Problem of the Month

Problem 7: April 2024

Hint

(a) In part (ii), solve for t in terms of x and then substitute into the equation involving y.

(b) At time t, how far is the particle from the origin?

(c) Starting with $y^2 = (\sin 2t)^2$, use trigonometric identities to eliminate all appearances of the variable t. Remember that $x = \cos t$.

(d) As Circle 2 rolls around Circle 1, let t be the angle made by the positive x-axis and the line segment connecting the origin and the centre of Circle 2. It will help to draw a reasonably accurate diagram with Circle 2 rolled part of the way around Circle 1 (perhaps an angle of $\frac{\pi}{3}$ or so). Once you have done this, mark the point on the circumference of Circle 2 that was originally at $(1, 0)$ by P (or some other label). The objective is to find the coordinates of P in terms of t. Since the circles roll without slipping, the arclength from the point of tangency along Circle 1 to $(1, 0)$ should equal the arclength from the point of tangency along Circle 2 to P.

(e) As Circle 2 rolls along the inside of Circle 1, it (usually) intersects the x-axis at two points. Convince yourself that one of these two points must be the origin, then think about the other point.

(f) Using a strategy similar to part (d), find a general formula for the coordinates of P in terms of the angle t. Do this either in general for r or do it separately for the three relevant values of r in this question.

  (i) Find identities for $\cos 3t$ in terms of $\cos t$ and $\sin 3t$ in terms of $\sin t$.

  (ii) Find the parametric equations for the position of P when $r = \frac{1}{3}$ if Circle 2 is rolled clockwise around Circle 1 instead of counterclockwise.