Problem of the Month
Problem 7: April 2023

Hint

(a) Suppose $d(\boldsymbol{a}, \boldsymbol{b}) = k$ for some k. Try to construct a path of length k in the natural graph from the vertex labelled $\boldsymbol{a}$ to the vertex labelled $\boldsymbol{b}$.

(b) For fixed $\boldsymbol{a} \in A_n$, how many $\boldsymbol{b} \in A_n$ have the property that $d(\boldsymbol{a}, \boldsymbol{b}) = k$?

(c) Find a function that works for $n = 2$ and use this to build one for $n = 3$. It might be useful to think of the natural graph of A_2 as a square and the natural graph of A_3 as a cube. As well, a cube can be thought of as two squares on top of each other with vertical edges connecting corresponding vertices in the top and bottom faces.