The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2023 Gauss Contests

(Grades 7 and 8)

Wednesday, May 17, 2023
(in North America and South America)

Thursday, May 18, 2023
(outside of North America and South America)

Solutions

Grade 7

1. Half of 24 is $24 \div 2 = 12$. Kiyana gives her friend 12 grapes.

   Answer: (D)

2. Reading from the graph, Friday had the highest temperature.

   Answer: (C)

3. At a cost of \$16.50 a basket, the cost to buy 4 baskets of strawberries is $4 \times \$16.50 = \$66.00$.

   Answer: (B)

4. The difference between 3 and -5 is $3 - (-5) = 3 + 5 = 8$. Therefore, it is now 8°C warmer.

   Answer: (A)

5. Since $5 \times 5 = 25$ and each of the given answers is greater than 25, then the integer that Sarah multiplied by itself must have been greater than 5.
   Further, $6 \times 6 = 36$ and each of the given answers is less than or equal to 36.
   Thus, of the given answers, only 36 could be the result of multiplying an integer by itself.
   Alternatively, we may have noted that the result of multiplying an integer by itself is a perfect square, and of the answers given, 36 is the only perfect square.

   Answer: (E)

6. Since the perimeter of $PQRS$ is 40 cm and $SR = 16$ cm, then the combined length of the remaining three sides is 40 cm − 16 cm = 24 cm.
   Each of the remaining three sides is equal in length, and so $PQ = \dfrac{24\text{ cm}}{3} = 8$ cm.

   Answer: (C)

7. Dividing 52 by each of the given denominators, we get that $\dfrac{52}{4} = 13$ is the only whole number.

   Answer: (C)

8. The line segment with greatest length that joins two points on a circle is a diameter of the circle. Since the circle has a radius of 4 cm, then its diameter has length 2×4 cm = 8 cm, and so the greatest possible length of the line segment is 8 cm.

   Answer: (B)

9. The number of integers in the list is 10. Of these integers, 5 are even (they are 10, 12, 14, 16, and 18). Thus, the probability that the chosen integer is even is $\dfrac{5}{10}$.

   Answer: (C)

10. Before adding tax, the combined cost of the three items is $\$4.20 + \$7.60 + \$3.20 = \$15.00$.
    The 5% tax on \$15.00 is $0.05 \times \$15.00 = \$0.75$, and so the total cost of the three items, after tax is added, is $\$15.00 + \$0.75 = \$15.75$.
    Note that we could have calculated the 5% tax on each individual item, however doing so is less efficient than calculating tax on the \$15.00 total.

    Answer: (D)

11. Since BCD is a straight line segment, then $\angle BCD = 180^\circ$.
Therefore, $\angle ACB = \angle BCD - \angle ACD = 180^\circ - 75^\circ = 105^\circ$.
Since the sum of the three angles in $\triangle ABC$ is 180°, then $\angle ABC = 180^\circ - 105^\circ - 35^\circ = 40^\circ$.
ANSWER: (B)

12. Of the 100 small identical squares, 28 are presently unshaded, and so $100 - 28 = 72$ are shaded.
So that 75% of the area of $WXYZ$ is shaded, 75 of the 100 small squares must be shaded.
Therefore, $75 - 72 = 3$ more of the small squares must be shaded.
ANSWER: (A)

13. Suppose we call the unknown vertex V.
The side of the rectangle joining the points $(2, 1)$ and $(2, 5)$ is vertical, and so the opposite side of the rectangle (the side joining $(4, 1)$ to V) must also be vertical.
This means that V has the same x-coordinate as $(4, 1)$, which is 4.
Similarly, the side of the rectangle joining the points $(2, 1)$ and $(4, 1)$ is horizontal, and so the opposite side of the rectangle (the side joining $(2, 5)$ to V) must also be horizontal.
This means that V has the same y-coordinate as $(2, 5)$, which is 5.
Therefore, the coordinates of the fourth vertex of the rectangle are $(4, 5)$.
ANSWER: (D)

14. The prime numbers that are less than 10 are $2, 3, 5$, and 7.
Thus, the only two different prime numbers whose sum is 10 are 3 and 7.
The product of these two numbers is $3 \times 7 = 21$.
ANSWER: (D)

15. The given list, $2, 9, 4, n, 2n$ contains 5 numbers.
The average of these 5 numbers is 6, and so the sum of the 5 numbers is $5 \times 6 = 30$.
That is, $2 + 9 + 4 + n + 2n = 30$ or $15 + 3n = 30$, and so $3n = 15$ or $n = 5$.
ANSWER: (D)

16. The sum of P and Q is equal to 5, and so P and Q are (in some order) either equal to 1 and 4, or they are equal to 2 and 3.
The difference between R and S is equal to 5, and so R and S must be (in some order) equal to 1 and 6.
Since R and S are equal to 1 and 6, then neither P nor Q can be 1, which means that P and Q cannot be equal to 1 and 4, and so they must be equal to 2 and 3.
The only numbers from 1 to 6 not accounted for are 4 and 5.
Since T is greater than U, then the number that replaces T is 5.
ANSWER: (E)

17. *Solution 1*
The area of $\triangle AED$ is equal to one-half its base times its height.
Suppose the base of $\triangle AED$ is AE, then its height is BD (AE is perpendicular to BD).
Since $AB = BC = 24$ cm and E and D are the midpoints of their respective sides, then $AE = 12$ cm and $BD = 12$ cm.
Thus, the area of $\triangle AED$ is $\frac{1}{2} \times 12 \text{ cm} \times 12 \text{ cm} = 72 \text{ cm}^2$.

Solution 2
The area of $\triangle AED$ is equal to the area of $\triangle ABD$ minus the area of $\triangle EBD$.
Suppose the base of $\triangle EBD$ is BD, then its height is EB.
Since $AB = BC = 24$ cm and E and D are the midpoints of their respective sides, then $EB = 12$ cm and $BD = 12$ cm.

Thus, the area of $\triangle EBD$ is $\frac{1}{2} \times 12 \text{ cm} \times 12 \text{ cm} = 72 \text{ cm}^2$.
The area of $\triangle ABD$ is equal to $\frac{1}{2} \times BD \times AB = \frac{1}{2} \times 12 \text{ cm} \times 24 \text{ cm} = 144 \text{ cm}^2$.
Thus, the area of $\triangle AED$ is $144 \text{ cm}^2 - 72 \text{ cm}^2 = 72 \text{ cm}^2$.

