The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2024 Fermat Contest

(Grade 11)

Wednesday, February 28, 2024
(in North America and South America)

Thursday, February 29, 2024
(outside of North America and South America)

Solutions

1. Calculating, $3\left(\frac{5}{3}-\frac{1}{3}\right)=3\cdot\frac{5}{3}-3\cdot\frac{1}{3}=5-1=4$.
   Alternatively, $3\left(\frac{5}{3}-\frac{1}{3}\right)=3\cdot\frac{4}{3}=4$.

   ANSWER: (D)

2. Simplifying, $4x^2-3x^2=x^2$. When $x=2$, this expression equals 4.
   Alternatively, when $x=2$, we have $4x^2-3x^2=4\cdot 2^2-3\cdot 2^2=16-12=4$.

   ANSWER: (C)

3. The volume of a $1\times 1\times 1$ cube is 1.
   The volume of a $2\times 2\times 2$ cube is 8.
   Thus, 8 of the smaller cubes are needed to make the larger cube.

   ANSWER: (E)

4. For there to be equal numbers of each colour of candy, there must be at most 3 red candies and at most 3 yellow candies, since there are 3 blue candies to start.
   Thus, Shuxin ate at least 7 red candies and at least 4 yellow candies.
   This means that Shuxin ate at least $7+4=11$ candies.
   We note that if Shuxin eats 7 red candies, 4 yellow candies, and 0 blue candies, there will indeed be equal numbers of each colour.

   ANSWER: (A)

5. Square $PQRS$ is made up of 16 equal-sized small squares.
   Of these, 2 are fully shaded and 8 are half-shaded.
   This shading is equivalent to fully shading $2+8\cdot\frac{1}{2}=2+4=6$ of the 16 small squares.
   Thus, square $PQRS$ is $\frac{6}{16}=\frac{3}{8}$ shaded.

   ANSWER: (E)

6. Using a calculator, $\sqrt{15}\approx 3.87$ and $\sqrt{50}\approx 7.07$.
   The integers between these real numbers are 4, 5, 6, 7, of which there are 4.
   Alternatively, we could note that integers between $\sqrt{15}$ and $\sqrt{50}$ correspond to values of $\sqrt{n}$ where n is a perfect square and n is between 15 and 50. The perfect squares between 15 and 50 are 16, 25, 36, 49, of which there are 4.

   ANSWER: (B)

7. *Solution 1*
   When a line is reflected in the y-axis, its y-intercept does not change (since it is on the line of reflection) and its slope is multiplied by -1.
   Therefore, the new line has slope -3 and y-intercept 6, which means that its equation is $y=-3x+6$.
   The x-intercept of this new line is found by setting $y=0$ and solving for x which gives $0=-3x+6$ or $3x=6$ or $x=2$.

   Solution 2
   The x-intercept of the original line is found by setting $y=0$ in the equation of the line and solving for x, which gives $0=3x+6$ or $3x=-6$ or $x=-2$.
   When the line is reflected in the y-axis, the x-intercept of the new line is the reflection of the original line in the y-axis, and thus is $x=2$.

   ANSWER: (A)

8. Using exponent laws, $1000^{20} = (10^3)^{20} = 10^{60}$ and so $n = 60$.

Answer: (B)

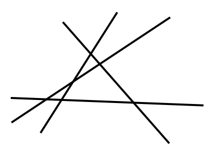
9. Since O is the centre of the circle, then $OA = OB = OC$.
This means that $\triangle AOB$ and $\triangle COB$ are both isosceles with $\angle ABO = \angle BAO = \angle BAC = 25^\circ$.
Thus, $\angle AOB = 180^\circ - \angle ABO - \angle BAO = 130^\circ$.
Since $\angle AOC$ is a straight angle, then $\angle BOC = 180^\circ - \angle AOB = 180^\circ - 130^\circ = 50^\circ$.

