The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

2024 Gauss Contests

(Grades 7 and 8)

Wednesday, May 15, 2024
(in North America and South America)

Thursday, May 16, 2024
(outside of North America and South America)

Solutions

Grade 7

1. Adding the digits of 2024, we get $2 + 0 + 2 + 4 = 8$.

   ANSWER: (B)

2. Substituting $n = 5$, we get $n + 2 = 5 + 2 = 7$.

   ANSWER: (C)

3. If a figure has a vertical line of symmetry, then when it is reflected in (or folded along) this vertical line, the two halves of the figure are identical to one another. Of the given shapes, such a vertical line is only possible for (E).

   ANSWER: (E)

4. As a percent, one-quarter is equivalent to 25%. Thus, Wednesday was chosen by exactly one-quarter of the students.

   ANSWER: (C)

5. A square with side length 5 has area $5 \times 5 = 25$.

   ANSWER: (E)

6. Since $\angle PQR$ is a straight angle, its measure is $180°$. Thus, the angles with measures $146°$ and $x°$ add to $180°$, and so the value of x is $180 - 146 = 34$.

   ANSWER: (E)

7. Katie's total time is equal to the sum of 3 minutes and 45 seconds and 4 minutes and 35 seconds. Adding 45 seconds and 35 seconds, we get $45 + 35 = 80$ seconds. Since there are 60 seconds in 1 minute, then 80 seconds is equal to 1 minute and 20 seconds. Thus, Katie's total time was $3 + 4 + 1 = 8$ complete minutes plus 20 seconds, for a total of 8 minutes and 20 seconds.

   ANSWER: (D)

8. *Solution 1*
   The sequence repeats every 5 symbols, and $23 = 4 \times 5 + 3$. Thus, the sequence of 5 symbols repeats 4 times, and 3 additional symbols follow. The 3rd symbol in the sequence is ⊠, and thus the 23rd symbol is ⊠.

   Solution 2
   If the sequence of 5 symbols is repeated 4 times, then $5 \times 4 = 20$ symbols are written. Following this, 3 more symbols are needed to reach 23, and since the 3rd symbol in the sequence is ⊠, the 23rd symbol is ⊠.

   ANSWER: (C)

9. Since $\frac{42 \text{ cm}}{2 \text{ cm}} = 21$, then Olivia cuts her string into 21 pieces.
   Since $\frac{42 \text{ cm}}{3 \text{ cm}} = 14$, then Jeff cuts his string into 14 pieces.
   Therefore, Olivia has $21 - 14 = 7$ more pieces of string than Jeff.

   ANSWER: (A)

10. From the given list, the numbers that are divisible by 2 are $2, 4, 6, 8$.
    The numbers that are divisible by 3 are $3, 6, 9$.
    The numbers that are divisible by both 2 and 3 are the numbers which appear in both of the previous two lists. The only number in both lists is 6.
    Thus, the numbers that are divisible by 2, or by 3, or by both 2 and 3 are $2, 3, 4, 6, 8, 9$.
    Since $2, 3, 4, 6, 8, 9$ are 6 of the 9 numbers listed, then the probability that the chosen number is divisible by 2, or by 3, or by both 2 and 3, is $\frac{6}{9}$.

    ANSWER: (C)

11. We rearrange the given subtraction to create the addition statement $Q6 + 49 = 8P$.
    Next, we consider the units digits.
    From the statement, the sum $6 + 9$ has a units digit of P which means that $P = 5$ (since $6 + 9 = 15$). Therefore, we have $Q6 + 49 = 85$.
    Rearranging this equation, we get $Q6 = 85 - 49 = 36$, and so $Q = 3$.
    Therefore, $P + Q = 5 + 3 = 8$.
    (We can check that $85 - 36 = 49$, as required.)

    ANSWER: (D)

12. The perimeter of a rectangle is made up of two widths and two lengths.
    Since each length is twice the width, then two lengths is equivalent to four widths, and so the perimeter is equal to $2 + 4 = 6$ times the width.
    Since the perimeter of the rectangle is 120 cm, then the width of the rectangle is $\frac{120 \text{ cm}}{6} = 20$ cm.

    ANSWER: (A)

13. Eloise spent a total of $765 and the mean price of each water pump was $85.
    Thus, Eloise purchased $\frac{\$765}{\$85} = 9$ water pumps.

    ANSWER: (C)

14. Since 385 has a units digit of 5, then it is divisible by 5. Dividing, we get $\frac{385}{5} = 77$.
    Since $77 = 7 \times 11$, then the three prime factors of 385 are 5, 7 and 11, and their sum is $5 + 7 + 11 = 23$.

    ANSWER: (D)

15. A circle with radius 2 has area $\pi \times 2^2 = 4\pi$.
    If the radius is tripled, then the new radius is $3 \times 2 = 6$.
    A circle with radius 6 has area $\pi \times 6^2 = 36\pi$.
    The area of the original circle divided by the area of the new circle is $\frac{4\pi}{36\pi} = \frac{4}{36} = \frac{1}{9}$.

