The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Hypatia Contest

(Grade 11)

Wednesday, April 5, 2023
(in North America and South America)

Thursday, April 6, 2023
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. A game is played in which each throw of a ball lands in one of two holes: the closer hole or the farther hole. A throw landing in the closer hole scores 2 points, while a throw landing in the farther hole scores 5 points. A player's total score is equal to the sum of the scores on their throws.

   (a) Jasmin had 3 throws that each scored 2 points and 4 throws that each scored 5 points. What was Jasmin's total score?

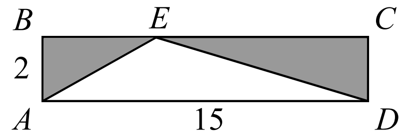
   (b) Sam had twice as many throws that scored 2 points as throws that scored 5 points. If Sam's total score was 36 points, how many throws did Sam take?

   (c) Tia had t throws that each scored 2 points and f throws that each scored 5 points. If Tia's total score was 37 points, determine all possible ordered pairs (t, f).

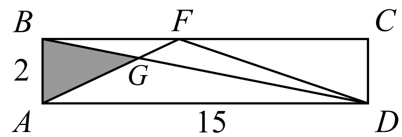
   (d) The game is changed so that each throw scores 6 or 21 points instead of 2 or 5. Explain whether or not it is possible to have a total score of 182 points.

2. In each question below, $ABCD$ is a rectangle with $AB = 2$ and $AD = 15$.

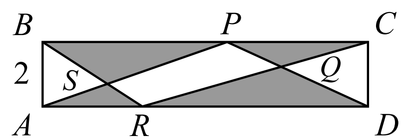
(a) Point E is on BC, as shown. What is the total area of the shaded regions?



(b) Point F is on BC, and BD intersects AF at G, as shown. If the area of $\triangle FGD$ is 5, what is the area of the shaded region?



(c) Point P is on BC and R is on AD. BR and AP intersect at S and PD and RC intersect at Q, as shown. If the area of $PQRS$ is 6, determine the total area of the shaded regions.



3. For any positive integer with three or more different, non-zero digits, let a *cousin* be defined as the result of switching two digits of the integer. For example, the integer 156 has three cousins:

- 516 (obtained by switching the 1st and 2nd digits),
- 651 (obtained by switching the 1st and 3rd digits), and
- 165 (obtained by switching the 2nd and 3rd digits).

(a) In no particular order, five of the six cousins of 6238 are listed below. Which cousin is missing from the list?

2638	6328
3268	6283
8236	

(b) In no particular order, the following list contains an original integer as well as all of its cousins. What is the original integer?

726 194	726 941	746 291	627 491
276 491	926 471	796 421	726 419
729 461	716 492	762 491	726 491
126 497	721 496	426 791	724 691

(c) Suppose that c and d are distinct, non-zero digits. The three-digit integer $cd3$ minus one of its cousins is equal to the three-digit integer $d95$. Determine the values of c and d and show that no other values are possible.

(d) Suppose that m and n are distinct, non-zero digits. The sum of the six cousins of the four-digit integer $mn97$ is equal to the five-digit integer $nmnm7$. Determine the values of m and n and show that no other values are possible.

4. The Great Math Company has a random integer generator which produces an integer from 1 to 9 inclusive, where each integer is generated with equal probability. Each member of the Multiplication Team uses this generator a certain number of times and then calculates the product of their integers.

(a) Amarpreet uses the generator 3 times. What is the probability that the product is a prime number?

(b) Braxton uses the generator 4 times. Determine the probability that the product is divisible by 5, but *not* divisible by 7.

(c) Camille uses the generator 2023 times. Let p be the probability that the product is *not* divisible by 6. Determine the ones digit of the integer equal to $p \times 9^{2023}$.

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For students...

Thank you for writing the 2023 Hypatia Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2023.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2023/2024 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results