Answer: (D)

10. After David is seated, there are 4 seats in which Pedro can be seated, of which 2 are next to David.
Thus, the probability that Pedro is next to David is $\frac{2}{4}$ or $\frac{1}{2}$.

Answer: (C)

11. Each of the 4 lines can intersect each of the other 3 lines at most once.
This might appear to create $4 \times 3 = 12$ points of intersection, but each point of intersection is counted twice – one for each of the 2 lines.
Thus, the maximum number of intersection points is $\dfrac{4 \times 3}{2} = 6$.
The diagram below demonstrates that 6 intersection points are indeed possible:



Answer: (D)

12. When a list of 5 numbers a, b, c, d, e has the property that $a + b + c = c + d + e$, it is also true that $a + b = d + e$.
With the given list of 5 numbers, it is likely easier to find two pairs with no overlap and with equal sum than to find two triples with one overlap and equal sum.
After some trial and error, we can see that $6 + 21 = 10 + 17$, and so the list 6, 21, 5, 10, 17 has the given property, which means that 5 is in the middle.
(We note that these two pairs are the only such pairs, after allowing for switching the numbers in each pair and/or switching the pairs.)

Answer: (A)

13. Expanding, $(x + m)(x + n) = x^2 + nx + mx + mn = x^2 + (m + n)x + mn$.
The constant term of this quadratic expression is mn, and so $mn = -12$.
Since m and n are integers, they are each divisors of -12 and thus of 12.
Of the given possibilities, only 5 is not a divisor of 12, and so m cannot equal 5.
We can check that each of the other four choices is a possible value of m.

Answer: (E)

14. We note first that $\triangle ACB$ has a right angle and a 60° angle and so it is a 30°-60°-90° triangle.
Since $AB = \sqrt{3}$, then using the known ratios of side lengths, we can see that $BC = 1$ and $AC = 2$.
Next, we note that $\triangle ACE$ has two 45° angles and so is an isosceles right-angled triangle.
This means that $CE = AC = 2$ and $\angle ACE = 90^\circ$.

Also, $AE = \sqrt{2}AC = 2\sqrt{2}$.
Further, since $\angle BCD$ is a straight angle, then

$$\angle ECD = 180^\circ - \angle ACB - \angle ACE = 180^\circ - 60^\circ - 90^\circ = 30^\circ$$

Since $\triangle CED$ has a 30° angle and a right-angle, it is also a 30°-60°-90° triangle.
Using the known ratios of sides, since $CE = 2$, we have $DE = 1$ and $CD = \sqrt{3}$.
Therefore, the perimeter of $ABDE$ is

$$AB + BC + CD + DE + AE = \sqrt{3} + 1 + \sqrt{3} + 1 + 2\sqrt{2} = 2 + 2\sqrt{2} + 2\sqrt{3}$$

ANSWER: (E)

15. We first note that $197 = 8 \cdot 24 + 5$.
This tells us that the time that is 197 hours from now is 8 days and 5 hours.
Since Anila's grandmother's activities are the same every day, then in 197 hours and 5 minutes she will be doing the same thing as she is doing in 5 hours and 5 minutes, at which point she is doing yoga.

ANSWER: (C)

16. Of the row and column products, only 135 and 160 are divisible by 5. This means that 5 must go in the square in the 2nd row, 3rd column.
Of the row and column products, only 21 and 56 are divisible by 7. This means that 7 must go in the square in the 1st row, 1st column.
Of the row and column products, only 108 and 135 are divisible by 9. This means that 9 must go in the square in the 2nd row, 2nd column.
So far, this gives the following grid:

7			56
	9	5	135
			48
21	108	160	

In the 2nd row, the product is 135 which means that the missing entry is $\frac{135}{5\cdot 9} = 3$.
In the 1st column, the product is 21 which means that the missing entry is $\frac{21}{7\cdot 3} = 1$.
The 3rd row, whose product is 48, thus includes 1 and two more integers between 1 and 9. The only divisor pair of 48 with both divisors less than 10 is $48 = 6 \cdot 8$.
Since 8 is not a divisor of 108, then N must be 6.