    ANSWER: (C)

16. After Brett pours half of his 300 mL of water out, he has 150 mL of water remaining in his glass.
    Juanita then pours 20% of her 300 mL or 0.20×300 mL $= 60$ mL of water into Brett's glass.
    The volume of water now in Brett's glass is 150 mL $+$ 60 mL $= 210$ mL.

    ANSWER: (A)

17. Since each unshaded section is 3 times the size of each shaded section, then together the size of 3 shaded sections is equal to the size of 1 unshaded section.
    Thus, the combined size of all sections is equal to $12 + 1 = 13$ unshaded sections.
    We can now imagine the spinner as having 13 equal sized sections of which 1 is shaded.
    The probability that the arrow stops in a shaded section is equal to the fraction of the spinner's area comprised of shaded sections, which is $\frac{1}{13}$.

    ANSWER: (D)

18. There are 3 different colours and 4 different numbers, and therefore $3 \times 4 = 12$ different kinds of robots that may be assembled. (These are R1, R2, R3, R4, B1, B2, B3, B4, G1, G2, G3, G4, where R, B, G represent the 3 colours red, blue, green.)
    Since there are 12 different kinds of robots, it is possible that the first 12 robots assembled are all different from one another.
    In this case, the 13th robot assembled would be the first robot to have the same colour and the same number as a previously assembled robot, and thus the greatest possible value of n is 13.
    Notes:
    (i) It is possible that the first duplicate occurs earlier, but we want the latest that it can occur.
    (ii) There must be a duplicate robot among the first 13 assembled, and so $n < 14$.
    (iii) This solution makes use of the *Pigeonhole principle* – a concept worth further investigation.

    ANSWER: (C)

19. Consider the following list of 5 integers, ordered from smallest to largest, and having a median of 10: $a, b, 10, c, d$.
    Since a is the smallest integer in the list and d is the largest, and the list has a range of 7, then d is 7 more than a.
    Since a and d differ by 7, then to find the smallest possible value of a, we can find the smallest possible value of d and subtract 7.
    The 5 integers in the list are different from one another, and so the smallest possible value of c is 11 (c must be greater than the median 10), and the smallest possible value of d is thus 12.
    Since d is 7 more than a, then the smallest possible integer in the list is $12 - 7 = 5$.
    (We note that $5, b, 10, 11, 12$, where b is greater than 5 and less than 10, is such a list.)

    ANSWER: (B)

20. Beginning at height 1 and moving up 6 settings at a time, the desk can stop at settings 7, 13, 19, 25, and 31.
    Beginning at height 31 and moving down 4 settings at a time, the desk can stop at settings 27, 23, 19, 15, 11, 7, and 3.
    The desk originally begins at an odd-numbered height, 1.
    Moving up 6 settings at a time, the desk can stop at only odd-numbered heights (since an even number added to an odd number is odd).
    Similarly, moving down 4 settings at a time, the desk can stop at only odd-numbered heights.
    Thus, it is not possible for the desk to stop at an even-numbered setting.
    To this point, we have shown that the desk is able to stop at the settings

    $$1, 3, 7, 11, 13, 15, 19, 23, 25, 27, 31,$$

    and is not able to stop at even-numbered settings.
    Next, we will show that it is possible for the desk to stop at the remaining odd-numbered settings, 5, 9, 17, 21, and 29.

Since the desk can stop at setting 13, then it can stop at settings 9 and 5 with one and two presses of the down button, respectively.

Similarly, since the desk can stop at setting 25, then it can stop at settings 21 and 17.

Finally, since the desk can stop at setting 23, then one press of the up button will take the desk to setting 29.

The desk can stop at all odd-numbered settings from 1 to 31 inclusive, and thus is able to stop at 16 different settings.

ANSWER: (B)

21. The three different integers selected from 1 to 6 and whose sum is 7 must be the integers 1, 2, 4. Thus, the vertical column contains the integers 1, 2, 4 in some order.

(Can you see why no other combination of three of the given integers has a sum of 7?)

The three different integers selected from 1 to 6 and whose sum is 11 must be 1, 4, 6 or 2, 4, 5 or 2, 3, 6.

If the integers in the horizontal row are 1, 4, 6, then there are two integers in common with those in the vertical column, namely 1 and 4.

Since there have to be five different integers used in the squares, then there cannot be two integers in common between the two lists, and so 1, 4, 6 cannot appear in the horizontal row.

Similarly, 2, 4, 5 cannot appear in the horizontal row.

Thus, the horizontal row must contain the integers 2, 3, 6 with 2 appearing in the centre square since it is the integer in common between the two lists.

The integer not appearing in any square is 5.

The figure shows a possible arrangement of the integers.
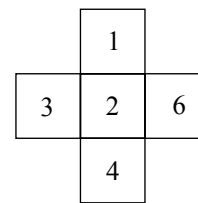
ANSWER: (E)

22. We begin by determining the number of inner toothpicks used to make a 20 by 24 grid of squares.

A grid containing 20 rows has 19 horizontal lines of inner toothpicks, each of which contains 24 toothpicks (since there are 24 columns).

Thus, the number of inner toothpicks positioned horizontally is $19 \times 24 = 456$.

