



le CENTRE d'ÉDUCATION
en MATHEMATIQUES et en
INFORMATIQUE
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Défi Team Up 2023

Juin 2023

Les corrigés



Défi Team Up 2023

Corrigé du travail d'équipe

Question	Réponse
1	8
2	9
3	le 12 novembre
4	3
5	16
6	11
7	la forêt
8	44
9	20
10	10
11	9
12	46.5
13	250
14	15
15	2178



Défi Team Up 2023

Corrigé du casse-tête de nombres croisés

8		1	9	1	9		3	
2	9	0		2		9	5	1
2		8	4		6	3		5
8	1		1	5		6	3	6
	2		6		2		3	
8	7	6		5	5		4	7
1		9	7		4	3		4
9	6	3		2		8	2	2
	2		3	1	3	8		5



Défi Team Up 2023

Corrigé du casse-tête logique

	Numéro de position					
	1	2	3	4	5	6
Nom de l'élève	Petr	Aria	Dhruv	Maggie	Finn	Leyla
Titre de l'œuvre d'art	<i>Chute de neige</i>	<i>Jaune</i>	<i>Bonheur</i>	<i>Silence</i>	<i>Trafic</i>	<i>Vendredi</i>
Type d'art	photographie	peinture acrylique	peinture à l'huile	croquis au crayon	aquarelle	dessin au pastel



Défi Team Up 2023

Corrigé du relais

Relais de pratique				
	Joueur 1	Joueur 2	Joueur 3	Joueur 4
Réponse	6	72	12	240

Relais A				
	Joueur 1	Joueur 2	Joueur 3	Joueur 4
Réponse	5	50	10	$\frac{6}{20}$ ou $\frac{3}{10}$

Relais B				
	Joueur 1	Joueur 2	Joueur 3	Joueur 4
Réponse	8	98	31	56

Relais C				
	Joueur 1	Joueur 2	Joueur 3	Joueur 4
Réponse	7	7620	168	21



The CENTRE for EDUCATION
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2023 Team Up Challenge

June 2023

Solutions



Team Paper

1. Since $3200 \div 400 = 8$, then $1 \times 8 = 8$ litres of cream is needed.

ANSWER: 8

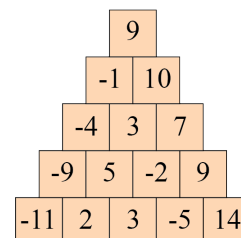
2. If n represents the unknown integer in the bottom row, then $n + 3 = 5$ and so $n = 2$.

The unknown integers in the next row will be $(-11) + 2 = -9$ and $(-5) + 14 = 9$.

The integers in the third row will be $(-9) + 5 = -4$, $5 + (-2) = 3$, and $(-2) + 9 = 7$.

The integers in the fourth row will be $(-4) + 3 = -1$ and $3 + 7 = 10$.

The integer at the top will then be $(-1) + 10 = 9$.



ANSWER: 9

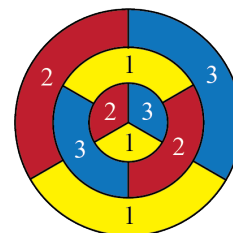
3. The second, third, and fourth lessons will occur on Tuesday, October 15th, 22nd, and 29th, respectively.

Since there are 31 days in October, Thursday would be October 31st and so Friday would be November 1st. Thus, the fifth lesson would be on Tuesday, November 5th.

The sixth (and last) lesson would be on Tuesday, November 12th.

ANSWER: November 12th

4. Since the three smallest regions are each bordering each other, then at least three colours are needed. In fact, it's possible to colour all the regions so that no two bordering regions are the same colour using exactly three colours. One way is shown in the diagram. Therefore the fewest number of colours needed is 3.



