Problem of the Month

Problem 2: November 2023

Hint

(a) (i) Because of the rotational symmetry of circles, there are lots of squares with their vertices lying on the circle with equation $x^2+y^2 = 1$. In fact, given any point on the circle, there is a square with that point as one of its vertices and the other three vertices somewhere else on the circle. It might be useful for part (v) to spend some time thinking about this.

(ii) Try looking for a square with the property that all four of its sides are parallel to either the x-axis or the y-axis. What are the coordinates of a square centred at the origin with its sides parallel to the axes?

(iii) It will be helpful to have an accurate sketch of the hexagon. Similar to part (ii), there is a square with its sides parallel to the axes.

(iv) After sketching the region, it shouldn't be hard to believe that there is a square with one of its sides on the line with equation $y = x$. The opposite side will lie on a line with the same slope.

(v) The figure has $180°$ rotational symmetry about the origin, so try looking for a square that is centred at the origin. If such a square exists, it should also have $108°$ rotational symmetry about the origin. Observations from part (i) might be useful when thinking about such squares.

(b) In our solution, we found it useful to coordinatize and assume that one the sides of the the triangle is on the x-axis in such a way that one of the vertices is at the origin.