A grid containing 24 columns has 23 vertical lines of inner toothpicks, each of which contains 20 toothpicks (since there are 20 rows).

Thus, the number of inner toothpicks positioned vertically is $23 \times 20 = 460$.

In total, there are $456 + 460 = 916$ inner toothpicks.

Next, we determine the total number of toothpicks used to make a 20 by 24 grid of squares.

There are 21 horizontal lines of toothpicks, each of which contains 24 toothpicks.

There are 25 vertical lines of toothpicks, each of which contains 20 toothpicks.

Thus, there are a total of $21 \times 24 + 25 \times 20 = 1004$ toothpicks used to make a 20 by 24 grid. (Alternately, we could have determined that there are 88 outer toothpicks, and so there are $916 + 88 = 1004$ toothpicks in total.)

The percentage of inner toothpicks used is $\frac{916}{1004} \times 100\%$, which is 91% when rounded to the nearest percent.

ANSWER: (E)

23. All six faces of the prism are painted which means that the 1 by 1 by 1 cubes in the interior of the prism are the only cubes that have no paint on them.
    Each of the three dimensions of the prism (length, width, height) must be at least 3, otherwise there are no 1 by 1 by 1 cubes without paint on them.
    The set of interior 1 by 1 by 1 cubes must also be in the shape of a rectangular prism.
    (You should confirm each of these last two sentences for yourself before reading on.)
    There are 50 interior 1 by 1 by 1 cubes, and so the volume of the interior prism is 50.
    Thus, we are looking for three positive integers, representing the length, width and height of the interior prism, whose product is 50.
    We may use the positive divisors of 50 (1, 2, 5, 10, 25, 50) to help identify the four possibilities: $1 \times 1 \times 50$, $1 \times 2 \times 25$, $1 \times 5 \times 10$, and $2 \times 5 \times 5$.
    These are the only ways to express 50 as the product of three positive integers.
    Next, we determine the dimensions of the original prisms given each set of dimensions for the interior prisms.
    Consider the interior prism with dimensions $1 \times 1 \times 50$.
    Recall that this is the prism that remains after all exterior (painted) cubes are removed.
    That is, 1 by 1 by 1 cubes have been removed from the top and bottom of the $1 \times 1 \times 50$ interior prism, from the left and right sides, as well as from the two ends (the front and back).
    This means that the dimensions of the original prism are each 2 greater than the dimensions of the interior prism. (You should try to visualize this.)
    We complete the following table to determine the dimensions and the volume of the original prism in each case.

Interior prism dimensions	Original prism dimensions	Volume of original prism
$1 \times 1 \times 50$	$3 \times 3 \times 52$	$V = 3 \times 3 \times 52 = 468$
$1 \times 2 \times 25$	$3 \times 4 \times 27$	$V = 3 \times 4 \times 27 = 324$
$1 \times 5 \times 10$	$3 \times 7 \times 12$	$V = 3 \times 7 \times 12 = 252$
$2 \times 5 \times 5$	$4 \times 7 \times 7$	$V = 4 \times 7 \times 7 = 196$

Therefore, the mean of all possible values of V is $\frac{468+324+252+196}{4} = \frac{1240}{4} = 310$.

ANSWER: (B)

24. Each Tiny three-digit integer belongs to exactly one of the following three cases.

Case 1: The units digit is 0

If the units digit of a Tiny integer is 0, then the tens digit must also be 0, otherwise, the units digit and tens digit can be switched to give a smaller integer.
In this case, there are no restrictions on the hundreds digit and thus there are 9 such Tiny integers. These are: 100, 200, 300, 400, 500, 600, 700, 800, 900.

Case 2: The units digit is not 0, but the tens digit is 0

If the hundreds digit is x and the units digit is z, then the integers in this case are of the form $x0z$, where $z \neq 0$. (If x is greater than z, then switching x and z creates a smaller integer.)
Integers of this form are Tiny exactly when x is greater than or equal to 1, and x is less than or equal to z. If $x = 1$, then z can be equal to any integer from 1 to 9 inclusive, and so there are 9 such Tiny integers. These are: $101, 102, 103, \ldots, 108, 109$.
If $x = 2$, then z can be equal to any integer from 2 to 9 inclusive, and so there are 8 such Tiny integers. These are: $202, 203, 204, \ldots, 208, 209$.
Continuing in this way, there are 7 Tiny integers when $x = 3$, 6 when $x = 4$, 5 when $x = 5$, 4 when $x = 6$, 3 when $x = 7$, 2 when $x = 8$, and finally 1 when $x = 9$.
In this case, there are $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ Tiny integers.

Case 3: The units digit and the tens digit are both not 0

If the hundreds digit is x (where x is greater than or equal to 1), the tens digit is y, and the units digit is z, then the integers in this case are of the form xyz. Integers of this form are Tiny exactly when x is less than or equal to y, and y is less than or equal to z.

For $x = 1$, we count the number of such Tiny integers in the table that follows.