ANSWER: 3

5. Since $x + y$ is even and less than 7, it follows that $x + y = 0$, $x + y = 2$, $x + y = 4$, or $x + y = 6$.

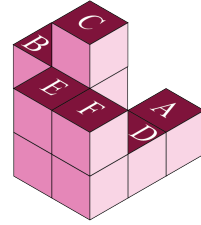
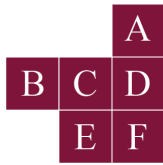
- If $x + y = 0$ then the only possible coordinates for (x, y) are $(0, 0)$.
- If $x + y = 2$ then (x, y) could be $(0, 2)$, $(1, 1)$, or $(2, 0)$.
- If $x + y = 4$ then (x, y) could be $(0, 4)$, $(1, 3)$, $(2, 2)$, $(3, 1)$, or $(4, 0)$.
- If $x + y = 6$ then (x, y) could be $(0, 6)$, $(1, 5)$, $(2, 4)$, $(3, 3)$, $(4, 2)$, $(5, 1)$, or $(6, 0)$.

Thus, there are 16 possibilities for the coordinates (x, y) .

ANSWER: 16



6. The top view shows that the linking cubes are arranged into 6 vertical columns. We label the columns using the letters $A, B, C, D, E,$ and $F,$ as shown.



Then, the three-dimensional figure has 3 layers: a top layer, a middle layer, and a bottom layer. We can count the number of cubes by looking at each of the three layers within each column.

- In column A there is 1 cube.
- In column B we can see 1 cube in the middle layer, but cannot see if there is a cube in the bottom layer. Since we are trying to find the maximum number of cubes, we assume there is a cube in the bottom layer. So the maximum number of cubes in column B is 2.
- In column C we can see 1 cube in the top layer and 1 cube in the middle layer, but we cannot see if there is a cube in the bottom layer. So the maximum number of cubes in column C is 3.
- In column D there is 1 cube.
- In column E there are 2 cubes.
- In column F there are 2 cubes.

Therefore, the maximum number of cubes in the figure is $1 + 2 + 3 + 1 + 2 + 2 = 11$.

ANSWER: 11

7. We can use a timeline to help us solve this problem, by putting places that Omar visited later to the right of places that he visited earlier. Since Omar went to the pool before he went to the forest, we place “pool” to the left of “forest” on the timeline. Then, since Omar went to the store after he went to both the pool and the forest, we place “store” to the right of “forest”.



Since Omar went to the store before the library, we place “library” to the right of “store”. Then, since Omar went to the store after he went to the movies, and he went to the movies after he went to the forest, we place “movies” in between “forest” and “store” on the timeline.



From the timeline, we can determine that Omar went to the forest second.

ANSWER: forest



8. First, label the points E, F and G , as shown.

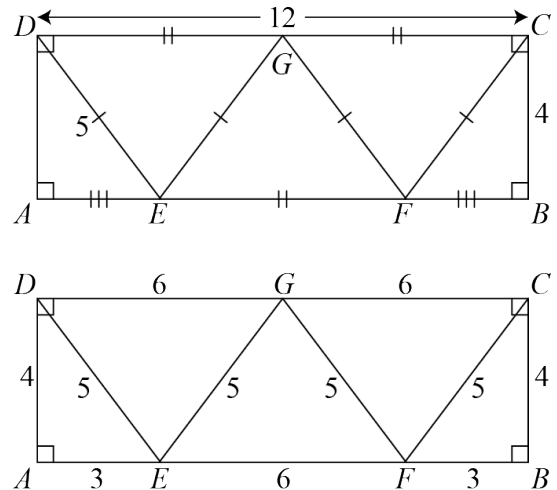
We can then determine the length of each line segment. Since $CD = 12$ and G is on CD such that $CG = GD$, then $CG = GD = 6$.

Also, $EF = CG = 6$.

All angles in $ABCD$ are right and so $ABCD$ is a rectangle. Thus, $AB = CD = 12$.

Since $EF = 6$ and $AE = FB$, then $AE = FB = 3$.

Finally, $CF = FG = EG = DE = 5$ and $AD = BC = 4$.