ANSWER: (C)

18. The water is in the shape of a rectangular prism with a 2 cm by 5 cm base and depth 6 cm.
Therefore, the volume of water is $2 \text{ cm} \times 5 \text{ cm} \times 6 \text{ cm} = 60 \text{ cm}^3$.
A face of the prism having the greatest area has dimensions 5 cm by 8 cm.
When the prism is tipped so that it stands on a 5 cm by 8 cm face, the water is once again in the shape of a rectangular prism with a 5 cm by 8 cm base and unknown depth.
Suppose that after the prism is tipped, the water's depth is d cm.
Since the volume of water is still 60 cm^3 when the prism is tipped, then $5 \text{ cm} \times 8 \text{ cm} \times d \text{ cm} = 60 \text{ cm}^3$ or $40d \text{ cm}^3 = 60 \text{ cm}^3$, and so $d = \dfrac{60}{40} = \dfrac{3}{2}$.
When the prism is tipped so that it stands on a face with the greatest area, the depth of the water is $\dfrac{3}{2}$ cm $= 1.5$ cm.

ANSWER: (D)

19. *Solution 1*
We begin by completing a table in which the ones digit of each possible product is listed.
For example, when the number on the first die is 3 and the number on the second die is 6, the entry in the table is 8 since $3 \times 6 = 18$ and the ones digit of 18 is 8.

Number on the Second Die

Number on the First Die

$\times$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	**0**	2
3	3	6	9	2	5	8
4	4	8	2	6	**0**	4
5	5	**0**	5	**0**	5	**0**
6	6	2	8	4	**0**	6

Of the 36 possible outcomes in the table above, 6 outcomes have a ones digit that is equal to 0.
Thus, the probability that the ones digit of the product is 0 is $\dfrac{6}{36} = \dfrac{1}{6}$.

Solution 2
Since the ones digit of the product is 0, then the product is divisible by 5 and is even.
Since the possible numbers in the product are 1, 2, 3, 4, 5, 6, then one of the numbers rolled must be 5.
Since the product is even (and 5 is not), then the other number rolled must be one of the three even numbers, namely 2, 4, 6.
Thus, the possible pairs of numbers that can be rolled to give a product whose ones digit is 0,

are $(5,2)$, $(5,4)$, $(5,6)$ or $(2,5)$, $(4,5)$, $(6,5)$. (We note that the first number in the ordered pair represents the first number rolled, while the second number in the pair is the second number rolled.)
Since there are 6 possible rolls for each of the two dice, then there are $6 \times 6 = 36$ possible ordered pairs representing all possible outcomes.
Since 6 of these ordered pairs represent a product whose ones digit is 0, then the required probability is $\frac{6}{36} = \frac{1}{6}$.

ANSWER: (D)

20. Since a and b are positive integers, then each of $\frac{a}{7}$ and $\frac{2}{b}$ is greater than 0.
The sum of $\frac{a}{7}$ and $\frac{2}{b}$ is equal to 1, and so each is less than 1.
Since $\frac{a}{7}$ is greater than 0 and less than 1, then the possible values of a are $1,2,3,4,5,6$.
By substituting each of these values for a into the equation one at a time, we can determine if there is a positive integer value of b for which the equation is true.
Substituting $a = 1$, we get $\frac{1}{7} + \frac{2}{b} = 1$ or $\frac{2}{b} = 1 - \frac{1}{7}$, and so $\frac{2}{b} = \frac{6}{7}$.
Since $\frac{2}{b} = \frac{6}{7}$, we can multiply the numerator and denominator of the first fraction by 3 (which is $6 \div 2$) to get $\frac{6}{3b} = \frac{6}{7}$.
This gives $3b = 7$ which does not have an integer solution ($b = \frac{7}{3}$).
Thus when $a = 1$, there is no positive integer value of b that satisfies the equation.

Substituting $a = 2$, we get $\frac{2}{7} + \frac{2}{b} = 1$ or $\frac{2}{b} = 1 - \frac{2}{7}$, and so $\frac{2}{b} = \frac{5}{7}$.
Since $\frac{2}{b} = \frac{5}{7}$, we can multiply the numerator and denominator of the first fraction by 5, and the numerator and denominator of the second fraction by 2 to get $\frac{10}{5b} = \frac{10}{14}$.
This gives $5b = 14$ which does not have an integer solution.
Thus when $a = 2$, there is no positive integer value of b that satisfies the equation.

Substituting $a = 3$, we get $\frac{3}{7} + \frac{2}{b} = 1$ or $\frac{2}{b} = 1 - \frac{3}{7}$, and so $\frac{2}{b} = \frac{4}{7}$.
Since $\frac{2}{b} = \frac{4}{7}$, we can multiply the numerator and denominator of the first fraction by 2 to get $\frac{4}{2b} = \frac{4}{7}$.
This gives $2b = 7$ which does not have an integer solution.
Thus when $a = 3$, there is no positive integer value of b that satisfies the equation.

Substituting $a = 4$ and simplifying, we get $\frac{2}{b} = \frac{3}{7}$.
Since $\frac{2}{b} = \frac{3}{7}$, we can multiply the numerator and denominator of the first fraction by 3, and the numerator and denominator of the second fraction by 2 to get $\frac{6}{3b} = \frac{6}{14}$.
This gives $3b = 14$ which does not have an integer solution.

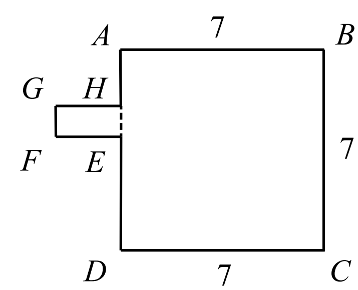
Thus when $a = 4$, there is no positive integer value of b that satisfies the equation.

Substituting $a = 5$ and simplifying, we get $\frac{2}{b} = \frac{2}{7}$.
Since the numerators are equal, then the denominators must be equal, and so $b = 7$ satisfies the equation.

Finally, substituting $a = 6$ and simplifying, we get $\frac{2}{b} = \frac{1}{7}$.
Since $\frac{2}{b} = \frac{1}{7}$, we can multiply the numerator and denominator of the second fraction by 2 to get $\frac{2}{b} = \frac{2}{14}$, and so $b = 14$.
Thus, there are two pairs of positive integers a and b that satisfy the given equation: $a = 5, b = 7$ and $a = 6, b = 14$.