Value of x	Value of y	Possible values of z	Number of Tiny integers
$x = 1$	$y = 1$	$z = 1, 2, 3, 4, \ldots, 9$	9
$x = 1$	$y = 2$	$z = 2, 3, 4, \ldots, 9$	8
$x = 1$	$y = 3$	$z = 3, 4, \ldots, 9$	7
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x = 1$	$y = 8$	$z = 8, 9$	2
$x = 1$	$y = 9$	$z = 9$	1

When $x = 1$, there are $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ Tiny integers in this case.

For $x = 2$, we may similarly count the number of Tiny integers.

Value of x	Value of y	Possible values of z	Number of Tiny integers
$x = 2$	$y = 2$	$z = 2, 3, 4, \ldots, 9$	8
$x = 2$	$y = 3$	$z = 3, 4, \ldots, 9$	7
$x = 2$	$y = 4$	$z = 4, \ldots, 9$	6
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x = 2$	$y = 8$	$z = 8, 9$	2
$x = 2$	$y = 9$	$z = 9$	1

When $x = 2$, there are $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$ Tiny integers in this case.

Notice that for each increase in the value of x by 1, the smallest possible value of y increases by 1 (to match the value of x), and so the smallest possible value of z also increases by 1 (to match the value of y).

This means that when $x = 3$, for example, the number of Tiny integers in the first row of the corresponding table is 1 less than the first row of the table for $x = 2$, and thus is 7.

That is, when $x = 3$, there are $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$ Tiny integers, and when $x = 4$, there are $6 + 5 + 4 + 3 + 2 + 1 = 21$ Tiny integers.

Continuing in this way, we summarize the count of Tiny integers for Case 3.

Value of x	Number of Tiny integers
$x = 1$	$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$
$x = 2$	$8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$
$x = 3$	$7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$
$x = 4$	$6 + 5 + 4 + 3 + 2 + 1 = 21$
$x = 5$	$5 + 4 + 3 + 2 + 1 = 15$
$x = 6$	$4 + 3 + 2 + 1 = 10$
$x = 7$	$3 + 2 + 1 = 6$
$x = 8$	$2 + 1 = 3$
$x = 9$	1

The number of Tiny three-digit integers in this case is

$$45 + 36 + 28 + 21 + 15 + 10 + 6 + 3 + 1 = 165$$

and so the total number of Tiny three-digit integers is $9 + 45 + 165 = 219$.

ANSWER: (E)

25. We begin by recognizing that 2 is the only even prime number.

    If x, y and z are each odd prime numbers, then both $x + y$ and $x + z$ are even prime numbers (since the sum of two odd numbers is even).

    However, both $x + y$ and $x + z$ are each at least $2 + 3 = 5$, and therefore each must be an odd prime number.

    This tells us that x, y, z cannot all be odd prime numbers, and so exactly one of them is equal to 2 (since they are all different from one another and 2 is the only even prime number).

    If $y = 2$, then each of x and z is odd and so $x + z$ is even, which is not possible.

    Similarly, if $z = 2$, then each of x and y is odd and so $x + y$ is even, which is not possible, and so we conclude that $x = 2$.

    Substituting $x = 2$, the list of 8 different prime numbers becomes:

    $$w, 2, y, z, 2 + y, 2 + z, 234 + z, 234 - z,$$

    and we note that $w + x + y = 234$ becomes $w + y = 232$.

    Since z and $2 + z$ are prime numbers that differ by 2, next we consider the consecutive odd prime numbers with z less than 50.

    These are: 3 and 5, 5 and 7, 11 and 13, 17 and 19, 29 and 31, 41 and 43.

    So then z is equal to one of 3, 5, 11, 17, 29, or 41.

    If $z = 3$, then $234 - z = 231$ which is divisible by 3 and thus not a prime number.

    If $z = 11$, then $234 + z = 245$ which is divisible by 5 and thus not a prime number.

    If $z = 17$, then $234 - z = 217$ which is divisible by 7 and thus not a prime number.

    If $z = 29$, then $234 - z = 205$ which is divisible by 5 and thus not a prime number.

    If $z = 41$, then $234 + z = 275$ which is divisible by 5 and thus not a prime number.

    Finally, if $z = 5$, then $234 - z = 229$ and $234 + z = 239$, and both of these are prime numbers.

    Alternately, we may have noted that if z has units digit 1, then $234 + z$ has units digit 5, and if z has units digit 9, then $234 - z$ also has units digit 5, and so each is divisible by 5, which is not possible since each is a prime number. We could have then removed $z = 11, 29, 41$ as possibilities and considered only $z = 3, 5, 17$ as we did above.

    The table below summarizes what we know about the 8 different prime numbers to this point.

w	x	y	z	$x + y$	$x + z$	$234 + z$	$234 - z$
	2		5	$2 + y$	7	239	229

    As shown previously, since y and $2 + y$ are consecutive odd prime numbers (with y less than 50), then y is equal to one of 3, 11, 17, 29, or 41 (recall that $z = 5$ and the 8 numbers must all be different).

    Since $w + y = 232$, then $w = 232 - y$.

    For which value(s) of y is $w = 232 - y$ a prime number different from those already in our list?

