



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING

*cemc.uwaterloo.ca*

# Cayley Contest

(Grade 10)

Wednesday, February 28, 2024  
(in North America and South America)

Thursday, February 29, 2024  
(outside of North America and South America)



UNIVERSITY OF  
**WATERLOO**

**Time:** 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

## Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as “7”) must be coded with a leading zero (“07”).
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

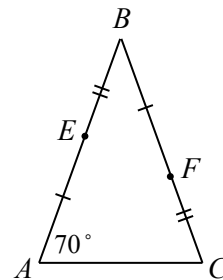
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*The name, grade, school and location, and score range of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

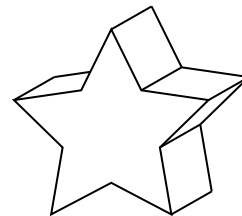
Scoring: There is *no penalty* for an incorrect answer.  
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**Part A: Each correct answer is worth 5.**

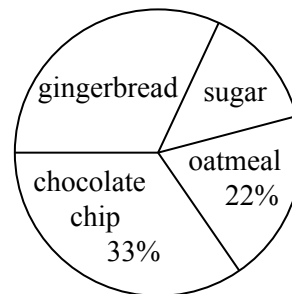
- The expression  $2 \times 0 + 2 \times 4$  is equal to  
(A) 0            (B) 2            (C) 4            (D) 6            (E) 8
- If  $x = 3$ , the value of  $-(5x - 6x)$  is  
(A) -33            (B) 3            (C) -1            (D) 33            (E) 11
- In  $\triangle ABC$ , points  $E$  and  $F$  are on  $AB$  and  $BC$ , respectively, such that  $AE = BF$  and  $BE = CF$ . If  $\angle BAC = 70^\circ$ , the measure of  $\angle ABC$  is  
(A)  $40^\circ$             (B)  $50^\circ$             (C)  $60^\circ$   
(D)  $70^\circ$             (E)  $30^\circ$



- At Wednesday's basketball game, the Cayley Comets scored 90 points. At Friday's game, they scored 80% as many points as they scored on Wednesday. How many points did they score on Friday?  
(A) 60            (B) 72            (C) 75            (D) 78            (E) 82
- In the diagram, the two identical bases of the prism are shaped like a star. The area of each star-shaped base is  $400 \text{ cm}^2$ . The depth of the prism (that is, the distance between the star-shaped bases) is 8 cm. The volume of the prism is  
(A)  $720 \text{ cm}^3$     (B)  $1520 \text{ cm}^3$     (C)  $3200 \text{ cm}^3$   
(D)  $3600 \text{ cm}^3$     (E)  $28\,800 \text{ cm}^3$



- Last year, Lloyd ate cookies in the percentages shown in the pie chart. The number of gingerbread cookies that he ate was two times the number of sugar cookies that he ate. What percentage of the cookies that he ate were gingerbread cookies?  
(A) 25%            (B) 28%            (C) 30%  
(D) 35%            (E) 38%



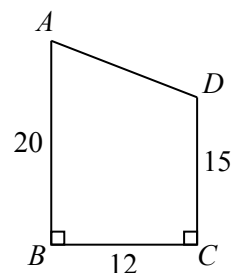
- If  $\frac{1}{6} + \frac{1}{3} = \frac{1}{x}$ , the value of  $x$  is  
(A) 9            (B) 6            (C) 18            (D) 2            (E) 3
- Which of the following integers is equal to a perfect square?  
(A)  $2^3$             (B)  $3^5$             (C)  $4^7$             (D)  $5^9$             (E)  $6^{11}$

9. The sum of five consecutive *odd* integers is 125. The smallest of these integers is  
 (A) 17            (B) 19            (C) 21            (D) 25            (E) 29
10. Two standard six-sided dice are rolled. What is the probability that the product of the two numbers rolled is 12?  
 (A)  $\frac{3}{36}$             (B)  $\frac{4}{36}$             (C)  $\frac{6}{36}$             (D)  $\frac{8}{36}$             (E)  $\frac{9}{36}$

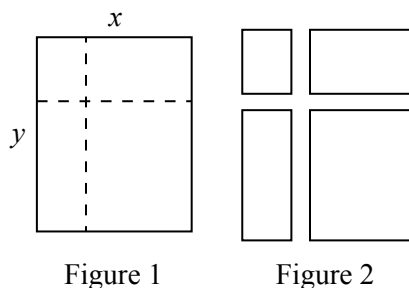
**Part B: Each correct answer is worth 6.**

