Problem of the Month
Problem 6: March 2023

Hint

(a)/(b) There is no hint given for these parts, but it might be useful in later parts to see if you notice any patterns in the distribution of the possible outputs of the function f.

(c) If $n^2 < m < (n+1)^2$, then $f(m) = d$ is equivalent to $n + \frac{d}{10} < \sqrt{m} < n + \frac{d+1}{10}$.

(d) Similar to the result in (c), if n is one more than a multiple of 5, then in the list

$$f(n^2+1), f(n^2+2), \ldots, f(n^2+2n)$$

every possible value from 0 through 9 appears exactly $\frac{n-1}{5}$ times, with the exception of 4 and 7 which appear $\frac{n-1}{5} + 1$ times each. Try to find and prove other similar results.