    If $y = 3$, then $w = 229$ which is not possible since $234 - z = 229$.

    If $y = 11$, then $w = 221$ which is divisible by 13 and therefore not a prime number.

    If $y = 17$, then $w = 215$ which is divisible by 5 and therefore not a prime number.

    If $y = 29$, then $w = 203$ which is divisible by 7 and therefore not a prime number.

    Finally, if $y = 41$, then $w = 191$ which is a prime number.

    The final list of 8 different prime numbers is shown below.

w	x	y	z	$x + y$	$x + z$	$234 + z$	$234 - z$
191	2	41	5	43	7	239	229

    The value of $w - y$ is $191 - 41 = 150$.

    ANSWER: (B)

Grade 8

1. The number of 5 cent coins needed to make 25 cents is $\dfrac{25 \text{ cents}}{5 \text{ cents}} = 5$.

   ANSWER: (E)

2. If a figure has a vertical line of symmetry, then when it is reflected in (or folded along) this vertical line, the two halves of the figure are identical to one another. Of the given shapes, such a vertical line is only possible for (E).

   ANSWER: (E)

3. Of the given numbers, only 1.32 and 1.03 are greater than 1. Since the tenths digit of 1.32, namely 3, is greater than the tenths digit of 1.03, which is 0, then 1.32 is the largest number in the list.

   ANSWER: (B)

4. If 50% of n is 2024, then half of n is 2024, and so n is equal to 2 times 2024, which is equal to 4048. (We may confirm that 50% of 4048, or half of 4048, is 2024 as required.)

   ANSWER: (D)

5. Reading from the graph, Ryan ran 2 km on Monday, 4 km on Tuesday, 6 km on Wednesday, 5 km on Thursday, and 3 km on Friday.
   Thus, the total distance that Ryan ran over the five days is $2 + 4 + 6 + 5 + 3 = 20$ km.

   ANSWER: (D)

6. When 11 is increased by 2, the result is 13.
   When 13 is then multiplied by 3, the final result is $13 \times 3 = 39$.

   ANSWER: (B)

7. If $15 + a = 10$, then $a = 10 - 15$ and so $a = -5$.

   ANSWER: (B)

8. Since the measure of a straight angle is 180°, then $40° + x° + x° = 180°$.
   Solving this equation, we get $40 + x + x = 180$ or $2x = 140$, and so $x = 70$.

   ANSWER: (D)

9. The ratio of the number of spoons to the number of forks is $1 : 2$, which means that for each spoon in the drawer, there are 2 forks in the drawer.
   That is, the spoons and forks in the drawer can be separated into groups of 3 (1 spoon and 2 forks), and thus the total number of spoons and forks in the drawer must be a multiple of 3.
   Each of the answers 12, 6, 18, and 3 is a multiple of 3.
   The only answer given that is not a multiple of 3 is 10, and so the total number of spoons and forks in the drawer cannot be 10.

   ANSWER: (D)

10. The large square with side length 6 has area $6 \times 6 = 36$.
    The shaded squares with side lengths 3, 2 and 1, have areas $3 \times 3 = 9$, and $2 \times 2 = 4$, and $1 \times 1 = 1$, respectively. The total area of the shaded regions is $9 + 4 + 1 = 14$.
    The total area of the unshaded region is equal to the area of the shaded regions subtracted from the area of the large square, or $36 - 14 = 22$.

    ANSWER: (C)

11. Continuing the sequence, we get 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, and so the smallest number greater than 100 that appears in the sequence is 123.

    ANSWER: (E)

12. Since 385 has a units digit of 5, then it is divisible by 5. Dividing, we get $\frac{385}{5} = 77$.
    Since $77 = 7 \times 11$, then the three prime factors of 385 are 5, 7 and 11, and their sum is $5 + 7 + 11 = 23$.

    ANSWER: (D)

13. We begin by splitting $ABCD$ into three equilateral triangles, as shown. (Confirm for yourself that this is the only way to do this.) Since the equilateral triangle in the middle shares a side with each of the two other equilateral triangles, then the sides of all three equilateral triangles are equal in length.
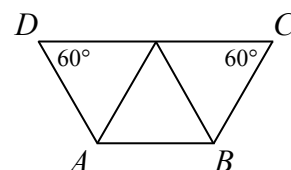
    The perimeter of $ABCD$ is made up of 5 side lengths of these equilateral triangles.

    Since the perimeter of $ABCD$ is 840 cm, then each side length of these equilateral triangles is $\frac{840 \text{ cm}}{5} = 168$ cm.

    Since AB is a side of an equilateral triangle, then $AB = 168$ cm.

    ANSWER: (C)

14. One container of ice cream can make 6 cones.
    Thus, 2 containers of ice cream can make $2 \times 6 = 12$ cones.
    Since 12 cones were made from 5 containers of ice cream, and it takes 2 containers of ice cream to make 12 cones, then $5 - 2 = 3$ containers of ice cream remain after the 12 cones are made.
    One container of ice cream can make 4 sundaes.
    Thus, the remaining 3 containers of ice cream can make $3 \times 4 = 12$ sundaes.