We can complete the square as follows:

7	2	4	56
3	9	5	135
1	6	8	48
21	108	160	

Answer: (C)

17. Since $b+d > a+d$, then $b > a$. This means that a does not have the greatest value.
Since $c+e > b+e$, then $c > b$. This means that b does not have the greatest value.
Since $b+d = c$ and each of b, c, d is positive, then $d < c$, which means that d does not have the greatest value.
Consider the last equation $a+c = b+e$ along with the fact that $a < b < c$.
From this, we see that $e = c + (a-b)$.
Since $a < b$, then $a-b$ is negative and so $e < c$.
This means that c has the greatest value.

Answer: (C)

18. Since $3x+2y = 6$, then $(3x+2y)^2 = 6^2$ or $9x^2+12xy+4y^2 = 36$.
Since $9x^2+4y^2 = 468$, then

$$12xy = (9x^2+12xy+4y^2) - (9x^2+4y^2) = 36 - 468 = -432$$

and so $xy = \frac{-432}{12} = -36$.
(With some additional work, we can find that the solutions to the system of equations are $(x,y) = (-4,9)$ and $(x,y) = (6,-6)$.)

Answer: (A)

19. Suppose that when the three dice are rolled, the numbers rolled are x, y and z.
Since there are 6 possibilities for each of x, y and z, there are $6 \cdot 6 \cdot 6 = 216$ possible outcomes.
Also, the sum, S, of the three rolls is at least $3 \cdot 1 = 3$ and at most $3 \cdot 6 = 18$.
The outcome "$S > 5$" is the complement of the outcome "$S \leq 5$".
Thus, the probability that $S > 5$ is 1 minus the probability that $S \leq 5$.
It is easier to compute the probability that $S \leq 5$ directly by listing the rolls that give this.
If $S = 3$, then $x+y+z = 3$ and so $(x,y,z) = (1,1,1)$.
If $S = 4$, then $x+y+z = 4$ and so x, y and z must be 1, 1 and 2 in some order. Thus, $(x,y,z) = (2,1,1)$ or $(1,2,1)$ or $(1,1,2)$.
If $S = 5$, then $x+y+z = 5$ and so x, y and z must be 1, 1 and 3, or 1, 2 and 2 in some order.
There are 3 arrangements in each case and so 6 triples in total.
Therefore, there are $1+3+6 = 10$ triples with $S \leq 5$, and so the probability that $S > 5$ is equal to $1 - \frac{10}{216} \approx 0.954$.
Of the given choices, this is closest to 0.95.

Answer: (B)