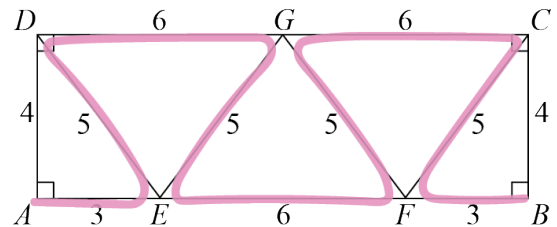


If a path included every line segment, then the length of such a path would be 52.

Starting at A , there are two possible line segments, AD and AE , that Ming can highlight. It is not possible for Ming to highlight both without having to highlight one a second time and so only one of AD and AE can be highlighted. Similarly, Ming can only highlight one of CB or FB . Thus, the length of the longest path is less than 52.

Case 1: Ming highlights the two segments of length 3

If Ming highlights the two line segments of length 3, then they cannot also highlight the two line segments of length 4. Thus, the path can have a length of at most $52 - 4 - 4 = 44$. A path from A to B with length 44 is shown.



Case 2: Ming highlights the two segments of length 4

If Ming highlights the two line segments of length 4, then they cannot also highlight the two line segments of length 3. Thus, the path can have a length of at most $52 - 3 - 3 = 46$.

Starting at A , Ming highlights AD and then at D they must choose between highlighting $DG = 6$ or $DE = 5$. Since Ming cannot highlight both, the length of the path will be at most $46 - 5 = 41$, which is less than the path of length 44 found in Case 1. Thus, the longest path does not include both segments of length 4.

Case 3: Ming highlights one segment of length 3 and one segment of length 4

If Ming highlights one segment of length 3 and one segment of length 4, then they cannot highlight the two other segments of length 3 and 4. Thus, the path can have a length of at most $52 - 3 - 4 = 45$. Starting at A if Ming highlights $AD = 4$, then (as in Case 2) at D they must choose between highlighting $DG = 6$ or $DE = 5$. Since Ming cannot highlight both, the length of the path will be at most $45 - 5 = 40$. Since the diagram is symmetric, if Ming ends the path by highlighting $CB = 4$, then they would have had to have chosen between highlighting $GC = 6$ or $FC = 5$ previously. Thus, the length of the path will be at most $45 - 5 = 40$. Thus, the longest path cannot be formed by highlighting one segment of length 3 and one segment of length 4. Therefore, the length of the longest connected path that Ming can draw is 44.

ANSWER: 44



9. Since each number in the sequence is determined by the previous number, we can use the second number, 36, to determine the third number in the sequence. We start at the top of the “repeat forever” block with num equal to 36. We add 2 and so num equals 38. Since $\text{num} < 20$ is false, we move on to the else statement and subtract 10. Then num equals 28. We print 28.

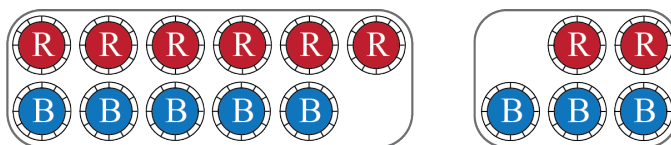
Continuing in this way, we determine that the first six numbers in the sequence are:

$$16, 36, 28, 20, 12, 28, \dots$$

As soon as we determine that the sixth number is 28, we stop. Since we have repeated an earlier number, then we know that the next number will be 20, and then 12, and then 28 again. The third and the sixth numbers are each 28, and the numbers repeat in such a way that every third number, or more specifically every number whose position in the sequence is a multiple of 3, will be 28. Since 2022 is a multiple of 3, the 2022nd number will be 28. Then, the 2023rd number will be 20.

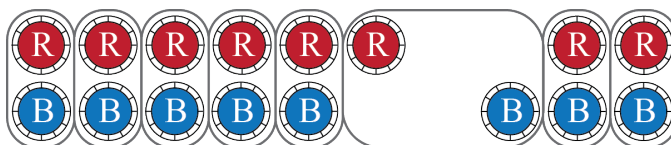
ANSWER: 20

10. It is helpful to draw a diagram to show the tokens in each group.



Notice, that if we look at all of the tokens together as one group, then there are $6 + 2 = 8$ red tokens and $5 + 3 = 8$ blue tokens, for a total of 16 tokens. In total, these tokens are worth $54 + 26 = 80$ points.

