house with six rooms, one of which contains a treasure. A floor plan of the house is shown below. Each room is labelled with a letter. Some rooms are connected by doors, which are locked. Each door is labelled with the symbol of the key that will unlock it.



You start in Room A and want to get to the treasure in Room F. What is the fewest number of keys you need to get to the treasure? What are the symbols on those keys?

Not printing this page? You can use our interactive worksheet.

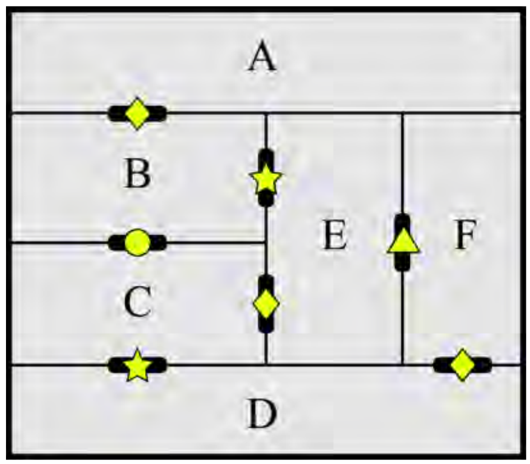
Problem of the Week

Problem A and Solution

Finding Treasure

Problem

You have been invited to an old house with six rooms, one of which contains a treasure. A floor plan of the house is shown below. Each room is labelled with a letter. Some rooms are connected by doors, which are locked. Each door is labelled with the symbol of the key that will unlock it.



You start in Room A and want to get to the treasure in Room F. What is the fewest number of keys you need to get to the treasure? What are the symbols on those keys?

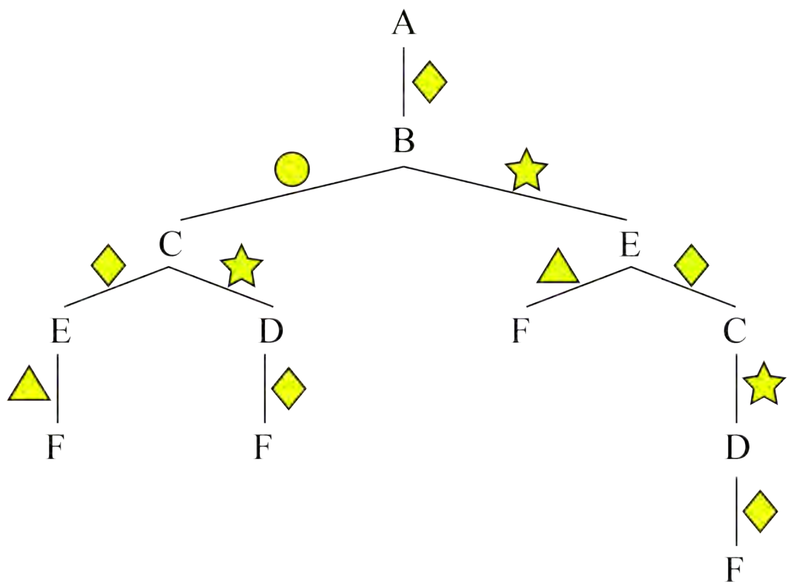
Not printing this page? You can use our interactive worksheet.

Solution

One way to solve this problem would be to list all the different paths that you can take from Room A to Room F, and record the keys required to open the doors along the way. Note that there would be no reason to return to a room that you have already visited on a particular path. Then you can compare the paths to see which one requires the least number of keys.

One way to organize this information is using a tree diagram. Since we start in Room A, Room A appears at the top of the tree (first level). Since Room A is connected by a door only to Room B, we draw a single line from A to B (in the second level of the tree). Since the door between Room A and Room B requires a diamond key, we label the line connecting A and B with a diamond. We next consider the different possible doors out of Room B and build the rest of the tree in a similar way.

The complete tree diagram is shown below.

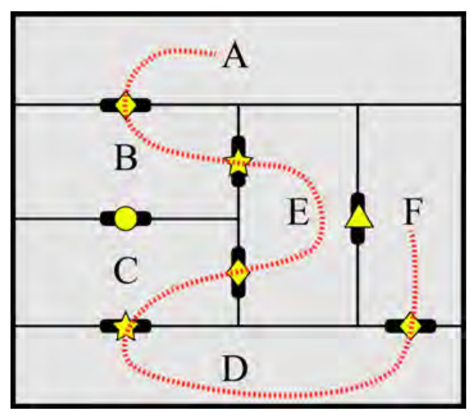


Using this tree diagram we can see the keys required for each path. For example, the path A → B → C → D → F requires three different keys: diamond, circle, and star.