    ANSWER: (C)

15. Given that n has a remainder of 8 when divided by 10, then n is 8 more than a multiple of 10. This means that n has a units digit of 8, which is 3 more than a units digit of 5, and thus n has a remainder of 3 when divided by 5.
    We may confirm that examples of numbers that are equal to 8 more than a multiple of 10, such as 8, 18, 28, 38, do indeed leave a remainder of 3 when divided by 5.

    ANSWER: (D)

16. Before the hole was drilled, the volume of the block of wood was 4 cm $\times$ 4 cm $\times$ 7 cm = 112 cm^3.
    The cylindrical hole has radius 1 cm and height equal to that of the block of wood, 7 cm.
    The volume of the cylindrical hole is thus $\pi \times (1 \text{ cm})^2 \times 7 \text{ cm} = 7\pi$ cm^3.
    In cm^3, the volume of the block of wood after the hole is drilled is $112 - 7\pi \approx 90.01$, which when rounded to the nearest cm^3, is 90 cm^3.

    ANSWER: (A)

17. There are 3 different colours and 4 different numbers, and therefore $3 \times 4 = 12$ different kinds of robots that may be assembled. (These are R1, R2, R3, R4, B1, B2, B3, B4, G1, G2, G3, G4, where R, B, G represent the 3 colours red, blue, green.)
    Since there are 12 different kinds of robots, it is possible that the first 12 robots assembled are all different from one another.
    In this case, the 13th robot assembled would be the first robot to have the same colour and the same number as a previously assembled robot, and thus the greatest possible value of n is 13.
    Notes:
    (i) It is possible that the first duplicate occurs earlier, but we want the latest that it can occur.
    (ii) There must be a duplicate robot among the first 13 assembled, and so $n < 14$.
    (iii) This solution makes use of the *Pigeonhole principle* – a concept worth further investigation.
    Answer: (C)

18. The angle of one complete rotation measures $360°$.
    Since the spinner is divided into 5 equal sections, each $\dfrac{360°}{5} = 72°$ rotation moves the arrow to the next dividing line between two sections.
    Rotating clockwise, the section labelled D is the 4th section.
    This means that if the angle of rotation is greater than $3 \times 72° = 216°$ and less than $4 \times 72° = 288°$, then the arrow stops in the section labelled D.
    Since each of the given answers is greater than $360°$ and less than $720°$, the arrow must have spun through more than one and less than two complete rotations.
    On the second spin around, the arrow will stop in the section labelled D if the angle of rotation is greater than $216° + 360° = 576°$ and less than $288° + 360° = 648°$.
    Only one of the given answers lies in this range, and so the angle of rotation must be $630°$.
    Answer: (C)

19. *Solution 1*
    In this solution, we work backward from each of the given choices. Since we are asked to find the smallest possible integer in the list, we begin with the smallest of the five choices, 39.

    If the smallest integer in the list is 39, then the largest integer in the list is $39 + 14 = 53$ (since the three integers have a range of 14).
    If the three integers have a mean of 50, then they have a sum of $50 \times 3 = 150$. Two of the integers are 39 and 53, and so the third (the middle) integer is $150 - 39 - 53 = 58$.
    Since 58 is greater than 53, this is not possible (the range of these three integers is $58 - 39 = 19$, not 14).

    If the smallest integer in the list is 40 (the next smallest answer given), then the largest integer in the list is $40 + 14 = 54$.
    If two of the integers are 40 and 54, then the third (the middle) integer is $150 - 40 - 54 = 56$.
    Since 56 is greater than 54, this is not possible (the range of these three integers is $56 - 40 = 16$, not 14).

    If the smallest integer in the list is 41 (the next smallest answer given), then the largest integer in the list is $41 + 14 = 55$.
    If two of the integers are 41 and 55, then the third (the middle) integer is $150 - 41 - 55 = 54$.
    We may confirm that the three integers 41, 54, 55 indeed have a range of 14 and a mean of 50.
    We have shown that 41 is the smallest of the five choices to satisfy the given conditions, and so 41 is the smallest possible integer in the list.

Solution 2

Assume that the list of 3 integers, ordered from smallest to largest, is a, b, c.

Since a is the smallest integer in the list and c is the largest, and the list has a range of 14, then c is 14 more than a or $c = a + 14$.

The three integers have a mean of 50, and so $\frac{a+b+c}{3} = 50$ or $a + b + c = 150$.

Substituting $c = a + 14$, the previous equation becomes $a + b + a + 14 = 150$ or $2a + b = 136$.

To find the smallest possible value of a, we determine the largest possible value of b, recalling that $a < b$ and $b < c$ and so $b < a + 14$.

Since $2a$ is even for all possible values of a, and 136 is even, then b must be even (since $2a + b = 136$).

We begin by choosing an arbitrary value of b and using this value to determine a and c.

If $b = 52$, then $2a = 136 - 52 = 84$ and so $a = 42$ and $c = 42 + 14 = 56$.

In this case, the three integers are $42, 52, 56$.

We continue to increase the value of b in order to determine the smallest possible value for a.