Notice, that these tokens can be divided into eight identical groups of two tokens, where each group contains one red and one blue token.



Since the eight groups are identical, they are each worth the same number of points.

So each group of two tokens (one red and one blue) is worth $80 \div 8 = 10$ points.

Therefore, if Antwan has one red and one blue token, then he has 10 points.

ANSWER: 10



11. There are 10 possible pairs of test containers. The 4 pairs of test containers that overflowed the container with capacity N will be the 4 pairs with the largest total volume. These 4 pairs of test containers, along with their total volumes, are given in the following table.

Test Containers	Total Volume (mL)
345 and 310	655
345 and 284	629
345 and 275	620
310 and 284	594

From the table, the smallest total volume that overflowed the container with capacity N is 594 mL. Thus, the container with capacity N cannot hold 594 mL and so $N < 594$.

The remaining 6 possible pairs of containers did not overflow the container with capacity N . The largest total volume of these remaining 6 pairs of containers is 585 mL. Thus, the container with capacity N can hold 585 mL (with possibly more capacity remaining). This tells us that $N \geq 585$.

The possible values of N are as follows:

$$585, 586, 587, 588, 589, 590, 591, 592, 593$$

Therefore, there are 9 possible values of N .

ANSWER: 9

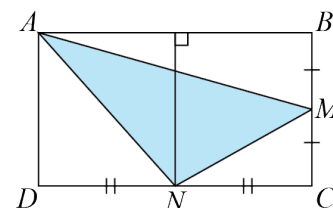
12. Converting to metres, the piece of “620 weight” paper measures 0.25 m by 0.3 m. Thus, the area of the paper is 0.075 m^2 . Since 0.075 m^2 is $0.075 \times 1 \text{ m}^2$, then the mass should be $0.075 \times 620 \text{ grams} = 46.5 \text{ grams}$.

ANSWER: 46.5

13. The probability of selecting a yellow marble is $100\% - 35\% - 50\% = 15\%$. Since there are 75 yellow marbles, the probability of selecting a yellow marble is also $\frac{75}{T}$, where T is the total number of marbles. These two expressions to calculate the probability of selecting a yellow marble must be equal, and so $15\% = \frac{15}{100} = \frac{75}{T}$. Since $15 \times 5 = 75$, it follows that $100 \times 5 = T$, so $T = 500$. Therefore, there are a total of 500 marbles in the box. Since the probability of selecting a purple marble is 50%, then 50% of 500, or 250 marbles are purple.

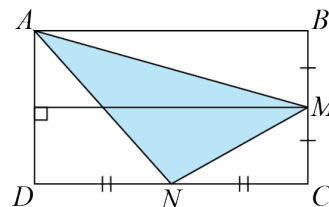
ANSWER: 250

14. A line can be drawn through N perpendicular to CD . This line divides the rectangle $ABCD$ into two smaller rectangles of equal area. The segment AN is a diagonal of one of those rectangles, and thus divides that rectangle in half. Thus, the area of $\triangle ADN$ is $\frac{1}{4}$ of the area of $ABCD$, or $\frac{1}{4}$ of 40 m^2 , which is 10 m^2 .

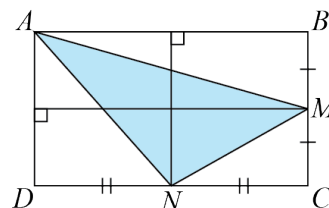




Similarly, a line drawn through M perpendicular to BC divides the rectangle $ABCD$ into two smaller rectangles of equal area. The segment AM is a diagonal of one of those rectangles, and thus divides the area of the rectangle in half. Thus, the area of $\triangle ABM$ is $\frac{1}{4}$ of the area of $ABCD$, or $\frac{1}{4}$ of 40 m^2 , which is 10 m^2 .