20. Suppose that the radius of cylinder is r and the height of the cylinder is h.
This means that the volume of the cylinder is $\pi r^2 h$; the volume of half of the cylinder is $\frac{1}{2}\pi r^2 h$.
Also, the radius of the cone is $\frac{1}{2}r$ and the height of the cone is h.
This means that the volume of the cone is $\frac{1}{3}\pi\left(\frac{1}{2}r\right)^2 h$ or $\frac{1}{12}\pi r^2 h$.
When the cone is divided into two pieces by a horizontal plane at half of its height, the top portion of the cone is a cone with the same proportions, but with dimensions $\frac{1}{2}$ of those of the larger cone.
This means that the volume of the top portion is $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ of that of the cone, which equals $\frac{1}{8} \cdot \frac{1}{12}\pi r^2 h$ or $\frac{1}{96}\pi r^2 h$.
To see this in another way, we note that this top portion of the cone has height $\frac{1}{2}h$ and should have radius $\frac{1}{2} \cdot \frac{1}{2}r$ (because the radius decreases proportionally to the height). This means that the volume of this portion is $\frac{1}{3}\pi\left(\frac{1}{4}r\right)^2 \cdot \frac{1}{2}h$ which is again $\frac{1}{96}\pi r^2 h$.
Using this information, the bottom portion of the cone has volume $\frac{7}{8} \cdot \frac{1}{12}\pi r^2 h = \frac{7}{96}\pi r^2 h$.
Now, when the cone is in the cylinder and the cylinder is filled with water to half of its height, the volume of the bottom half of the cylinder is filled with the bottom portion of the cone and with the water.
Therefore, the volume of water is the difference between half of the volume of the cylinder and the volume of the bottom portion of the cone, or $\frac{1}{2}\pi r^2 h - \frac{7}{96}\pi r^2 h = \frac{48}{96}\pi r^2 h - \frac{7}{96}\pi r^2 h = \frac{41}{96}\pi r^2 h$.
When the cone is removed, the water then occupies a cylinder with radius r and volume $\frac{41}{96}\pi r^2 h$.
If the depth of the water in this configuration is d, then $\pi r^2 d = \frac{41}{96}\pi r^2 h$ and so $d = \frac{41}{96}h$, which means that the depth of the water is $\frac{41}{96}$ of the height of the cylinder.

Answer: (B)

21. Since the second column includes the number 1, then step (ii) was never used on the second column, otherwise each entry would be at least 2.
To generate the 1, 3 and 2 in the second column, we thus need to have used step (i) 1 time on row 1, 3 times on row 2, and 2 times on row 3.
This gives:

1	1	1
3	3	3
2	2	2

We cannot use step (i) any more times, otherwise the entries in column 2 will increase. Thus, $a = 1 + 3 + 2 = 6$.
To obtain the final grid from this current grid using only step (ii), we must increase each entry in column 1 by 6 (which means using step (ii) 3 times) and increase each entry in column 3 by 4 (which means using step (ii) 2 times). Thus, $b = 3 + 2 = 5$.
Therefore, $a + b = 11$.

Answer: 11

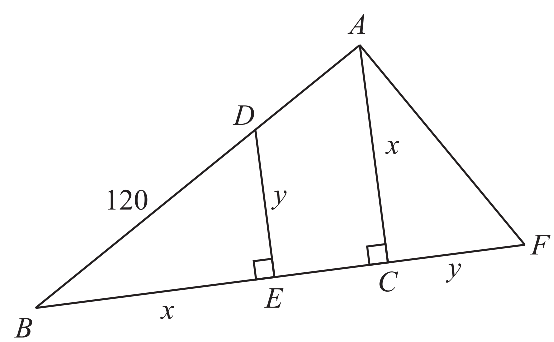
22. We note that

$$ac + bd - ad - bc = ac - ad - bc + bd = a(c - d) - b(c - d) = (a - b)(c - d)$$

Since each of a, b, c, d is taken from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then $a - b \leq 9$ since the greatest possible difference between two numbers in the set is 9.
Similarly, $c - d \leq 9$.
Now, if $a - b = 9$, we must have $a = 10$ and $b = 1$.
In this case, c and d come from the set $\{2, 3, 4, 5, 6, 7, 8, 9\}$ and so $c - d \leq 7$.
Therefore, if $a - b = 9$, we have $(a - b)(c - d) \leq 9 \cdot 7 = 63$.
If $a - b = 8$, then either $a = 9$ and $b = 1$, or $a = 10$ and $b = 2$.
In both cases, we cannot have $c - d = 9$ but we could have $c - d = 8$ by taking the other of these two pairs with a difference of 8.
Thus, if $a - b = 8$, we have $(a - b)(c - d) \leq 8 \cdot 8 = 64$.
Finally, if $a - b \leq 7$, the original restriction $c - d \leq 9$ tells us that $(a - b)(c - d) \leq 7 \cdot 9 = 63$.
In summary, the greatest possible value for $ac + bd - ad - bc$ is 64 which occurs, for example, when $a = 9$, $b = 1$, $c = 10$, and $d = 2$.