11. Arturo has an equal number of \$5 bills, of \$10 bills, and of \$20 bills. The total value of these bills is \$700. How many \$5 bills does Arturo have?  
 (A) 16            (B) 24            (C) 12            (D) 20            (E) 28
12. The Cayley Corner Store sells three types of toys: Exes, Wyes and Zeds. All Exes are identical, all Wyes are identical, and all Zeds are identical. The mass of 2 Exes equals the mass of 29 Wyes. The mass of 1 Zed equals the mass of 16 Exes. The mass of 1 Zed equals the mass of how many Wyes?  
 (A) 3.625            (B) 1.103            (C) 232            (D) 464            (E) 928

13. In the diagram, quadrilateral  $ABCD$  has  $AB = 20$ ,  $BC = 12$ , and  $CD = 15$ . Also,  $AB$  and  $CD$  are perpendicular to  $BC$ . The perimeter of quadrilateral  $ABCD$  is  
 (A) 47            (B) 59            (C) 84  
 (D) 72            (E) 60



14. Ten numbers have an average (mean) of 87. Two of those numbers are 51 and 99. The average of the other eight numbers is  
 (A) 90            (B) 89            (C) 88            (D) 91            (E) 92
15. A rectangle has width  $x$  and length  $y$ , as shown in Figure 1. The rectangle is cut along the horizontal and vertical dotted lines in Figure 1 to produce four smaller rectangles as shown in Figure 2.

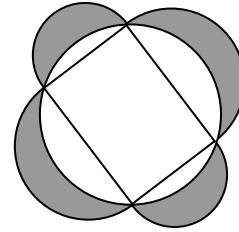


- The sum of the perimeters of these four rectangles in Figure 2 is 24. The value of  $x + y$  is  
 (A) 6            (B) 8            (C) 9.6            (D) 12            (E) 16

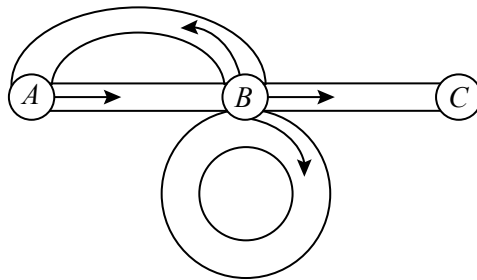


22. For how many integers  $m$  does the line with the equation  $y = mx$  intersect the line segment with endpoints  $(20, 24)$  and  $(4, 202)$ ?

23. Four semi-circles are arranged so that their diameters form a 6 by 8 rectangle. A circle is drawn through the four vertices of the rectangle. In the diagram, the region inside the four semi-circles but outside the circle is shaded. The total area of the shaded region is  $A$ . What is the integer closest to  $A$ ?

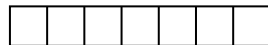


24. A park has four paths, as shown in the map below. It takes 2 minutes to walk along the path from  $A$  to  $B$ , 3 minutes to walk along the path from  $B$  to  $A$ , 3 minutes to walk along the path from  $B$  to  $C$ , and 3 minutes to walk around the path that begins and ends at  $B$ . Rasheeqa goes for a walk, starting at  $A$ , walking only in the directions indicated along the paths, never stopping to rest, and finishing at  $C$ .



If the walk takes a total of  $t$  minutes, how many possible values of  $t$  are there with  $t \leq 100$ ?

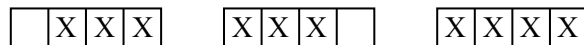
25. Erin has an empty  $1 \times 7$  grid consisting of  $1 \times 1$  squares:



and follows the process below to construct a pattern:

- (i) Place an X in any empty square.
- (ii) If three or more consecutive squares each contain an X, stop and do not add any more X's; otherwise, go to step (i) and continue the process.

For example, in a smaller  $1 \times 4$  grid, there are 3 different patterns that can be constructed:



(The last pattern may be obtained by placing X's, in order, in squares 1, 2, 4, and then 3.) By applying this process starting with the empty  $1 \times 7$  grid, how many different possible patterns can Erin construct?



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Thank you for writing the 2024 Cayley Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to find

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- Free copies of past contests
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Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to

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- Look at our free online courseware
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- Find your school's contest results