A line drawn through M perpendicular to BC and a line through N perpendicular to CD together divide $ABCD$ into four smaller rectangles of equal area. The segment MN is a diagonal of one of those rectangles, and thus divides the area of the rectangle in half. Since $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$, the area of $\triangle MCN$ is $\frac{1}{8}$ of the area of $ABCD$, or $\frac{1}{8}$ of 40 m^2 , which is 5 m^2 .



Therefore, the area of $\triangle AMN$ is $40 - 10 - 10 - 5 = 15 \text{ m}^2$.

ANSWER: 15

15. Notice that since $3000 \times 4 = 12\,000$, which is a five-digit number, it follows that $ABCD < 3000$ and thus $A = 1$ or $A = 2$. Since A is the ones digit of $DCBA$, which is a multiple of 4, it follows that A must be even and thus $A = 2$.

$$\begin{array}{r} 2BCD \\ \times 4 \\ \hline DCB2 \end{array}$$

Since $ABCD > 2000$, it follows that $DCBA > 2000 \times 4 = 8000$. Therefore, $D = 8$ or $D = 9$. Furthermore, we know that $D \times 4$ has ones digit $A = 2$, and so $D = 8$ is the only possibility.

$$\begin{array}{r} 2BC8 \\ \times 4 \\ \hline 8CB2 \end{array}$$

Since $2300 \times 4 = 9200$, then $B < 3$ and so $B = 0$, $B = 1$, or $B = 2$. However, B is also the ones digit of $4C + 3$ which is odd for every value of C . Thus, $B = 1$ is the only possibility.

$$\begin{array}{r} 21C8 \\ \times 4 \\ \hline 8C12 \end{array}$$

Since $4C + 3$ has a ones digit of 1, then $C = 2$ or $C = 7$. Testing both values, we determine that $C = 7$ is the only value that satisfies the given equation.

Therefore, the four-digit number $ABCD$ is 2178.

ANSWER: 2178



Crossnumber Puzzle

8		1	9	1	9		3	
2	9	0		2		9	5	1
2		8	4		6	3		5
8	1		1	5		6	3	6
	2		6		2		3	
8	7	6		5	5		4	7
1		9	7		4	3		4
9	6	3		2		8	2	2
	2		3	1	3	8		5

Across

- The sum of the digits in $\boxed{8228}$ is $8 + 2 + 2 + 8 = 20$. From the grid, the thousands digit and the tens digit of this number are 1. Since $20 - 1 - 1 = 18$, the sum of the remaining two digits is 18. This is only possible if the hundreds digit and ones digit are both 9. Thus, the number must be 1919.
- In 1 metre there are 100 centimetres, and so it follows that in 2.9 metres there are $2.9 \times 100 = 290$ centimetres.
- From the grid, the tens digit of this number is 5 and the ones digit is 1. The only three-digit number in this sequence with tens digit 5 and ones digit 1 is 951.
- From the grid, the tens digit is 8. The only two-digit number with a tens digit of 8 that is the sum of three consecutive even integers is $26 + 28 + 30 = 84$.



10. In 1 week there are 7 days, and so it follows that in 9 weeks there are $9 \times 7 = 63$ days.
11. From the grid, the ones digit of this number is 1. The only two-digit number with a ones digit of 1 that is the product of two equal integers is 81.
13. Since $\frac{3}{11} = \frac{15}{\boxed{55}}$, the number is 15.
14. From the grid, the hundreds digit is 6 and the ones digit is 6. They have a product of 36. Since $\boxed{108} \div 36 = 3$, the tens digit must be 3. Thus, the number is 636.
17. From the grid, the tens digit is 7 and the hundreds digit is 8. Since 7 is the median of 8 and 6, the ones digit must be 6, and the number is 876.
19. The sum is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$.
20. The smallest prime number greater than $\boxed{43}$ is 47.
22. The largest prime number less than 100 is 97.
23. The result is $\boxed{55} - \boxed{12} = 43$.
25. From the grid, the hundreds digit is 9 and the ones digit is 3. Since 9 is the largest a digit can be, it must be the sum of the other two digits. As such, the number must be 963.
28. From the grid, the hundreds digit is 8 and the ones digit is 2. Since the mode of the digits is 2, the tens digit must be 2. Thus, the number is 822.
29. The perimeter is equal to $2 \times (\boxed{876} + \boxed{693}) = 2 \times 1569 = 3138$.