Answer: (E)

21. Since $ABCD$ is a square and its side lengths are integers, then its area is equal to a perfect square.
Since the product of the areas of $ABCD$ and $EFGH$ (the rectangle) is equal to 98, then the area of $ABCD$ is a divisor of 98.
The positive divisors of 98 are 1, 2, 7, 14, 49, and 98.
There are exactly two divisors of 98 that are perfect squares, namely 1 and 49.
Since the area of $ABCD$ is greater than the area of $EFGH$, then the area of $ABCD$ is 49, and so the area of $EFGH$ is 2 (since $49 \times 2 = 98$).
Square $ABCD$ has area 49, and so $AB = BC = CD = DA = 7$.
The perimeter of $ABCDEFGH$ is equal to



$$\begin{aligned}
& AB + BC + CD + DE + EF + FG + GH + HA \\
= \ & 7 + 7 + 7 + DE + EF + EH + GH + HA \quad \text{(since } EH = FG\text{)} \\
= \ & 21 + DE + EH + HA + EF + GH \quad \text{(reorganizing)} \\
= \ & 21 + DA + EF + GH \quad \text{(since } DE + EH + HA = DA\text{)} \\
= \ & 21 + 7 + EF + GH \quad \text{(since } DA = 7\text{)} \\
= \ & 28 + 2 \times GH \quad \text{(since } EF = GH\text{)}
\end{aligned}$$

Since the side lengths are integers and the area of $EFGH$ is 2, then either $GH = 1$ (and $FG = 2$), or $GH = 2$ (and $FG = 1$).
If $GH = 1$, then the perimeter of $ABCDEFGH$ is $28 + 2 \times 1 = 30$.
Since 30 is not given as a possible answer, then $GH = 2$ and the perimeter is $28 + 2 \times 2 = 32$.

Answer: (B)

22. If a Gareth sequence begins 10, 8, then the 3rd number in the sequence is $10 - 8 = 2$, the 4th is $8 - 2 = 6$, the 5th is $6 - 2 = 4$, the 6th is $6 - 4 = 2$, the 7th is $4 - 2 = 2$, the 8th is $2 - 2 = 0$, the 9th is $2 - 0 = 2$, the 10th is $2 - 0 = 2$, and the 11th is $2 - 2 = 0$.
Thus, the resulting sequence is $10, 8, 2, 6, 4, 2, 2, 0, 2, 2, 0, \ldots$.
The first 5 numbers in the sequence are $10, 8, 2, 6, 4$, the next 3 numbers are $2, 2, 0$, and this block of 3 numbers appears to repeat.
Since each new number added to the end of this sequence is determined by the two previous numbers in the sequence, then this block of 3 numbers will indeed continue to repeat. (That

is, since the block repeats once, then it will continue repeating.)
The first 30 numbers of the sequence begins with the first 5 numbers, followed by 8 blocks of $2, 2, 0$, followed by one additional 2 (since $5 + 8 \times 3 + 1 = 30$).
The sum of the first 5 numbers is $10 + 8 + 2 + 6 + 4 = 30$.
The sum of each repeating block is $2 + 2 + 0 = 4$, and so the sum of 8 such blocks is $8 \times 4 = 32$.
Thus, the sum of the first 30 numbers in the sequence is $30 + 32 + 2 = 64$.

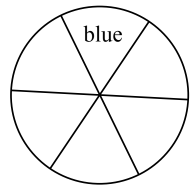
Answer: (E)

23. Suppose that the length, or the width, or the height of the rectangular prism is equal to 5.
The product of 5 with any of the remaining digits has a units (ones) digit that is equal to 5 or it is equal to 0.
This means that if the length, or the width, or the height of the rectangular prism is equal to 5, then at least one of the two-digit integers (the area of a face) has a units digit that is equal to 5 or 0.
However, 0 is not a digit that can be used, and each digit from 1 to 9 is used exactly once (that is, 5 cannot be used twice), and so it is not possible for one of the dimensions of the rectangular prism to equal 5.
Thus, the digit 5 occurs in one of the two-digit integers (the area of a face).
The digit 5 cannot be the units digit of the area of a face, since this would require that one of the dimensions be 5.
Therefore, one of the areas of a face has a tens digit that is equal to 5.
The two-digit integers with tens digit 5 that are equal to the product of two different one-digit integers (not equal to 5) are $54 = 6 \times 9$ and $56 = 7 \times 8$.
Suppose that two of the dimensions of the prism are 7 and 8, and so one of the areas is 56.
In this case, the digits $5, 6, 7$, and 8 have been used, and so the digits $1, 2, 3, 4$, and 9 remain.
Which of these digits is equal to the remaining dimension of the prism?
It cannot be 1 since the product of 1 and 7 does not give a two-digit area, nor does the product of 1 and 8.
It cannot be 2 since the product of 2 and 8 is 16 and the digit 6 has already been used.
It cannot be 3 since $3 \times 7 = 21$ and $3 \times 8 = 24$, and so the areas of two faces share the digit 2.
It cannot be 4 since $4 \times 7 = 28$ and the digit 8 has already been used.
Finally, it cannot be 9 since $9 \times 7 = 63$ and the digit 6 has already been used.
Therefore, it is not possible for 7 and 8 to be the dimensions of the prism, and thus 6 and 9 must be two of the three dimensions.
Using a similar systematic check of the remaining digits, we determine that 3 is the third dimension of the prism.
That is, when the dimensions of the prism are $3, 6$ and 9, the areas of the faces are $3 \times 6 = 18$, $3 \times 9 = 27$, and $6 \times 9 = 54$, and we may confirm that each of the digits from 1 to 9 has been used exactly once.
Since the areas of the faces are 18, 27 and 54, the surface area of the rectangular prism is $2 \times (18 + 27 + 54)$ or $2 \times 99 = 198$.

Answer: (D)

24. *Solution 1*

Begin by colouring the section at the top blue.
Since two circles have the same colouring if one can be rotated to match the other, it does not matter which section is coloured blue, so we arbitrarily choose the top section.

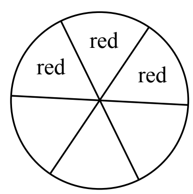


There are now 5 sections which can be coloured green.
After choosing the section to be coloured green, there are 4 sections remaining which can be coloured yellow.
Each of the remaining 3 sections must then be coloured red.
Thus, the total number of different colourings of the circle is $5 \times 4 = 20$.