ANSWER: 64

23. *Solution 1*
Suppose that $BE = AC = x$ and $DE = y$.
Extend BC to point F so that $BC = DE = y$.

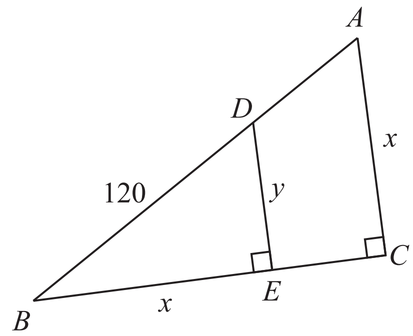


Since $BC + DE = 288$, then $BF = BC + CF = BC + DE = 288$.
Also, $\triangle BED$ is congruent to $\triangle ACF$ by side-angle-side.
Therefore,

$$\angle BAF = \angle BAC + \angle ACF = \angle BDE + \angle DBE = 90^\circ$$

since DE and AC are parallel.
Next, $\triangle BED$ is similar to $\triangle BAF$ since both are right-angled and they share an angle at B.
Therefore, $\dfrac{DE}{BD} = \dfrac{FA}{BF}$ and so $\dfrac{DE}{120} = \dfrac{120}{288}$, which gives $DE = \dfrac{120 \cdot 120}{288} = 50$, as required.

Solution 2
Suppose that $BE = AC = x$ and $DE = y$.
Since $DE + BC = 288$, then $BC = 288 - y$.



We note that $\triangle BED$ is similar to $\triangle BCA$ because each is right-angled and their angles at B are common.
Therefore, $\dfrac{BE}{DE} = \dfrac{BC}{AC}$ and so $\dfrac{x}{y} = \dfrac{288 - y}{x}$.
Manipulating, we obtain $x^2 = y(288 - y)$ and so $x^2 = 288y - y^2$ or $x^2 + y^2 = 288y$.
Also, using the Pythagorean Theorem in $\triangle BED$ gives $x^2 + y^2 = 120^2$.
Since $x^2 + y^2 = 288y$ and $x^2 + y^2 = 120^2$, then $288y = 120^2$ which gives $2 \cdot 12 \cdot 12 \cdot y = 120 \cdot 120$ and so $2y = 10 \cdot 10$ or $y = 50$.
Therefore, $DE = 50$.

ANSWER: 50

24. Throughout this solution, we use the fact that if N is a positive integer with $N > 1$ and N has prime factorization $p_1^{a_1} p_2^{a_2} \cdots p_m^{a_m}$ for some distinct prime numbers $p_1, p_2, \dots, p_m$ and positive integers $a_1, a_2, \dots, a_m$, then the number of positive divisors of N including 1 and N is equal to $(1 + a_1)(1 + a_2) \cdots (1 + a_m)$.
We are told that N is a positive multiple of 2024.
Now, $2024 = 8 \cdot 253 = 2^3 \cdot 11 \cdot 23$.
This means that N has at least 3 prime factors (namely 2, 11 and 23) and that at least one of these prime factors has an exponent of at least 3.
Let D be the number of positive divisors that N has. We are told that $100 < D < 110$.
Since $D = (1 + a_1)(1 + a_2) \cdots (1 + a_m)$ and N has at least 3 prime factors, then D is a positive integer that can be written as the product of at least 3 positive integers each greater than 2.
D cannot equal 101, 103, 107, or 109, since each of these is prime (and so cannot be written as the product of 3 integers each at least 2).
D also cannot equal 106 because $106 = 2 \cdot 53$ (both 2 and 53 are prime), which means that 106 cannot be written as the product of three integers each greater than 1.
The possible values of D that remain are 102, 104, 105, 108.
We note that $102 = 2 \cdot 3 \cdot 17$ and $104 = 2^3 \cdot 13$ and $105 = 3 \cdot 5 \cdot 7$ and $108 = 2^2 \cdot 3^3$.