Down

1. From the grid, the hundreds digit of this number is 2. It follows that the tens digit is 2. From the grid, the ones digit is 8. It follows that the thousands digit is 8. Thus, the number must be 8228.
2. $\boxed{63} + \boxed{55} - 10 = 118 - 10 = 108$.
3. A cube has a total of 12 edges.
4. A rectangle with perimeter $\boxed{156}$ and length $\boxed{43}$ has width 35 since $2 \times (\boxed{43} + 35) = \boxed{156}$. Thus, the number is 35.
6. From the grid, the hundreds digit is 9 and the tens digit is 3. Since 3, 6, and 9 are the only single-digit positive multiples of 3, the ones digit must be 6. Thus, the number is 936.
7. 80% of $195 = 0.8 \times 195 = 156$.
9. From the grid, the hundreds digit is 4 and the tens digit is 1. The only number with these digits that is divisible by 4 and 13 is 416.



12. The number of quarters is $\$31.75 \div \$0.25 = 127$.
15. From the grid, the hundreds digit of this number is 3 and the ones digit is 4.
The only three-digit number in this sequence with hundreds digit 3 and ones digit 4 is 334.
16. The sum of the digits in $\boxed{416}$ is $4 + 1 + 6 = 11$. From the grid, the tens digit of this number is 5 and the ones digit is 4. Since $11 - 5 - 4 = 2$, this number is 254.
17. Since $\frac{8}{\boxed{63}} = \frac{104}{819}$, the number is 819.
18. From the grid, the hundreds digit is 6 and the tens digit is 9. Since the digits are the same as the digits in $\boxed{936}$, the ones digit must be 3. The number is 693.
21. The volume is equal to $\boxed{55} \times \boxed{15} \times 9 = 7425$.
24. From the grid, the hundreds digit is 3. The only number that is a multiple of $\boxed{97}$ and has a hundreds digit of 3 is 388.
26. From the grid, the tens digit is 6. If the difference is 4 then the ones digit is either 4 less than 6 or 4 more than 6. But $6 + 4 = 10$, which isn't a single digit. Thus, the ones digit is $6 - 4 = 2$ and the number is 62.
27. The six faces on a standard die have 1, 2, 3, 4, 5, and 6 dots. The total number of dots is $1 + 2 + 3 + 4 + 5 + 6 = 21$.



Logic Puzzle

We start by considering clues (1) and (6):

- (1) The piece titled *Traffic*, which is not a photograph, is next to Maggie’s piece.
- (6) Leyla’s piece was placed in position 6 next to a piece titled *Traffic*.

Since Leyla’s piece is in position 6, it follows that the piece titled *Traffic* must be in position 5. Then Maggie’s piece must be in position 4.

Next we consider clues (2), (5), and (7):

- (2) Aria’s piece titled *Yellow* is next to the photograph.
- (5) Dhruv’s piece is next to both Aria’s piece and a pencil sketch titled *Quiet*.
- (7) The photograph was not taken by Dhruv or Maggie.

Since Aria’s and Dhruv’s pieces are next to each other, then they must be in two of the first three positions. Since Aria’s piece is also next to the photograph, and since the photograph was not taken by Maggie or Dhruv, it follows that Aria’s piece, Dhruv’s piece, and the photograph must be in the first three positions, in some order. The only way for Aria’s piece to be next to both the photograph and Dhruv’s piece is if Aria’s piece, titled *Yellow*, is in position 2.