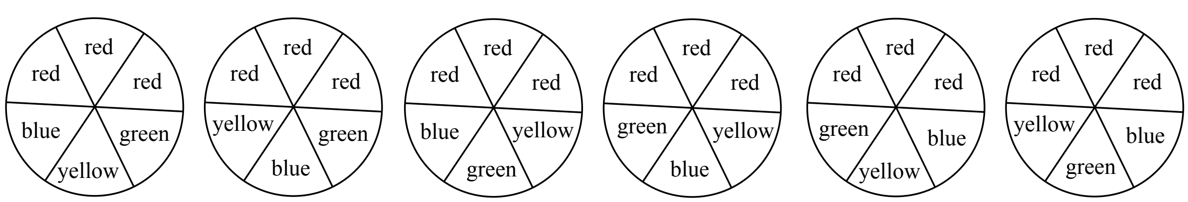
Solution 2

We begin by considering the locations of the three sections coloured red, relative to one another.
The three red sections could be adjacent to one another, or exactly two red sections could be adjacent to one another, or no red section could be adjacent to another red section.
We consider each of these 3 cases separately.

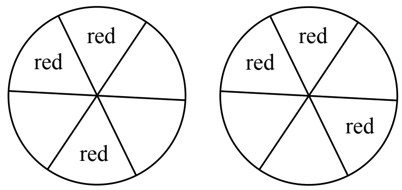
<u>Case 1</u>: All three red sections are adjacent to one another
Begin by colouring any three adjacent sections red.



Since two circles have the same colouring if one can be rotated to match the other, it does not matter which three adjacent sections are coloured red.
Consider the first section that follows the three red sections as we move clockwise around the circle.
There are 3 choices for the colour of this section: blue, green or yellow.
Continuing to move clockwise to the next section, there are now 2 choices for the colour of this section.
Finally, there is 1 choice for the colour of the final section, and thus there are $3 \times 2 \times 1 = 6$ different colourings of the circle in which all three red sections are adjacent to one another.
These 6 colourings are shown below.

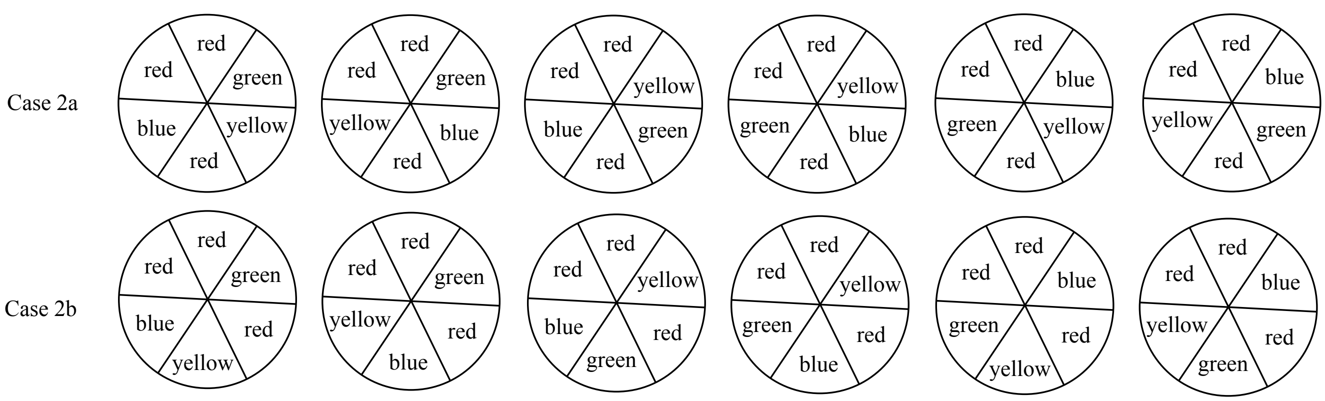


Case 2: Exactly two red sections are adjacent to one another
There are two different possible arrangements in which exactly two red sections are adjacent to one another.
In the first of these, the next two sections that follow the two adjacent red sections as we move clockwise around the circle, are both not red. We call this Case 2a.
In the second of these, the section that follows the two adjacent red sections as we move clockwise around the circle is not red, but the next section is. We call this Case 2b.
The arrangements for Cases 2a and 2b are shown below.

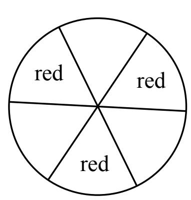


Case 2a Case 2b

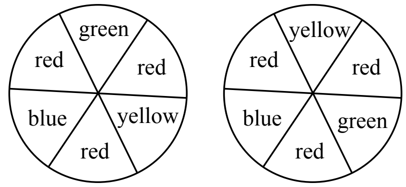
Notice that the first of these two circles cannot be rotated to match the second.
The number of colourings in Case 2a and in Case 2b are each equal to the number of colourings as in Case 1.
That is, there are 3 choices for the first uncoloured section that follows the two red sections as we move clockwise around the circle.
Continuing to move clockwise to the next uncoloured section, there are now 2 choices for the colour of this section.
Finally, there is 1 choice for the colour of the final section, and thus there are $3 \times 2 \times 1 = 6$ different colourings of the circle in Case 2a as well as in Case 2b.
These 12 colourings are shown below.



Case 3: No red section is adjacent to another red section
Begin by colouring any three non-adjacent sections red.



Since two circles have the same colouring if one can be rotated to match the other, it does not matter which three non-adjacent sections are coloured red.
In this case, there are 2 possible colourings as shown below.

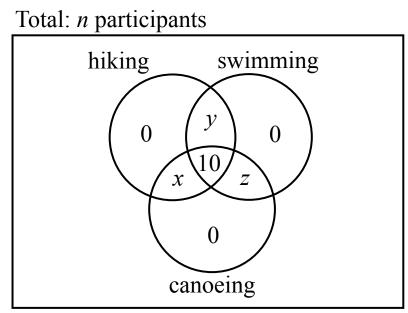


A circle with any other arrangement of the green, yellow and blue sections can be rotated to match one of the two circles above.

The total number of different colourings of the circle is $6 + 12 + 2 = 20$.

ANSWER: (E)

25. We can represent the given information in a Venn diagram by first introducing some variables.
Let x be the number of students that participated in hiking and canoeing, but not swimming.
Let y be the number of students that participated in hiking and swimming, but not canoeing.
Let z be the number of students that participated in canoeing and swimming, but not hiking.
Since 10 students participated in all three activities and no students participated in fewer than two activities, we complete the Venn diagram as shown.



