Grade 6 Math Circles

Probability - Problem Set

1. Suppose we flip a coin three times.
   (a) What is the probability of getting Heads on the first coin flip?
   (b) What is the probability of getting Tails on the third coin flip?
   (c) What is the probability of not getting Tails on the second coin flip?
   (d) What is the probability of getting Heads on all three coin flips?
2. A sack contains marbles of different colours. There are 25 marbles in total, and the probability of drawing a red marble from the sack is 0.44.
   (a) How many red marbles are in the sack?
   (b) How many marbles in the sack are not red?
   (c) What is the probability of drawing a marble that is not red?
3. An aquarium contains two kinds of fish: clownfish and pufferfish. Let the event A = the fish is a pufferfish, with $|A| = 81$ and $P(A) = 0.54$.
   (a) How many fish are there in total?
   (b) What is $\overline{A}$ and $P(\overline{A})$?
   (c) Use any method to determine $|\overline{A}|$.
4. Use the formulas for the relationship between the probabilities of the union and intersection of events to answer the following.
   (a) Determine $P(A)$, for $P(A \cap B) = 0.25$, $P(A \cup B) = 0.52$ and $P(B) = 0.4$.
   (b) Determine $P(C \cup D)$, for $P(D) = 0.13$, $P(C) = 0.26$ and $P(C \cap D) = 0.09$.
   (c) Determine $P(E \cap F)$, for $P(E) = 0.38$, $P(E \cup F) = 0.77$ and $P(F) = 0.39$.
5. **TRUE** or **FALSE**: Is it possible to have the following probabilities? Justify your answer.

$$P(A) = 0.43 \qquad P(B) = 0.17 \qquad P(A \cup B) = 0.41 \qquad P(A \cap B) = 0.19$$

6. At an exclusive social event, each guest is given a wristband that is either blue, green, orange or yellow. Of the 500 guests that attend the event, 127 have blue wristbands, 98 have green wristbands, 143 have orange wristbands, and 132 have yellow wristbands. For a randomly selected person at the event, we have the events B = the person has a blue wristband, G = the

person has a green wristband, O = the person has an orange wristband, and Y = the person has a yellow wristband.

(a) Determine the probability of each of the events.

(b) Are the events is disjoint or not? Justify your answer.

(c) What is the probability that a randomly selected person has either a blue wristband or a yellow wristband?

(d) What is the probability that a randomly selected person doesn't have a blue wristband and doesn't have a yellow wristband?

7. In a candy jar, the candy is categorized by the following attributes that have no influence on one another: the candy is either sweet or sour; and the candy is either hard or soft. The events are A = the candy is soft, and B = the candy is sweet. There are a total of 1875 pieces of candy in the jar, with $P(A) = 0.36$ and $P(\overline{B}) = 0.52$.

(a) Determine S, $|A|$, $|\overline{A}|$, $|B|$ and $|\overline{B}|$.

(b) Are the events A and B independent or dependent? Explain.

(c) Determine $P(A \cap B)$, $P(A \cap \overline{B})$, $P(\overline{A} \cap B)$ and $P(\overline{A} \cap \overline{B})$.

(d) What is the sum of the probabilities from part (c)? Why is this the case?

Bonus Problems

8. Is it possible for two events A and B with non-zero probabilities to be both disjoint and independent? Explain. If it is possible then provide an example.

9. Suppose you are on a game show and are presented with 10 doors to pick from, where you win whatever is behind the door you pick. Behind one of these doors is a cash prize and behind the rest of the doors is nothing. Each door has the same probability of having the money behind it. Before you are able to pick a door, 6 of the doors that have nothing behind them are removed. You then pick one of the remaining doors. Once you've made your choice, even more doors with nothing behind them are removed until there are only 2 left: the door you picked, and one other door. You are then given the option of staying with the door you picked, or swapping it with the other door. What is the probability of winning the cash prize if you stay with your door? What is the probability of winning the cash prize if you swap doors? Show your work.