Problem of the Month

Problem 1: October 2023

Given a positive integer n, the *digit sum* of n is the sum of the base 10 digits of n. We will denote the digit sum of n by $D(n)$. For example, $D(1409) = 1 + 4 + 0 + 9 = 14$.

Suppose that m is a positive integer. We will call a list of consecutive positive integers

$$a, a + 1, a + 2, \ldots, a + k$$

an *m-list* if none of $D(a)$, $D(a + 1)$, $d(a + 2)$, and so on up to $D(a + k)$ is a multiple of m. For example, the list $997, 998, 999, 1000, 1001, 1002$ is a 4-list because the digit sums of the integers in the list are 25, 26, 27, 1, 2, and 3, respectively, none of which is a multiple of 4.

This problem explores the maximum length of an m-list for a few values of m.

(a) Show that the maximum length of a 2-list is 2. To do this, you must show that there is a 2-list of length 2 and you must also show that no list of three or more consecutive positive integers can be a 2-list.

(b) Show that the maximum length of a 7-list is 12.

(c) Determine the maximum length of a 9-list.

(d) Determine the maximum length of an 11-list.