Suppose that the total number of students participating in the school trip was n.
Since 50% of all students participated in at least hiking and canoeing, then $\frac{50}{100}n$ or $\frac{n}{2}$ participated in at least hiking and canoeing.
Since this number of students, $\frac{n}{2}$, is an integer, then n must be divisible by 2.
Similarly, $\frac{60}{100}n$ or $\frac{3n}{5}$ students participated in at least hiking and swimming.
Since this number of students, $\frac{3n}{5}$, is an integer, then n must be divisible by 5 (since 3 and 5 have no factors in common).
This means that n is divisible by both 2 and 5, and thus n is divisible by 10.
From the Venn diagram, we see that $x + 10 = \frac{n}{2}$, and $y + 10 = \frac{3n}{5}$.
Since the total number of participants is n, we also get that $x+y+z+10 = n$ or $z = n-10-x-y$.
We may now use these equations,

$$x = \frac{n}{2} - 10,\ y = \frac{3n}{5} - 10,\ \text{and } z = n - 10 - x - y$$

and the fact that n is divisible by 10, to determine all possible values of z.
We can then use the values of z to determine all possible values of the positive integer k, where k% participated in at least canoeing and swimming.

Since n is a positive integer that is divisible by 10, its smallest possible value is 10.
However, substituting $n = 10$ into $x = \frac{n}{2} - 10$, we get $x = 5 - 10$ and so $x = -5$ which is not possible. (Recall that x is the number of students that participated in hiking and canoeing, but not swimming, and so $x \geq 0$.)
Next, we try $n = 20$.

When $n = 20$, $x = 10 - 10$ and so $x = 0$.
When $n = 20$, $y = \dfrac{3 \times 20}{5} - 10$ or $y = 12 - 10$, and so $y = 2$.
Finally, when $n = 20$, $x = 0$, and $y = 2$, we get $z = 20 - 10 - 0 - 2 = 8$.
When $z = 8$, the number of students who participated in at least canoeing and swimming is $8 + 10 = 18$ (since 10 students participated in all three), and so the percentage of students who participated in at least canoeing and swimming is $\dfrac{18}{20} \times 100\% = 90\%$, and so $k = 90$.
In the table below, we continue in this way by using successively greater multiples of 10 for the value of n.

n	$x = \frac{n}{2} - 10$	$y = \frac{3n}{5} - 10$	$z = n - 10 - x - y$	$k = \frac{z+10}{n} \times 100$
20	0	2	8	$k = \frac{8+10}{20} \times 100 = 90$
30	5	8	7	$k = \frac{7+10}{30} \times 100 \approx 56.7$
40	10	14	6	$k = \frac{6+10}{40} \times 100 = 40$
50	15	20	5	$k = \frac{5+10}{50} \times 100 = 30$
60	20	26	4	$k = \frac{4+10}{60} \times 100 \approx 23.3$
70	25	32	3	$k = \frac{3+10}{70} \times 100 \approx 18.6$
80	30	38	2	$k = \frac{2+10}{80} \times 100 = 15$
90	35	44	1	$k = \frac{1+10}{90} \times 100 \approx 12.2$
100	40	50	0	$k = \frac{0+10}{100} \times 100 = 10$

For values of n that are greater than 100, we get that $z < 0$, which is not possible.
Therefore, the sum of all such positive integers k is $90 + 40 + 30 + 15 + 10 = 185$.

Answer: (B)

Grade 8

1. The fraction $\frac{1}{4}$ is equivalent to $1 \div 4 = 0.25$.

   Answer: (B)

2. Reading from the graph, the forecast wind speed is less than 20 km/h on Monday, Tuesday, Wednesday, and Sunday.
   Thus, Jack will be able to sail alone on 4 days during this 7-day period.

   Answer: (A)

3. We note that $15 \times 10 = 150$, $15 \times 2 = 30$, $15 \times 3 = 45$, and $15 \times 4 = 60$.
   Since there is no integer n for which $15 \times n = 25$, then 25 is not a multiple of 15.

   Answer: (B)

4. Ordering the given list of integers from least to greatest, we get $-9, -7, 0, 9, 10$.
   The third integer in the ordered list is 0.

   Answer: (D)

5. *Solution 1*
   If $2n = 14$, then $n = \dfrac{14}{2} = 7$.
   When $n = 7$, the value of $10n$ is $10 \times 7 = 70$.

   Solution 2
   Multiplying both sides of the given equation by 5, we get $5 \times 2n = 5 \times 14$, and so $10n = 70$.

   Answer: (C)

6. There are 6 possible outcomes when Tallulah rolls a single standard die once.
   She loses if she rolls 2 of these 6 outcomes, and so the probability that she loses is $\dfrac{2}{6} = \dfrac{1}{3}$.

   Answer: (A)

7. *Solution 1*
   We may convert the given addition problem to a subtraction problem.
   That is, since $1013 + PQPQ = 2023$, then $PQPQ = 2023 - 1013$.
   The difference between 2023 and 1013 is $2023 - 1013 = 1010$, and so $P = 1$, $Q = 0$, and $P + Q = 1 + 0 = 1$.

   Solution 2
   The ones (units) digit of the sum 2023 is 3.
   Thus, the ones digit of the sum $3 + Q$ must equal 3.
   Since Q is a digit, the only possible value of Q is 0.
   The tens digit of the sum 2023 is 2.
   Since there is no "carry" from the ones column to the tens column, the ones digit of the sum $1 + P$ must equal 2.
   Since P is a digit, the only possible value of P is 1.
   We may confirm that when $P = 1$ and $Q = 0$, we get $1013 + 1010 = 2023$ as required.
   The value of $P + Q$ is $1 + 0 = 1$.

   Answer: (B)

8. Suppose the salad dressing initially contains 300 mL of oil.
Since the ratio of oil to vinegar is $3:1$, then the salad dressing initially contains one-third as much vinegar as oil, or $\frac{1}{3} \times 300$ mL $= 100$ mL of vinegar (note that $300:100 = 3:1$).
If the volume of vinegar is doubled, the new volume of vinegar is 2×100 mL $= 200$ mL, and so the new ratio of oil to vinegar is $300:200 = 3:2$.
Note: We chose to begin with 300 mL of oil, but performing the above calculations with any starting volume of oil will give the same $3:2$ ratio.

Answer: (A)

9. Before including tax, the combined cost of the three items is $\$4.20 + \$7.60 + \$3.20 = \$15.00$.
The 5% tax on \$15.00 is $0.05 \times \$15.00 = \$0.75$, and so the total cost of the three items, including tax, is $\$15.00 + \$0.75 = \$15.75$.
Note that we could have calculated the 5% tax on each individual item, however doing so is less efficient than calculating tax on the \$15.00 total.