That leaves Dhruv’s piece in either position 1 or position 3. If Dhruv’s piece is in position 1, then it cannot be next to a piece called *Quiet* since Aria’s piece, titled *Yellow*, is in position 2. Thus, Dhruv’s piece must be in position 3. It follows that Maggie’s piece, in position 4, is a pencil sketch titled *Quiet*.

It then follows that the photograph must be in position 1.

The following partially-completed table contains the information we have determined so far.

	Position Number					
	1	2	3	4	5	6
Student’s Name		Aria	Dhruv	Maggie		Leyla
Title of Art Piece		<i>Yellow</i>		<i>Quiet</i>	<i>Traffic</i>	
Type of Art	photograph			pencil sketch		

Next we consider clue (4):

- (4) The acrylic painting is next to both the photograph titled *Snowfall*, and the oil painting titled *Happiness*.

This tells us *Snowfall* is in position 1, the acrylic painting is in position 2, and then the oil painting titled *Happiness* is in position 3.



The following partially-completed table contains the information we have determined so far.

	Position Number					
	1	2	3	4	5	6
Student's Name		Aria	Dhruv	Maggie		Leyla
Title of Art Piece	<i>Snowfall</i>	<i>Yellow</i>	<i>Happiness</i>	<i>Quiet</i>	<i>Traffic</i>	
Type of Art	photograph	acrylic painting	oil painting	pencil sketch		

Next we consider clue (8):

(8) The piece titled *Friday*, which is not a watercolour painting, is next to Finn's piece.

Since the only title that has not yet been filled in is in position 6, it follows that this piece must be *Friday*. Then Finn's piece is in position 5. Since *Friday* is not a watercolour painting, it follows that *Traffic* must be the watercolour painting. Then *Friday* must be the pastel drawing.

Finally we consider clue (3):

(3) The title of Petr's piece is not *Friday*.

The only student's name that has yet to be filled in is in position 1, so this must be Petr's piece.

This completes the logic puzzle.

	Position Number					
	1	2	3	4	5	6
Student's Name	Petr	Aria	Dhruv	Maggie	Finn	Leyla
Title of Art Piece	<i>Snowfall</i>	<i>Yellow</i>	<i>Happiness</i>	<i>Quiet</i>	<i>Traffic</i>	<i>Friday</i>
Type of Art	photograph	acrylic painting	oil painting	pencil sketch	watercolour painting	pastel drawing



Relay

(Note: Where possible, the solutions are written as if the value of N is not initially known, and then N is substituted at the end.)

Practice Relay

P1: From the graph, Noah read 6 books.

P2: Sabrina uses $5 + 7 = 12$ beads to make one necklace. To make N necklaces, she uses $12 \times N$ beads in total.

Since the answer to the previous question is 6, then $N = 6$, and so $12 \times N = 12 \times 6 = 72$.

P3: Zoe got $\$35 + \$25 = \$60$ in total for the bicycle and the skateboard. The remaining $\$N - \60 is the amount she got for the ice skates.

Since the answer to the previous question is 72, then $N = 72$, and so $72 - 60 = 12$.

P4: If Kai walks around his garden one time, he walks $5 + 5 + 5 + 5 = 20$ metres. If he walks around his garden N times, he walks $20 \times N$ metres.

Since the answer to the previous question is 12, then $N = 12$, and so $20 \times 12 = 240$.

ANSWER: 6, 72, 12, 240

Relay A

P1: It takes Ari $7 + 10 = 17$ minutes to walk to school, and 12 minutes to walk to the library. Thus, it takes $17 - 12 = 5$ more minutes for Ari to walk to school than to the library.

P2: We can start by making a table to record the total number of toothpicks used as Gabe creates more figures.

Figure Number (N)	Number of Toothpicks in this Figure	Total Number of Toothpicks Used
1	4	4
2	7	11
3	10	21
4	13	34
5	16	50
6	19	69

Since the answer to the previous question is 5, then $N = 5$, and so Gabe used 50 toothpicks in total.