However, the path A → B → E → C → D → F requires only two keys: diamond and star.

This path is shown on the map.

When we check the other three paths in the tree, they all require three different keys, so this path requires the least number of keys.



We also note that to leave Room A we require a diamond key, and to leave Room B we require a different key. We can conclude that we require *at least* two keys to get from Room A to Room F. Since we found a path that requires exactly two keys, this is the least number of keys required.

Therefore, the fewest number of keys required to get to the treasure is two. The symbols on the keys are the diamond and the star.

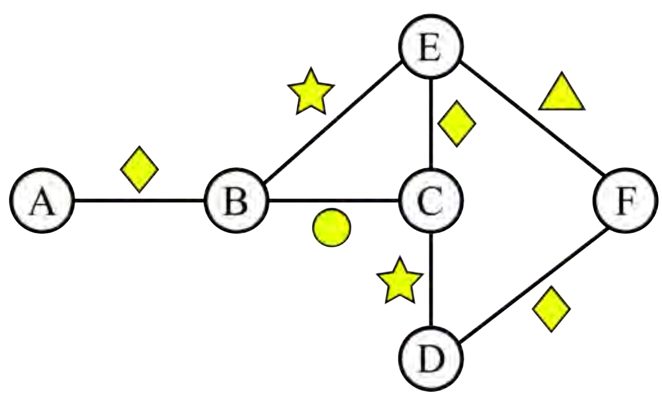
Teacher's Notes

When looking at solving real-world problems, the first step is often to create a mathematical model of the situation. We sometimes refer to this process as *abstraction*.

One form of abstraction is called a graph. A graph is a mathematical model that uses circles we call *nodes* or *vertices* to represent objects, and lines we call *edges* to represent links between the objects. The edges between nodes may have labels on them to indicate how the nodes are linked logically. The edges may also have arrows indicating that a link between nodes has a specific direction. Edges with arrows indicate this is a *directed* graph.

We can use a graph to represent many different real-world systems. For example we could represent cities on a map as nodes and the roads between them as edges. Another example is to represent webpages on the internet as nodes, and the links you click to get from one page to another as edges.

The key to abstraction is to keep the important information and ignore the irrelevant details. We could represent the information in this problem with a graph. The nodes could represent the rooms, and the edges could represent the doors connecting the rooms. Each edge would be labelled with the symbol for the key required to unlock the door. Since you can move in either direction through a door from one room to another, we would use an *undirected* graph. Here is a graph representing the information in this problem.



The graph has all the information we need to find paths from Room A to Room F and the keys necessary to get from one room to another.

Problem of the Week
Problem A
Family Facts

Myra has a school project to describe her family. As part of this, she wants to find out the ages of all the people in her family.

Myra knows that the following facts are true today.

Myra is 9 years old.

Myra's sister Vivienne is 3 years older than her.

Myra was 2 years old when her baby brother Jacob was born and today is Jacob's birthday.

Myra's mother Aven is 36 years old and her father Pierce is 5 years older than her mother.

Myra's grandmother Teagan was 55 years old when Jacob was born.

Myra's grandfather John was 49 when Vivienne was born and they have the same birthday.

What are the ages of everyone in Myra's family today?

Problem of the Week

Problem A and Solution

Family Facts

Problem

Myra has a school project to describe her family. As part of this, she wants to find out the ages of all the people in her family.

Myra knows that the following facts are true today.

Myra is 9 years old.

Myra's sister Vivienne is 3 years older than her.

Myra was 2 years old when her baby brother Jacob was born and today is Jacob's birthday.

Myra's mother Aven is 36 years old and her father Pierce is 5 years older than her mother.

Myra's grandmother Teagan was 55 years old when Jacob was born.

Myra's grandfather John was 49 when Vivienne was born and they have the same birthday.

What are the ages of everyone in Myra's family today?

Solution

Since Myra is 9 years old and Vivienne is 3 years older, then Vivienne is $9 + 3 = 12$ years old today.

Since Myra was 2 years old when Jacob was born and today is Jacob's birthday, then Jacob is $9 - 2 = 7$ years old today.