Answer: (D)

10. When $(1, 3)$ is reflected in the y-axis, the reflected point is $(-1, 3)$.
In general, when a point is reflected in the y-axis, its x-coordinate changes sign, and its y-coordinate does not change.
Thus, the vertices of the reflected rectangle are $(-1, 3)$, $(-1, 1)$, $(-4, 1)$, and $(-4, 3)$.
Of the given possibilities, $(-3, 4)$ is not a vertex of the reflected rectangle.

Answer: (C)

11. In the leftmost rectangle, the length of the path along the rectangle's diagonal, d, and the sides with lengths 3 and 4, form a right-angled triangle.
Using the Pythagorean Theorem, we get $d^2 = 3^2 + 4^2$, and so $d = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ (these are "3-4-5" right-angled triangles).
The path from A to B includes one such diagonal, two vertical sides each of length 4, and three horizontal sides each of length 3, and thus has length $5 + (2 \times 4) + (3 \times 3) = 22$.

Answer: (A)

12. Since $\angle PQR$ is a straight angle, its measure is 180°, and so $\angle SQR = 180^\circ - 125^\circ = 55^\circ$.
Since $SQ = SR$, then $\angle SRQ = \angle SQR = 55^\circ$.
The sum of the angles inside $\triangle SQR$ is 180°, and so $x = 180 - 55 - 55 = 70$.

Answer: (B)

13. The number of peaches in the original pile is two more than a multiple of three.
Of the choices given, 29 is the only number which is two more than a multiple of three ($29 = 3 \times 9 + 2$).

Answer: (D)

14. The sum of each block of 5 repeating integers is $4 - 3 + 2 - 1 + 0 = 2$.
In the first 23 integers, there are 4 such blocks of 5 integers, plus 3 additional integers (since $23 = 4 \times 5 + 3$).
The sum of the 4 blocks of 5 repeating integers is $4 \times 2 = 8$, and the next three integers in the list following these 4 blocks are $4, -3, 2$.
Thus, the sum of the first 23 integers is $8 + 4 - 3 + 2 = 11$.

Answer: (D)

15. The circumference of each of Bindu's bike tires is $2 \times \pi \times 30$ cm $= 60\pi$ cm.
If the bike tires rotate exactly five times, then the distance travelled by Bindu's bike is $5 \times 60\pi$ cm $= 300\pi$ cm.

ANSWER: (D)

16. *Solution 1*
The sum of all 8 numbers in the list is $41 + 35 + 19 + 9 + 26 + 45 + 13 + 28 = 216$.
When the 8 numbers are arranged in pairs, there are 4 pairs.
The sum of the numbers in each pair is the same, and so this sum is $\frac{216}{4} = 54$.
Therefore, the number paired with 13 is $54 - 13 = 41$.
Note: We may confirm that $45 + 9 = 54$, $41 + 13 = 54$, $35 + 19 = 54$, and $28 + 26 = 54$.

Solution 2
Since the sum of the numbers in each pair is the same, then the largest number in the list must be paired with the smallest number, the second largest with the second smallest, and so on.
(Can you reason why this must be true?)
That is, the largest and smallest numbers in the list, 45 and 9, must be paired.
The second largest and second smallest numbers, 41 and 13, must be paired, and so the number paired with 13 is 41.
Note: We may confirm that $45 + 9 = 54$, $41 + 13 = 54$, $35 + 19 = 54$, and $28 + 26 = 54$.

ANSWER: (E)

17. The mean (average) is determined by adding the 30 recorded temperatures, and dividing the sum by 30.
The sum of the temperatures for the first 25 days was $25 \times 21^\circ\text{C} = 525^\circ\text{C}$.
The sum of the temperatures for the last 5 days was $5 \times 15^\circ\text{C} = 75^\circ\text{C}$.
Thus, the mean of the recorded temperatures was $\frac{525^\circ\text{C} + 75^\circ\text{C}}{30} = \frac{600^\circ\text{C}}{30} = 20^\circ\text{C}$.

ANSWER: (C)

18. We begin by listing, in order, the smallest 2-digit positive divisors of 630.
We note that by first writing 630 as a product of its prime factors ($630 = 2 \times 3^2 \times 5 \times 7$), it may be easier to determine these divisors.
The smallest five 2-digit positive divisors of 630 are $10, 14, 15, 18$, and 21.
The next largest 2-digit positive divisor of 630 is 30, and we notice that $21 \times 30 = 630$.
That is, 21 and 30 are a pair of 2-digit positive integers whose product is 630, and they are consecutive in the ordered list of positive divisors.
Thus, each of $10, 14, 15$, and 18 must be paired with a divisor of 630 that is greater than 30.
We may check that the divisors that pair with each of $10, 14, 15$, and 18 is a 2-digit positive integer by dividing 630 by the smaller divisor.
That is, $\frac{630}{18} = 35$, $\frac{630}{15} = 42$, $\frac{630}{14} = 45$, and $\frac{630}{10} = 63$.
Thus, the pairs of 2-digit positive integers whose product is 630 are 21 and 30, 18 and 35, 15 and 42, 14 and 45, and 10 and 63, and so there are 5 such pairs.

ANSWER: (D)

19. Between 9 a.m. and 10 a.m., Ryan cut $\frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{4}{8} = \frac{3}{8}$ of his lawn.
Ryan cut $\frac{3}{8}$ of his lawn in 1 hour (60 minutes), and so he cut $\frac{1}{8}$ of his lawn in $\frac{60}{3}$ minutes or 20 minutes.
At 10 a.m., Ryan had cut $\frac{7}{8}$ of his lawn, and thus had $1 - \frac{7}{8} = \frac{1}{8}$ of his lawn left to cut.
Since Ryan cuts $\frac{1}{8}$ of his lawn in 20 minutes, then he finished at 10:20 a.m.

ANSWER: (C)

20. Begin by placing four tiles in the squares of the first row.
The only restriction is that the row must contain one tile of each colour.
Thus in the first row, there are 4 choices of tile colour for the first column, 3 choices for the second, 2 for the third, and 1 choice for the fourth.
That is, there are $4 \times 3 \times 2 \times 1 = 24$ different ways to cover the squares in the first row using one tile of each of the four colours.
For example, using R for red, B for black, G for green, and Y for yellow, the first row could contain tiles coloured $G\ \ Y\ \ R\ \ B$, in that order.
Suppose that the first row does contain tiles coloured $G\ \ Y\ \ R\ \ B$, in that order.
We will show that there is only one way to arrange the remaining 12 tiles in the grid.