P3: The percentage of strawberry ice cream cones sold is $100\% - 50\% - 30\% = 20\%$. Since there were N ice cream cones sold in total, the number of strawberry ice cream cones is $\frac{20}{100} \times N$, or $0.2 \times N$.

Since the answer to the previous question is 50, then $N = 50$, and so $0.2 \times 50 = 10$.



P4: Vijay has four \$20 bills, six \$10 bills, and N \$5 bills. Thus, he has $4 + 6 + N = 10 + N$ bills in total. The probability of choosing a \$10 bill at random is $\frac{6}{10+N}$.

Since the answer to the previous question is 10, then $N = 10$, and so $\frac{6}{10+10} = \frac{6}{20} = \frac{3}{10}$.

ANSWER: 5, 50, 10, $\frac{6}{20}$ or $\frac{3}{10}$

Relay B

P1: One basket of twelve apples, one basket of six apples, and two individual apples costs $\$4.50 + \$2.50 + \$0.50 + \$0.50 = \$8$.

Note that a basket of six apples is cheaper than six individual apples. Also a basket of twelve apples is cheaper than two baskets of six apples. Two baskets of twelve apples costs \$9, which is more than \$8. Thus, the least expensive total price is \$8.

P2: After dividing by 2 we obtain $16 \div 2 = 8$.

After adding 5 we obtain $8 + 5 = 13$.

After multiplying by N we obtain $13 \times N$.

After subtracting 6 we obtain $13 \times N - 6$.

Since the answer to the previous question is 8, then $N = 8$, and so $13 \times 8 - 6 = 104 - 6 = 98$.

P3: Converting to centimetres, 350 mm is 35 cm. Then, a total of $35 + 32 = 67$ cm of rope has been cut. The remaining piece of rope is $(N - 67)$ cm.

Since the answer to the previous question is 98, then $N = 98$, and so $98 - 67 = 31$.

P4: If the average of all the four ages is 25, then the sum of the four ages is $25 \times 4 = 100$. Then, the age of Santiago's grandmother is $100 - 5 - 8 - N = 87 - N$.

Since the answer to the previous question is 31, then $N = 31$, and so $87 - 31 = 56$.

ANSWER: 8, 98, 31, 56

Relay C

P1: Since the playlist has 35 songs, and the probability that the next song is one of Aminah's favourites is $\frac{1}{5}$, then it follows that $\frac{1}{5}$ of the songs on the playlist are her favourites. Then, the number of Aminah's favourite songs on the playlist is $\frac{1}{5} \times 35 = 7$.

P2: The number of minutes in 2 hours is $2 \times 60 = 120$. This corresponds to $120 \times 60 = 7200$ seconds.

The number of seconds in N minutes is $N \times 60$.

The number of seconds in 2 hours and N minutes is then $7200 + N \times 60$.

Since the answer to the previous question is 7, then $N = 7$, and so

$7200 + 7 \times 60 = 7200 + 420 = 7620$.

P3: The possibilities for the units digit are 4 or 8.

The possibilities for the tens digit are 3, 6, or 9.

The hundreds digit must be 1.

The possible three-digit numbers are then 134, 138, 164, 168, 194, or 198.



Since the answer to the previous question is 7620, then $N = 7620$, and so the sum of the digits is $7 + 6 + 2 + 0 = 15$. The only possible three-digit number whose digits sum to 15 is 168.

P4: The fraction of papers delivered by Emil's brother, sister, and cousin is

$\frac{1}{8} + \frac{1}{4} + \frac{1}{2} = \frac{1}{8} + \frac{2}{8} + \frac{4}{8} = \frac{7}{8}$. Thus, Emil's dad delivered $1 - \frac{7}{8} = \frac{1}{8}$ of the papers. Since there were N papers in total, Emil's dad delivered $\frac{1}{8} \times N$ papers.

Since the answer to the previous question is 168, then $N = 168$, and so $\frac{1}{8} \times 168 = 21$.

ANSWER: 7, 7620, 168, 21