Since Aven is 36 years old and Pierce is 5 years older, then Pierce is $36 + 5 = 41$ years old today.

Since Teagan was 55 years old when Jacob was born and we know that was 7 years ago, then Teagan is $55 + 7 = 62$ years old today.

Since John was 49 years old when Vivienne was born on his birthday and Vivienne is 12 years old today, then John is $49 + 12 = 61$ years old today.

In summary, the ages of everyone in Myra's family today are as follows.

- Myra's brother Jacob is 7 years old.
- Myra is 9 years old.
- Myra's sister Vivienne is 12 years old.
- Myra's mother Aven is 36 years old.
- Myra's father Pierce is 41 years old.
- Myra's grandfather John is 61 years old.
- Myra's grandmother Teagan is 62 years old.

Teacher's Notes

In math word problems we often see some extraneous information that is used to make the text more interesting. This means that we need to determine what facts are necessary to solve the problem. In this problem there are some details that may seem unnecessary, but they are actually critically important to calculating the definitive answer.

There would be more than one possible answer to this problem if we did not know that it was Jacob's birthday today. Suppose we did not know that today was Jacob's birthday. Knowing that Myra was 2 years old when Jacob was born is not be enough information to know how old Jacob is on other days of the year. For example, if her birthday was one day after Jacob's and today was Myra's birthday, then Myra's age today would actually be 3 years more than Jacob's age.

When we are solving problems, we want them to be *deterministic*. This means there is a single, correct solution and we can be confident that we have the right answer. Unfortunately, real life problems do not always fall into this category.

Problem of the Week

Problem A

Track and Field

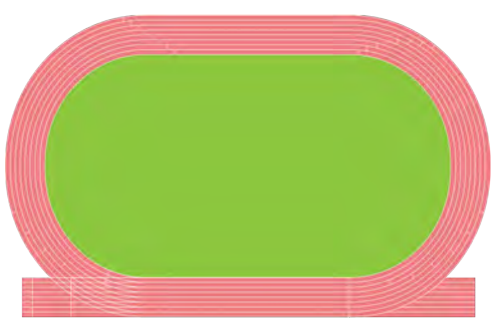
Andre, Bilal, Glenna, and Juanita are all friends. They have given each other nicknames as well. The four nicknames are Boss, Buzz, Cosmo, and Tiger. The friends are competing in the track and field day at their school. Using the following clues, determine the nickname of each of the friends.

Clues:

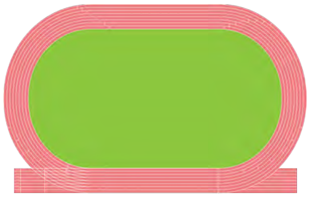
1. Juanita beat Tiger in the 400 m race, but Juanita lost to Buzz in the high jump.
2. Glenna and Tiger tied in the long jump, but Boss beat Glenna in the 1500 m race.
3. Cosmo, Tiger and Andre found the water station together after the first event.
4. Cosmo and Juanita ran separate legs on the relay race.

The table below, with a column for each name and a row for each nickname, may be helpful in solving this problem.

	Andre	Bilal	Glenna	Juanita
Boss				
Buzz				
Cosmo				
Tiger				



Theme Computational Thinking



Problem of the Week

Problem A and Solution

Track and Field

Problem

Andre, Bilal, Glenna, and Juanita are all friends. They have given each other nicknames as well. The four nicknames are Boss, Buzz, Cosmo, and Tiger. The friends are competing in the track and field day at their school. Using the following clues, determine the nickname of each of the friends.

Clues:

1. Juanita beat Tiger in the 400 m race, but Juanita lost to Buzz in the high jump.
2. Glenna and Tiger tied in the long jump, but Boss beat Glenna in the 1500 m race.
3. Cosmo, Tiger and Andre found the water station together after the first event.
4. Cosmo and Juanita ran separate legs on the relay race.

A table with a column for each name and a row for each nickname may be helpful in solving this problem.

Solution

We will give the final answer and then show the solution. Andre's nickname is Buzz, Bilal's nickname is Tiger, Glenna's nickname is Cosmo, and Juanita's nickname is Boss.

Now, each clue eliminates at least one possible person/nickname pair. If *Person A* is doing something with *Person B*, then *Person A* and *Person B* must be different people. We can put an ✘ in the table where we know someone does not match a nickname.