Consider the first square in the second row, that is, the square directly below the tile coloured G.
The tile placed in this square cannot be coloured G since it shares an edge with the tile coloured G in row 1.
Also, the tile placed in this square cannot be coloured Y since it touches the corner of the square containing the tile coloured Y in row 1.
Assume that the tile in the first square of row 2 is coloured B, as shown.

G	Y	R	B
B			

Next, consider the colour of the tiles that could be placed in the second square of row 2.
The tile in this square cannot be coloured Y since it shares an edge with the tile coloured Y in row 1.
Also, the tile in this square cannot be coloured G since it touches the corner of the square containing the tile coloured G in row 1.
Further, the tile in this square cannot be coloured R since it touches the corner of the square containing the tile coloured R in row 1.
Since the first square in this row contains a tile coloured B, then we have no possible tile that can be placed in the second square of row 2.
This means that the tile in the first square of row 2 cannot be coloured B, and thus it must be coloured R, as shown.

G	Y	R	B
R			

The tile in the second square of row 2 cannot be coloured Y or G (as noted earlier), and thus must be coloured B.
Continuing to move right along row 2, the next tile cannot be coloured Y since it touches the corner of the square containing the tile coloured Y in row 1, and so the tile in this square must be coloured G, with the final tile in the row being coloured Y.

That is, the positions of the 4 tiles in row 2 are completely determined by the tiles in row 1.
Thus for each of the 24 different ways to place the tiles in row 1, there is exactly one way to place the tiles in row 2.
Repeating the argument, the same is then true for the tiles in row 3 and row 4; that is, there

is exactly one choice for the location of each of the coloured tiles within each of these two rows as well.

The 4×4 in our example above is completed here, as shown.

You should justify for yourself that each of rows 3 and 4 must contain tiles exactly as shown.

For each of the 24 different ways to cover the squares in the first row using one tile of each of the four colours, there is exactly one way to cover all remaining squares in the grid.

Thus, there are 24 different ways that the tiles can be arranged.

G	Y	R	B
R	B	G	Y
G	Y	R	B
R	B	G	Y

ANSWER: (B)

21. Since OM is a radius of the circle, then $OM = 87$.
$\triangle MNO$ is a right-angled triangle, and so by the Pythagorean Theorem, we get $OM^2 = MN^2 + NO^2$ or $87^2 = 63^2 + NO^2$, and so $NO^2 = 87^2 - 63^2 = 3600$.
Since $NO > 0$, then $NO = \sqrt{3600} = 60$.

Since OP is also a radius, then $OP = 87$, and so $NP = NO + OP = 60 + 87 = 147$.
The area of $\triangle PMN$ is equal to $\frac{1}{2} \times NP \times MN = \frac{1}{2} \times 147 \times 63 = 4630.5$.

ANSWER: (D)

22. Nasrin's mean (average) speed is determined by dividing the total distance travelled, which is 9 km, by the total time.
It took Nasrin 2 hours and thirty minutes, or 150 minutes, to canoe into her camp.
On the return trip, it took her $\dfrac{1}{3} \times 150$ minutes or 50 minutes.
Thus, the total time for Nasrin to paddle to camp and back was 200 minutes.
Converting to hours, 200 minutes is 3 hours and 20 minutes, and since 20 minutes is $\dfrac{20}{60} = \dfrac{1}{3}$ hours, it took Nasrin $3\frac{1}{3}$ hours in total.
Thus, Nasrin's mean speed as she paddled to camp and back was $\dfrac{9 \text{ km}}{3\frac{1}{3} \text{ h}}$ or $\dfrac{9 \text{ km}}{\frac{10}{3} \text{ h}}$, which is equal to $9 \times \dfrac{3}{10}$ km/h $= \dfrac{27}{10}$ km/h $= 2.7$ km/h.

ANSWER: (E)

23. To begin, the volume of water in Cylinder B is $\pi \times (8 \text{ cm})^2 \times 50 \text{ cm} = 3200\pi \text{ cm}^3$.
After some water is poured from Cylinder B into Cylinder A, the total volume of water in the two cylinders will be $3200\pi \text{ cm}^3$ (since no water is lost).
Let h cm be the height of the water in each of the two cylinders when the height of the water in both cylinders is the same.
At this time, the volume of water in Cylinder B is $\pi \times (8 \text{ cm})^2 \times h \text{ cm} = 64\pi h \text{ cm}^3$.
At this time, the volume of water in Cylinder A is $\pi \times (6 \text{ cm})^2 \times h \text{ cm} = 36\pi h \text{ cm}^3$.
Thus, the total volume of water in the two cylinders is $64\pi h \text{ cm}^3 + 36\pi h \text{ cm}^3 = 100\pi h \text{ cm}^3$, and so $100\pi h = 3200\pi$ or $h = \dfrac{3200\pi}{100\pi} = 32$.
When the height of the water in both cylinders is the same, that height is 32 cm.

ANSWER: (C)

24. *Solution 1*

We begin by multiplying the given equation through by 20 to get $20 \times \frac{a}{4} + 20 \times \frac{b}{10} = 20 \times 7$, or $5a + 2b = 140$.

Since $5a = 140 - 2b$ and both 140 and $2b$ are even, then $5a$ is even, which means that a is even.

We start by trying $a = 20$ and $b = 20$ which is a solution, since $5 \times 20 + 2 \times 20 = 140$.

This pair for a and b satisfies all of the conditions except $a < b$.

We may find other solutions to the equation $5a + 2b = 140$ by adding two 5s and subtracting five 2s (this is the same as adding 10 and subtracting 10), or by subtracting two 5s and adding five 2s.

Adding two 5s is equivalent to increasing the value of a by 2.

Subtracting five 2s is equivalent to decreasing the value of b by 5.

Consider $a = 20 + 2 = 22$ and $b = 20 - 5 = 15$.

This is a solution since $5 \times 22 + 2 \times 15 = 140$.

However, in this case $a > b$ and every time we add two 5s and subtract five 2s, a becomes greater and b becomes smaller.

Thus, we need to go in the other direction.

Consider $a = 20 - 2 = 18$ and $b = 20 + 5 = 25$.

This is a solution since $5 \times 18 + 2 \times 25 = 140$.

Here, $a < b$ and $a + b = 43$, so this pair satisfies all of the conditions.

Next, consider $a = 18 - 2 = 16$ and $b = 25 + 5 = 30$.

This is a solution since $5 \times 16 + 2 \times 30 = 140$.

Here, $a < b$ and $a + b = 46$, so this pair satisfies all of the conditions.

Notice that the sum $a + b$ increases by 3 on each of these steps.