If $b = 54$, then $2a = 136 - 54 = 82$ and so $a = 41$ and $c = 41 + 14 = 55$.

In this case, the three integers are $41, 54, 55$.

If $b = 56$, then $2a = 136 - 56 = 80$ and so $a = 40$ and $c = 40 + 14 = 54$.

In this case, the three integers are $40, 56, 54$, which is not possible since $56 > 54$.

Continuing to increase the value of b will continue to give values of c that are less than b.

Decreasing the value of b will give values of a that are greater than 41.

Therefore, the smallest possible integer in the list is 41.

ANSWER: (E)

20. Since 21 pieces of fruit are not apples, then the number of pears added to the number of bananas is 21.

    Since 25 pieces of fruit are not pears, then the number of apples added to the number of bananas is 25.

    Since 28 pieces of fruit are not bananas, then the number of apples added to the number of pears is 28.

    In adding these three totals $21 + 25 + 28$, we are counting the number of pears twice, the number of bananas twice, and the number of apples twice (since each appears in exactly two of the three statements above).

    That is, $21 + 25 + 28 = 74$ is twice the total number of pieces of fruit, and so the number of pieces of fruit in the box is $\frac{74}{2} = 37$.

    ANSWER: (D)

21. Expressing the product $6 \times 5 \times 4 \times 3 \times 2 \times 1$ in terms of prime factors, we get

    $$6 \times 5 \times 4 \times 3 \times 2 \times 1 = (2 \times 3) \times 5 \times (2 \times 2) \times 3 \times 2$$

    This product has 4 factors of 2, 2 factors of 3, and 1 factor of 5, and thus is equal to $2^4 \times 3^2 \times 5^1$.
    The value of $a + b + c$ is $4 + 2 + 1 = 7$.

    ANSWER: (B)

22. The ratio of the number of quarters to the number of dimes to the number of nickels is $9 : 3 : 2$.

    This means that for some positive integer k, the number of quarters is $9k$, the number of dimes is $3k$, and the number of nickels is $2k$.

    The value of each quarter is \$0.25 or 25 cents, and so the value of the quarters in the jar, in cents, is $25 \times 9k = 225k$.

The value of each dime is \$0.10 or 10 cents, and so the value of the dimes in the jar, in cents, is $10 \times 3k = 30k$.

The total value of the quarters and dimes is \$17.85 or 1785 cents, and so $225k + 30k = 1785$ or $255k = 1785$, and so $k = \frac{1785}{255} = 7$.

The number of nickels is $2k = 14$, and so the value of the nickels is $\$0.05 \times 14 = \$0.70$.

<div align="right">ANSWER: (C)</div>

23. The smallest five positive integers, each having a divisor of d, are d, $2d$, $3d$, $4d$, and $5d$.

Thus, the smallest possible sum of five different positive integers whose greatest common divisor is d is $d + 2d + 3d + 4d + 5d = 15d$.

We know that the sum is at least $15d$ and is equal to 264, which means that $15d \leq 264$, and so $d \leq \frac{264}{15}$ or $d \leq 17.6$.

Each of the five integers is divisible by d, and so the sum of the five integers, 264, is divisible by d.

Thus, we want the largest possible divisor of 264 that is less than or equal to 17.

Since $264 = 2^3 \times 3 \times 11$, the divisors of 264 that are less than or equal to 17 are: $1, 2, 3, 4, 6, 8, 11,$ and 12, and so the largest possible value of d is 12.

The sum of the digits of the largest possible value of d is $1 + 2 = 3$.

(We note that 12, 24, 36, 48, and 144 are five such integers whose greatest common divisor is 12 and whose sum is 264.)

<div align="right">ANSWER: (B)</div>

24. There are 6 different locations at which the path splits, and we label these splits 1 to 6, as shown.

We begin by determining the probability that a ball lands in the bin labelled A.

There is exactly one path that leads to bin A.

This path travels downward to the left at each of the three splits labelled 1, 2 and 3.

At each of these splits, the probability that a ball travels to the left is $\frac{1}{2}$, and so the probability that a ball lands in bin A is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Next, we determine the probability that a ball lands in the bin labelled C.

There are exactly three paths that lead to bin C.

One of these paths travels downward to the right at each of the three splits labelled 1, 4 and 6.

Thus, the probability that a ball lands in bin C by following this path is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

A second path to bin C travels downward to the right at split 1, to the left at split 4, to the right at split 5, and to the right at split 6.

The probability that a ball follows this path is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$.

The third and final path to bin C travels left at split 1, and to the right at each of the three splits 2, 5 and 6.

The probability that a ball follows this path is also $\frac{1}{16}$.

The probability that a ball lands in bin C is the sum of the probabilities of travelling each of these three paths or

$$\tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{16} = \tfrac{2+1+1}{16} = \tfrac{4}{16} = \tfrac{1}{4}$$

Finally, we determine the probability that a ball lands in bin B.

