Grade 7/8 Math Circles
Solving Systems of Equations Using Matrices - Problem Set

1. Determine if the following systems of equations are underdetermined, overdetermined or balanced. Provide a brief explanation.

   (a) $7x - 20y = 4$
   $4x + 2y = 7$
   $3y - z = 100$

   (b) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{array}\right]$

   (c) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right]$

   (d) $y - 5x = -3$
   $10x + 7y - z = 0$

   (e) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{array}\right]$

   (f) $7 = -14x$
   $2x = -1$

2. Convert the following system of equations into an augmented matrix:

$$3a + 2b - 4c + 7d = 15$$
$$a - 3b - d = 0$$
$$4b + 4c = 5$$

3. Convert the following augmented matrix into a system of equations, with variables of your choosing:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 100 \\ 1 & -2 & 0 & 3 & 36 \\ 0 & -6 & 4 & -9 & -50 \\ 0 & 0 & 0 & 1 & 20 \end{array}\right]$$

4. Determine a unique solution for the following systems of equations using substitution. If there is no unique solution, state how many solutions the system has.

   (a) $18y - 2x = 6$
       $6x - 54y = 2$

   (b) $3x - y = 13$
       $3y - 2x = -4$

   (c) $4x + y = 7$
       $9y - 2x = -13$
       $2x - 9y = 13$

5. Determine a unique solution for the following systems of equations using matrices. If there is no unique solution, state how many solutions the system has.

   (a) $-3x + 10y = 6$
       $6y - x = 2$

   (b) $-8x + 4y = 30$
       $y = 2x$

   (c) $4x + y + z = 7$
       $9y - 2x + z = -13$
       $2x - 9y - z = 13$

Bonus Questions

6. At a county festival there are two types of tickets with different prices: **Adult**, which costs $5, and **Child**, which costs $2. On Saturday, 1000 people entered the festival and $3350 was collected in ticket sales.

   (a) Determine the number of adults and the number of children that visited the festival on Saturday using substitution.

(b) Determine the number of adults and the number of children that visited the festival on Saturday using matrices.

7. There are four teams competing in a sports tournament, which are simply known as Red Team, Blue Team, Pink Team, and Orange Team. Each team has many loyal fans that attend the tournament to cheer on their favourite team. We want to figure out exactly how many fans their are each team, but we only have the following information:

- The Red Team and Orange have 8100 fans combined.
- The number of Blue Team fans multiplied by 3 is 1700 more than the number of Pink Team fans multiplied by 2.
- The Orange Team has 400 fans more than the Pink Team.
- The combined number of Blue Team and Orange team fans is 1800 more than the number of Red Team fans.
- There are 14000 fans in total at the tournament.

Using either substitution or matrices, determine how many fans there are for each team.