Problem of the Month

Problem 4: January 2024

Hint

(a) Any list that compresses $[1 : 9]$ must contain 1. Think about the largest possible number of integers in $f(A)$ when A is a list of length k.

(b) First, try to find a list that compresses $[1 : 63]$ that is as short as possible. It might help to read about the *binary representation* of positive integers.

(c) Work out a few more examples like the one in (b). It is possible to compress $[1 : n]$ using a list A that consists entirely or almost entirely of powers of 2.

(d) For $k \geq 3$ and $m \geq 2$, if A compresses $[m : m + k - 1]$, then A must contain m and $m + 1$.

(e) The answer is 39. Do not worry about trying to compress $[5 : k]$ using as short a list as possible. As well, *inductive* thinking could be useful here. Suppose you can show that there is some k with the property that $[5 : k]$, $[5 : k + 1]$, $[5 : k + 2]$, $[5 : k + 3]$, and $[5 : k + 4]$ are all compressible. Can you deduce that $[5 : n]$ is compressible for all $n \geq k$?