There are several ways to determine the correct nicknames; here is one way. We can start with the first clue: *Juanita beat Tiger in the 400 m race, but Juanita lost to Buzz in the high jump.*

Using this information, we can update the table by adding two ✘s in the column for Juanita: an ✘ in the cell for Buzz and an ✘ in the cell for Tiger.

	Andre	Bilal	Glenna	Juanita
Boss				
Buzz				✘
Cosmo				
Tiger				✘

From the second, third, and fourth clues, we can add five new ✘s to the table as follows.

	Andre	Bilal	Glenna	Juanita
Boss			✘	
Buzz				✘
Cosmo	✘			✘
Tiger	✘		✘	✘

At this point, we know that *Juanita* must have the nickname *Boss*, since this is the only nickname left in her column. We can put a check in that box, and eliminate *Boss* as the possible nickname for everyone else, by putting an ✘ in the remaining empty cells in the row for Boss.

	Andre	Bilal	Glenna	Juanita
Boss	✘	✘	✘	✔
Buzz				✘
Cosmo	✘			✘
Tiger	✘		✘	✘

Now we can conclude that *Andre* must have the nickname *Buzz*, since this is the only nickname left in his column. We can put a check in that box, and eliminate *Buzz* as the possible nickname for everyone else.

	Andre	Bilal	Glenna	Juanita
Boss	✘	✘	✘	✔
Buzz	✔	✘	✘	✘
Cosmo	✘			✘
Tiger	✘		✘	✘

Next we can conclude that *Glenna* must have the nickname *Cosmo*, since this is the only nickname left in her column. We can put a check in that box, and eliminate *Cosmo* as the possible nickname for *Bilal*, who is the only person left. The only nickname left is *Tiger*. That must be *Bilal's* nickname. The final version of the table is below.

	Andre	Bilal	Glenna	Juanita
Boss	✘	✘	✘	✔
Buzz	✔	✘	✘	✘
Cosmo	✘	✘	✔	✘
Tiger	✘	✔	✘	✘

So Andre's nickname is Buzz, Bilal's nickname is Tiger, Glenna's nickname is Cosmo, and Juanita's nickname is Boss.

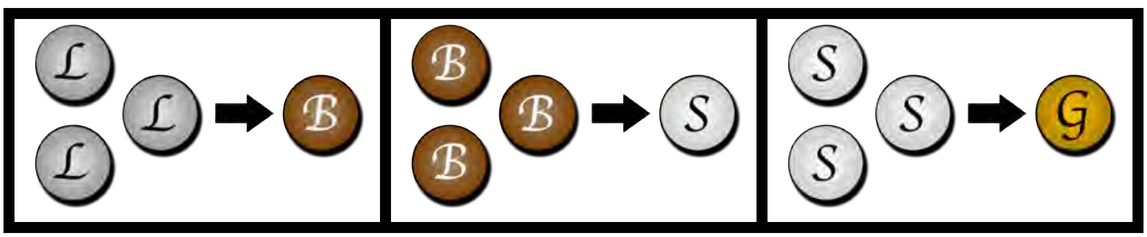
Problem of the Week

Problem A

Tabulating Tokens

In the Game of Threes, players gather tokens. There are four kinds of tokens: lead, bronze, silver, and gold. Bronze tokens are 10 times more valuable than lead, silver tokens are 10 times more valuable than bronze, and gold tokens are 10 times more valuable than silver.

In the game, whenever you have three of the same kind of token, you can trade them for one of the next most valuable token.



The goal is to get as many of the most valuable tokens as possible. For example it would be better to have 1 gold token than 20 lead tokens, since gold tokens are $10 \times 10 \times 10 = 1000$ times more valuable than lead tokens.

(a) Suppose you start out with 2 lead, 1 bronze, 2 silver, and 1 gold tokens, and on your next turn you gather 1 more of each type. After making the best possible trades, how many lead, bronze, silver, and gold tokens would you have?

(b) Suppose you start out with 43 lead tokens. After making the best possible trades, how many lead, bronze, silver, and gold tokens would you have?