Case 1: $D = 102$
Since the prime factors of N include at least 2, 11 and 23, then the prime factorization of N includes factors of 2^a, 11^b and 23^c for some positive integers a, b and c with $a \geq 3$.
If a fourth prime power p^e was also a factor of N, then D would be divisible by $(1 + a)(1 + b)(1 + c)(1 + e)$. ($D$ could have more factors if N had more prime factors.)
Since $D = 102 = 2 \cdot 3 \cdot 17$ has only 3 prime factors, it cannot be written as the product of 4 integers each greater than 1.

Thus, N cannot have a fourth prime factor.
This means that $N = 2^a 11^b 23^c$, which gives $D = (1+a)(1+b)(1+c) = 2 \cdot 3 \cdot 17$.
This means that $1+a$, $1+b$ and $1+c$ are equal to 2, 3 and 17, in some order, and so a, b and c are equal to 1, 2 and 16, in some order.
For D to be as small as possible, the largest exponent goes with the smallest prime, the next largest exponent with the next smallest prime, and so on. (Can you see why this makes N as small as possible?)
Therefore, the smallest possible value of N in this case is $N = 2^{16}11^2 23^1 = 182\,386\,688$.

<u>Case 2: $D = 105$</u>
Using a similar argument, we can determine that $N = 2^a 11^b 23^c$ with $1+a$, $1+b$ and $1+c$ equal to 3, 5 and 7 in some order, meaning that a, b and c equal 2, 4, 6 in some order.
Therefore, the minimum value of N is this case is $N = 2^6 11^4 23^2 = 495\,685\,696$.

<u>Case 3: $D = 104$</u>
Since $D = 2^3 \cdot 13$ has 4 prime factors, then N cannot have more than 4 prime factors. (If N had 5 or more prime factors, then the product equal to D would include at least 5 integers, each at least 2.)
Therefore, $N = 2^a 11^b 23^c$ and $D = (1+a)(1+b)(1+c)$, or $N = 2^a 11^b 23^c p^e$ for some prime $p \neq 2, 11, 23$ and $D = (1+a)(1+b)(1+c)(1+e)$.
This means that $(1+a)(1+b)(1+c) = 2^3 \cdot 13$ or $(1+a)(1+b)(1+c)(1+e) = 2^3 \cdot 13$.
In the case that N has three prime factors, we note that $104 = 26 \cdot 2 \cdot 2 = 13 \cdot 4 \cdot 2$ are the only two ways of writing 104 as the product of 3 integers each of which is at least 2.
These give corresponding minimum values of N of $N = 2^{25} \cdot 11 \cdot 23 = 8\,489\,271\,296$ and $N = 2^{12} \cdot 11^3 \cdot 23 = 125\,390\,848$.
In the case that N has four prime factors, then $(1+a)(1+b)(1+c)(1+e) = 2 \cdot 2 \cdot 2 \cdot 13$ means that a, b, c and e are 1, 1, 1, 12 in some order.
This in turn means that the corresponding smallest possible value of N is

$$N = 2^{12} \cdot 3 \cdot 11 \cdot 23 = 3\,108\,864$$

We note here that the prime power p^e has become 3^1 in order to minimize both p (since $p > 2$) and its exponent.