There are six different paths that lead to bin B, and we could determine the probability that a ball follows each of these just as we did for bins A and C.
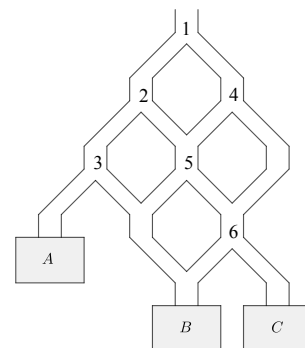
However, it is more efficient to recognize that a ball must land in one of the three bins, and thus the probability that it lands in bin B is 1 minus the probability that it lands in bin A minus the probability that it lands in bin C, or

$$1 - \tfrac{1}{8} - \tfrac{1}{4} = \tfrac{8-1-2}{8} = \tfrac{5}{8}$$

The probability that the two balls land in different bins is equal to 1 minus the probability that the two balls land in the same bin.

The probability that a ball lands in bin A is $\tfrac{1}{8}$, and so the probability that two balls land in bin A is $\tfrac{1}{8} \times \tfrac{1}{8} = \tfrac{1}{64}$.

The probability that a ball lands in bin C is $\tfrac{1}{4}$, and so the probability that two balls land in bin C is $\dfrac{1}{4} \times \tfrac{1}{4} = \tfrac{1}{16}$.

The probability that a ball lands in bin B is $\tfrac{5}{8}$, and so the probability that two balls land in bin B is $\tfrac{5}{8} \times \tfrac{5}{8} = \tfrac{25}{64}$.

Therefore, the probability that the two balls land in different bins is equal to

$$1 - \tfrac{1}{64} - \tfrac{1}{16} - \tfrac{25}{64} = \tfrac{64-1-4-25}{64} = \tfrac{34}{64} = \tfrac{17}{32}$$

ANSWER: (A)

25. The smallest possible value of d is closest to 6.40.

On a flat surface, the shortest distance between two points is along a straight line between the points. The ant must walk on the surface of the figure, and so to determine a straight line distance between P and Q, we can "flatten" the figure.

In the figures below, we show a straight line path whose distance is closest to 6.40 from each of the two perspectives.
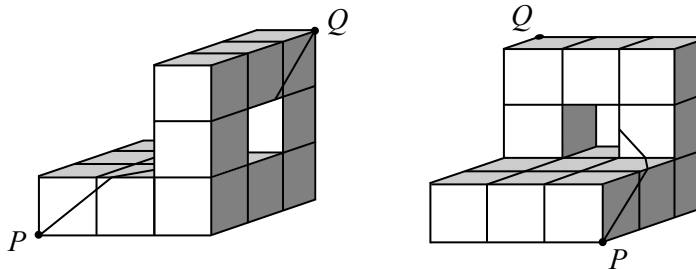

To help see this path more clearly, we strip away all but the 5 cubes that the ant walks on, below.
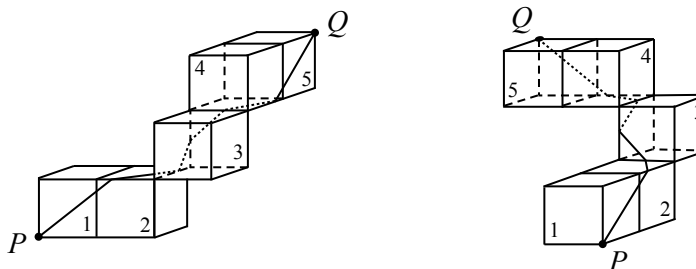

To see that this path is along a straight line from P to Q, we draw a partial net of the previous diagrams. This partial net includes each face that the ant walks on.

The numbers shown in the diagram below indicate that the face is from the cube with the matching number in the diagrams above.
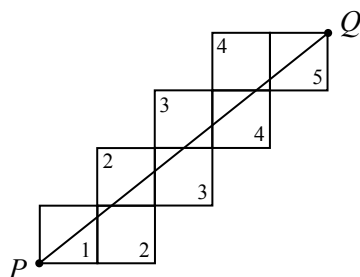
To determine the length of PQ, we position R so that PR is perpendicular to QR, as shown below. Triangle PQR is a right-angled triangle with $PR = 5$ and $QR = 4$, and so by the Pythagorean Theorem, we get $PQ = \sqrt{5^2 + 4^2} = \sqrt{41}$, and so the distance d is closest to 6.40.
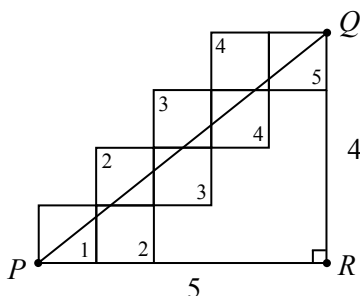
Since 6.40 is the smallest of the five choices given, and we have shown that there is a path of this length from P to Q on the figure's surface, then the smallest possible value of d is 6.40.
Each of the other four given answers, 6.43, 6.48, 6.66, and 6.71 is a result of the ant travelling along other paths from P to Q. For example, there exists a second possible straight line path on a net of this figure for which PQ is approximately 6.71. Can you determine this path, and each of the other three paths from P to Q whose lengths are equal to 6.43, 6.48, and 6.66? There are also paths different from the one shown above for which $PQ = \sqrt{41}$. Can you find these?

ANSWER: (B)