The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
*cemc.uwaterloo.ca*

# Canadian Intermediate Mathematics Contest

***Wednesday, November 13, 2024***
***(in North America and South America)***

***Thursday, November 14, 2024***
***(outside of North America and South America)***

UNIVERSITY OF WATERLOO

**Time:** 2 hours



**Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.**

Do not open this booklet until instructed to do so.
There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

**PART A**

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

**PART B**

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Write your name, school name, and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

**At the completion of the contest, insert your student information form inside your answer booklet.**

*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

*The name, grade, school and location, and score range of some top-scoring students will be published on the website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.*

## *Canadian Intermediate Mathematics Contest*

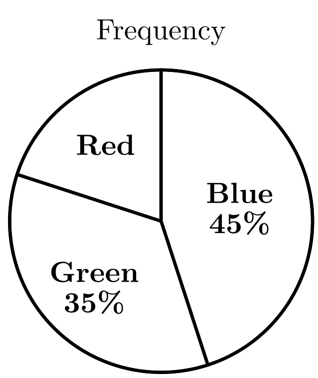
NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the $x$-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

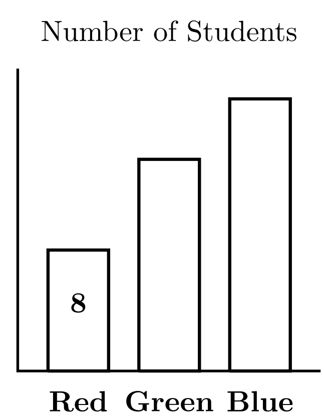
# PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

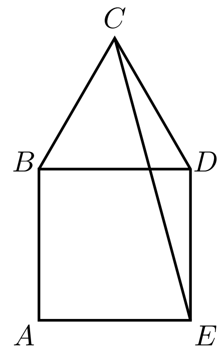
1. A container in the shape of a cube has dimensions 4 cm by 4 cm by 4 cm. The container is sitting on a horizontal table on one of its faces and is filled with water to a depth of 2 cm. In $\text{cm}^3$, what is the volume of the water in the container?

2. A group of students were asked to choose their favourite of the three colours red, green and blue. Unfortunately, some of the results of the survey were lost. All of the remaining data are expressed in both the pie graph and the bar graph shown below. What is the total number of students that were surveyed?



3. In the diagram, $ABDE$ is a square and $\triangle BCD$ is equilateral. What is the measure of $\angle ECB$?




4. The positive integers $a$ and $b$ satisfy $2a < b$. The sum $\frac{a}{4} + \frac{b}{2}$ is greater than 27 and less than 28. What is the greatest possible value of $a$?

5. Lakshmi creates a table with 95 rows and 7 columns. She places a positive integer in each of the $95 \times 7 = 665$ cells according to the following rules.

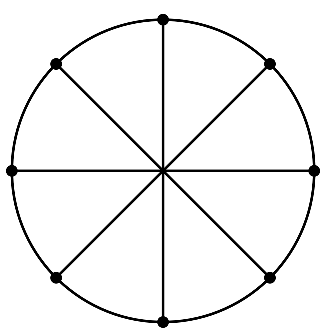
- Every integer in the first (leftmost) column is 5.
- The cells in the second column contain the consecutive integers from 5 through 99 in order with 5 at the top and 99 at the bottom.
- Every integer in the remaining columns is equal to the sum of the integers in the two cells directly to its left in the same row.

The first three rows of the table are shown below.

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 5 | 10 | 15 | 25 | 40 | 65 |
| 5 | 6 | 11 | 17 | 28 | 45 | 73 |
| 5 | 7 | 12 | 19 | 31 | 50 | 81 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

How many different two-digit integers are in exactly 5 of the cells in the table?

6. Eight points are equally spaced around the circumference of a circle and pairs of points are connected by 4 diameters, as shown. The eight points are to be labelled randomly using the integers from 1 through 8, each exactly once. What is the probability that at least one of the four diameters has a multiple of 3 at one of its ends and a multiple of 2 at its other end?

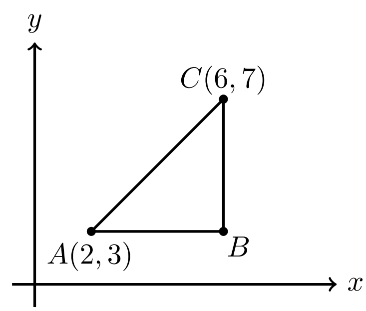


# PART B

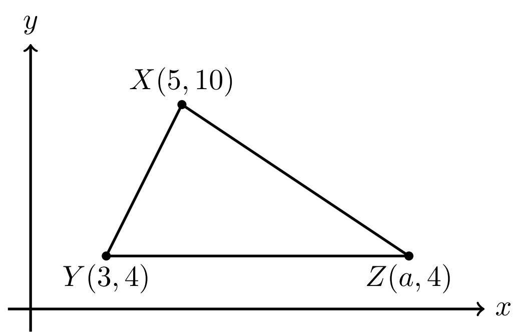
For each question in Part B, your solution must be well-organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

> *Useful Fact for Part B*:
>
> The sum of the first $k$ perfect squares is equal to $\dfrac{k(k+1)(2k+1)}{6}$.
>
> That is, $1^2 + 2^2 + 3^2 + \cdots + k^2 = \dfrac{k(k+1)(2k+1)}{6}$.