<u>Case 4: $D = 108$</u>
Since $D = 2^2 \cdot 3^3$ has 5 prime factors, then N cannot have more than 5 prime factors.
If N has 5 prime factors, then we need to use the factorization $D = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$.
This is not possible, however, because the power 2^a must have $a \geq 3$ which would mean that one of the five factors of D would have to be at least 4.
If N has 4 prime factors, then D must be partitioned as $9 \cdot 3 \cdot 2 \cdot 2$ or $6 \cdot 3 \cdot 3 \cdot 2$ or $4 \cdot 3 \cdot 3 \cdot 3$. (Since two of the prime factors have to be combined, either two 2s, two 3s, or a 2 and a 3 are combined.)
These give minimum values of $N = 2^8 \cdot 3^2 \cdot 11 \cdot 23 = 582\,912$ and $N = 2^5 \cdot 3^2 \cdot 11^2 \cdot 23 = 801\,504$ and $N = 2^3 \cdot 3^2 \cdot 11^2 \cdot 23^2 = 4\,408\,648$.
If N has 3 prime factors, then we must use one of the factorizations $D = 27 \cdot 2 \cdot 2$ or $D = 18 \cdot 3 \cdot 2$ or $D = 12 \cdot 3 \cdot 3$ or $D = 9 \cdot 4 \cdot 3$ or $D = 6 \cdot 6 \cdot 3$.

These gives corresponding minimum values

$$N = 2^{26} \cdot 11 \cdot 23 = 16\,978\,542\,592$$
$$N = 2^{17} \cdot 11^2 \cdot 23 = 364\,773\,376$$
$$N = 2^{11} \cdot 11^2 \cdot 23^2 = 131\,090\,432$$
$$N = 2^{8} \cdot 11^3 \cdot 23^2 = 180\,249\,344$$
$$N = 2^{5} \cdot 11^5 \cdot 23^2 = 2\,726\,271\,328$$

Combining Cases 1 through 4, the minimum possible value of N is 582 912.
The sum of the digits of 582 912 is $5 + 8 + 2 + 9 + 1 + 2 = 27$.

Answer: 27

25. Suppose that, for some integer $n \geq 2$, we have $a_n = x$ and $a_{n-1} = y$.
The equation $a_n + a_{n-1} = \frac{5}{2}\sqrt{a_n \cdot a_{n-1}}$ can be re-written as $x + y = \frac{5}{2}\sqrt{xy}$.
Since $x > 0$ and $y > 0$, squaring both sides of the equation gives an equivalent equation which is $(x+y)^2 = \frac{25}{4}xy$.
Manipulating algebraically, we obtain the following equivalent equations:

$$\begin{aligned}(x+y)^2 &= \tfrac{25}{4}xy \\ 4(x^2 + 2xy + y^2) &= 25xy \\ 4x^2 - 17xy + 4y^2 &= 0 \\ (4x - y)(x - 4y) &= 0\end{aligned}$$

Therefore, the given relationship is equivalent to $4x = y$ or $x = 4y$.
Returning to the sequence notation, we now know that it is the case that $4a_n = a_{n-1}$ (that is, $a_n = \frac{1}{4}a_{n-1}$) or $a_n = 4a_{n-1}$.
Putting this another way, each term in the sequence can be obtained from the previous term either by multiplying by 4 or by dividing by 4.
We are told that $a_1 = 4$ and $a_{11} = 1024$. We note $\dfrac{a_{11}}{a_1} = \dfrac{1024}{4} = 256 = 4^4$.
We can think of moving along the sequence from a_1 to a_{11} by making 10 "steps", each of which involves either multiplying by 4 or dividing by 4.

If there are m steps in which we multiply by 4 and $10 - m$ steps in which we divide by 4, then $\dfrac{4^m}{4^{10-m}} = 4^4$ which gives $4^{2m-10} = 4^4$ or $2m - 10 = 4$ and so $m = 7$.
In other words, the sequence involves 7 steps of multiplying by 4 and 3 steps of dividing by 4. These steps completely define the sequence.
The number of possible sequences, S, equals the number of ways of arranging these 10 steps, which equals $\dbinom{10}{3}$.
(If combinatorial notation is unfamiliar, we could systematically count the number of arrangements instead.)
Therefore, $S = \dfrac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 5 \cdot 3 \cdot 8 = 120$. The rightmost two digits of S are 20.

Answer: 20