Themes Algebra, Computational Thinking

Problem of the Week

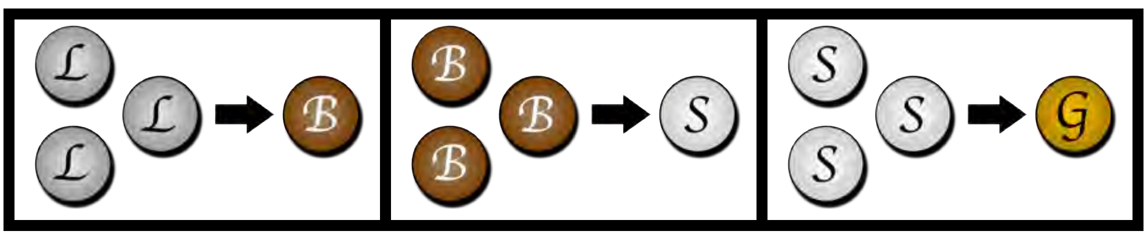
Problem A and Solution

Tabulating Tokens

Problem

In the Game of Threes, players gather tokens. There are four kinds of tokens: lead, bronze, silver, and gold. Bronze tokens are 10 times more valuable than lead, silver tokens are 10 times more valuable than bronze, and gold tokens are 10 times more valuable than silver.

In the game, whenever you have three of the same kind of token, you can trade them for one of the next most valuable token.



The goal is to get as many of the most valuable tokens as possible. For example it would be better to have 1 gold token than 20 lead tokens, since gold tokens are $10 \times 10 \times 10 = 1000$ times more valuable than lead tokens.

(a) Suppose you start out with 2 lead, 1 bronze, 2 silver, and 1 gold tokens, and on your next turn you gather 1 more of each type. After making the best possible trades, how many lead, bronze, silver, and gold tokens would you have?

(b) Suppose you start out with 43 lead tokens. After making the best possible trades, how many lead, bronze, silver, and gold tokens would you have?

Solution

(a) After gathering 1 more coin of each type, we will have 3 lead, 2 bronze, 3 silver, and 2 gold tokens. Now we can start making some trades. The following table shows the results after each trade:

Tokens Before Trade	Trade	Tokens After Trade
3 lead, 2 bronze, 3 silver, 2 gold	3 lead for 1 bronze	0 lead, 3 bronze, 3 silver, 2 gold
0 lead, 3 bronze, 3 silver, 2 gold	3 bronze for 1 silver	0 lead, 0 bronze, 4 silver, 2 gold
0 lead, 0 bronze, 4 silver, 2 gold	3 silver for 1 gold	0 lead, 0 bronze, 1 silver, 3 gold

This means we end up with 0 lead, 0 bronze, 1 silver, and 3 gold tokens after making the best possible trades. Note that we could have made a trade of 3 silver for 1 gold as our first or second trade and it would not have changed the final token tally.

(b) If we start with 43 lead tokens, we can trade as many groups of three as possible into bronze tokens. We can use skip counting or division to determine that there are 14 groups of 3 lead tokens with 1 lead token left over. Now we can take the 14 groups of 3 lead tokens and trade them for 14 bronze tokens. Next, we can determine that there are 4 groups of 3 bronze tokens with 2 bronze tokens left over. Now we can trade the 4 groups of 3 bronze tokens for 4 silver tokens. Finally, we can trade 3 silver tokens for 1 gold token with 1 silver token left over.

Here is a summary table of the trades:

Tokens Before Trade	**Trade**	**Tokens After Trade**
43 lead, 0 bronze, 0 silver, 0 gold	42 lead for 14 bronze	1 lead, 14 bronze, 0 silver, 0 gold
1 lead, 14 bronze, 0 silver, 0 gold	12 bronze for 4 silver	1 lead, 2 bronze, 4 silver, 0 gold
1 lead, 2 bronze, 4 silver, 0 gold	3 silver for 1 gold	1 lead, 2 bronze, 1 silver, 1 gold

This means we end up with 1 lead, 2 bronze, 1 silver, and 1 gold token.

Teacher's Notes

The rules of this game mimic rules that can be used in a *context-free grammar*. Each rule describes a substitution that can take place as you move from a starting point to an end point.

A context-free grammar is used in other disciplines including:

- Linguistics - a way to describe the grammar rules of a natural language
- Biology - a way to describe biological change such as the growth of plants
- Computer Science - a way to describe the rules of a programming language

When you follow the rules from a starting point (such as starting with 43 lead coins) to a successful end point, this is called a *derivation*. This process is also called *parsing*. In Computer Science, parsing is an important step in converting code that people write in a programming language such as Java or Python into a form that a computer can understand and execute.