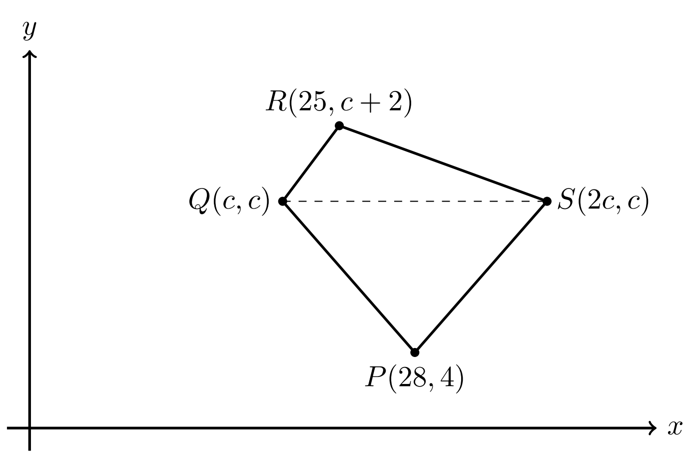
1. (a) In the diagram, $A(2,3)$ and $C(6,7)$ are two of the vertices of $\triangle ABC$. If $AB$ is horizontal and $BC$ is vertical, determine the area of right-angled $\triangle ABC$.




(b) In the diagram, $\triangle XYZ$ has vertices $X(5,10)$, $Y(3,4)$ and $Z(a,4)$, for some real number $a > 3$, which means that side $YZ$ is horizontal. If the area of $\triangle XYZ$ is 24, determine the value of $a$.




(c) Quadrilateral $PQRS$ has vertices $P(28,4)$, $Q(c,c)$, $R(25,c+2)$, and $S(2c,c)$. The diagonal $QS$ is horizontal and divides $PQRS$ into two triangles, as shown. There is one positive integer $c$ with the property that the area of $PQRS$ is 180. Determine this value of $c$.



2. (a) Beryl ran 30 km. She ran the first 20 km at 12 km/h and the last 10 km at 10 km/h. Determine the total amount of time, in hours, that it took for Beryl to run 30 km.

   (b) Carol walked 10 km in 2 hours and 18 minutes. For the first $x$ km, she walked at 6 km/h. For the remaining $(10 - x)$ km, she walked at 4 km/h. Determine the value of $x$.

   (c) Daryl rode his bicycle for a total of 3 hours. He rode $a$ km at 24 km/h followed by $b$ km at 16 km/h. Determine the number of pairs $(a, b)$ of positive integers for which this is possible.

   (d) Errol competed in an endurance competition that took 5 hours. He ran $r$ km at 12 km/h, then he jogged $j$ km at 8 km/h, and finally walked $w$ km at 4 km/h. Determine the number of triples $(r, j, w)$ of positive integers for which this is possible.

3. Given an increasing list of consecutive integers, the *3-sign sum* of the list is the sum of the integers in the list, in order, except that every third integer is subtracted instead of added. For example, the 3-sign sum of the list 3, 4, 5, 6, 7, 8, 9 is $3 + 4 - 5 + 6 + 7 - 8 + 9 = 16$.

   (a) Determine the 3-sign sum of 8, 9, 10, 11, 12, 13, 14, 15.

   For a positive integer $n$, a *slice* of the list 1, 2, 3, ..., $n - 1$, $n$ is an increasing list of at least 1 and at most $n$ consecutive integers, each of which is between 1 and $n$ inclusive. For example, 1, 2 and 2, 3, 4 are both slices of the list 1, 2, 3, 4, 5. As another example, the list 1, 2, 3 has a total of six slices. They are given in the left column of the table below with their 3-sign sums in the right column.

| Slice | 3-sign sum |
|---|---|
| 1, 2, 3 | $1 + 2 - 3 = 0$ |
| 1, 2 | $1 + 2 = 3$ |
| 2, 3 | $2 + 3 = 5$ |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |

   For a positive integer $n$, the *Ghimire number* of $n$, denoted $G_n$, is the sum of the 3-sign sums of all slices of 1, 2, 3, ..., $n - 1$, $n$. For example, using the information from the table above, $G_3 = 0 + 3 + 5 + 1 + 2 + 3 = 14$.

   (b) For each integer $n \geq 1$, show that $\dfrac{G_{3n} - 2G_{3n-1} + G_{3n-2}}{3}$ is a perfect square.

   (c) Determine the remainder when $G_{2025} - G_{2024}$ is divided by 27.

2024
Canadian
Intermediate
Mathematics
Contest
(English)