



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Hypatia Contest

(Grade 11)

Thursday, April 4, 2024

(in North America and South America)

Friday, April 5, 2024

(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 75 minutes

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Do not open this booklet until instructed to do so.

Number of questions: 4

Each question is worth 10 marks

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 

- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by 

- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks



WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x -intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. At Radford Motors, 4050 trucks were sold. Of the trucks sold, 32% were white, 24% were grey, and 44% were black.



(a) How many white trucks were sold?



(b) If $\frac{1}{4}$ of the grey trucks sold were electric, how many trucks sold were both grey and electric?



(c) In addition to the 4050 trucks that were sold, there were k unsold trucks, all of which were black. In total, 46% of all trucks, sold and unsold, were black. Determine the value of k .

2. For a positive 3-digit integer n , $f(n)$ is equal to the sum of n and the digits of n . For example, $f(351) = 351 + 3 + 5 + 1 = 360$.

Note: The decimal representation of the 3-digit number abc is $a \cdot 10^2 + b \cdot 10 + c$. For example, $836 = 8 \cdot 10^2 + 3 \cdot 10 + 6$.



(a) What is the value of $f(132)$?



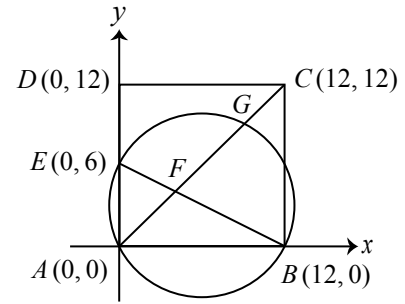
(b) If $f(n) = 175$, what is the value of n ?



(c) If $f(n) = 204$, determine all possible values of n .

3. In the diagram, $ABCD$ is a square with side length 12. The midpoint of AD is E , and BE intersects AC at F . The circle with diameter BE passes through A , and intersects AC at G .

Note: A circle with centre (h, k) and radius r has equation $(x - h)^2 + (y - k)^2 = r^2$.



(a) What are the coordinates of F ?



(b) What is the area of $\triangle AEF$?



(c) Determine the area of quadrilateral $GDEF$.

4. A *Hewitt number* is a positive integer that is the sum of the cubes of three consecutive positive integers. The smallest Hewitt number is $1^3 + 2^3 + 3^3 = 36$.



(a) How many Hewitt numbers between 10 000 and 100 000 are divisible by 10?



(b) Determine how many of the smallest 2024 Hewitt numbers are divisible by 216.



(c) Consider the following statement:

There are two distinct Hewitt numbers whose sum is equal to $9 \cdot 2^k$ for some positive integer k .

Show that this statement is true by finding two such Hewitt numbers or prove that it is false by demonstrating that there cannot be two such Hewitt numbers.



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For students...

Thank you for writing the 2024 Hypatia Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2024.

Visit our website cemc.uwaterloo.ca to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

- Obtain information about our 2024/2025 contests
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