This means that doing this 17 more times gets us to $a = 16 - 17 \times 2 = -18$ and $b = 30 + 17 \times 5 = 115$.

This is a solution since $5 \times (-18) + 2 \times 115 = 140$.

Notice that it is still the case that $a < b$ and $a + b < 100$.

Repeating this process one more time, we get $a = -20$ and $b = 120$, which gives $a + b = 100$ and so there are no more pairs that work.

Since we know that a has to be even and we are considering all possible even values for a, there can be no other pairs that work.

In total, there are $1 + 1 + 17 = 19$ pairs of integers a and b that satisfy each of the given conditions and the given equation.

Solution 2

We begin by rearranging the given equation to isolate b.

Doing so, we get

$$\begin{aligned}
\frac{a}{4} + \frac{b}{10} &= 7 \\
\frac{b}{10} &= 7 - \frac{a}{4} \\
10 \times \frac{b}{10} &= 10 \times 7 - 10 \times \frac{a}{4} \qquad \text{(multiplying each term by 10)} \\
b &= 70 - \frac{10a}{4} \\
b &= 70 - \frac{5a}{2}
\end{aligned}$$

Since b is an integer, then $70 - \frac{5a}{2}$ is an integer, which means that $\frac{5a}{2}$ must be an integer.

Since 2 does not divide 5, then 2 must divide a and so a is even.

Since $a < b$ and $b = 70 - \frac{5a}{2}$, then

$$\begin{aligned}
a &< 70 - \frac{5a}{2} \\
2 \times a &< 2 \times 70 - 2 \times \frac{5a}{2} \qquad \text{(multiplying each term by 2)} \\
2a &< 140 - 5a \\
7a &< 140 \\
a &< 20
\end{aligned}$$

Further, since $a + b < 100$ and $b = 70 - \frac{5a}{2}$, then

$$\begin{aligned}
a + 70 - \frac{5a}{2} &< 100 \\
2 \times a + 2 \times 70 - 2 \times \frac{5a}{2} &< 2 \times 100 \qquad \text{(multiplying each term by 2)} \\
2a + 140 - 5a &< 200 \\
-60 &< 3a \\
-20 &< a
\end{aligned}$$

Thus a is an even integer that is greater than -20 and less than 20.
Since there are 19 even integers from -18 to 18 inclusive, we suspect that there are 19 pairs of integers a and b that satisfy the given equation.
It is a good idea (and good practice) to at least check that the largest and smallest of these values of a do indeed satisfy each of the given conditions.

When $a = -18$, we get $b = 70 - \frac{5(-18)}{2}$ or $b = 70 - 5(-9)$, and so $b = 115$.
This pair satisfies the given conditions that $a < b$ and $a + b < 100$.
Substituting $a = -18$ and $b = 115$ into the given equation, we get

$$\frac{a}{4} + \frac{b}{10} = \frac{-18}{4} + \frac{115}{10} = \frac{-9}{2} + \frac{23}{2} = \frac{14}{2} = 7,$$

and thus $a = -18$ and $b = 115$ is a solution.

When $a = 18$, we get $b = 70 - \frac{5(18)}{2}$ or $b = 70 - 5(9)$, and so $b = 25$.
This satisfies the given conditions that $a < b$ and $a + b < 100$.
Substituting $a = 18$ and $b = 25$ into the given equation, we get

$$\frac{a}{4} + \frac{b}{10} = \frac{18}{4} + \frac{25}{10} = \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7,$$

and thus $a = 18$ and $b = 25$ is also a solution.

At this point we can be confident that for each of the 19 even integer values of a from -18 to 18 inclusive, there is an integer b for which the pair of integers a and b satisfy each of the given conditions and the given equation.

Answer: (B)

25. For any triangle, the sum of the lengths of two sides is always greater than the length of the third side. This property is known as the *triangle inequality*.
If for example the side lengths of a triangle are a, b and c, then the triangle inequality says that

$$a + b > c \text{ and } a + c > b \text{ and } b + c > a.$$

We begin by considering the number of different ways to choose three integers from $3, 4, 10, 13$ (without using n), and then forming a triangle whose side lengths are equal to those integers.
Consider choosing the integers $3, 4, 10$.
Since $3 + 4 < 10$, then it is not possible to form a triangle whose side lengths are $3, 4, 10$.
Consider choosing the integers $3, 4, 13$.
Since $3 + 4 < 13$, then it is not possible to form a triangle whose side lengths are $3, 4, 13$.
Consider choosing the integers $3, 10, 13$.
Since $3 + 10 = 13$, then it is not possible to form a triangle whose side lengths are $3, 10, 13$.
However, the remaining possible choice of three side lengths, $4, 10, 13$, does satisfy the triangle inequality since $4 + 10 > 13$ and $4 + 13 > 10$ and $10 + 13 > 4$.
Thus, without using the value of n, there is exactly one way to choose three integers and form a triangle whose side lengths are equal to those integers.
This means that we need to determine values of n for which there are exactly three different ways to choose two of the integers $3, 4, 10, 13$ and form a triangle whose side lengths are equal to those two integers and n.
There are six possible ways to choose two integers from the list $3, 4, 10, 13$.
Thus for each value of n, the triangles we need to consider have side lengths: $3, 4, n$ or $3, 10, n$ or $3, 13, n$ or $4, 10, n$ or $4, 13, n$ or $10, 13, n$.
For each value of n, we need exactly three of these six triangles to satisfy the triangle inequality.
Next, we use the triangle inequality to determine the restrictions on n for each of the six possible groups of triangles.

In a triangle with side lengths $3, 4, n$, we get $3 + n > 4$ or $n > 1$, and $3 + 4 > n$ or $n < 7$, and $4 + n > 3$ or $n > -1$.
To satisfy all three inequalities, n must be greater than 1 and less than 7.
Thus, the possible values of n for which a triangle has side lengths $3, 4, n$ are $n = 2, 3, 4, 5, 6$.
Since n must be different from all other numbers in the list, then $n = 2, 5, 6$.

In a triangle with side lengths $3, 10, n$, we get $3 + n > 10$ or $n > 7$, and $3 + 10 > n$ or $n < 13$, and $10 + n > 3$ or $n > -7$.
To satisfy all three inequalities, n must be greater than 7 and less than 13.
Thus, the possible values of n for which a triangle has side lengths $3, 10, n$ are $n = 8, 9, 11, 12$ ($n \neq 10$ since 10 is in the list).

In a triangle with side lengths $3, 13, n$, we get $3 + n > 13$ or $n > 10$, and $3 + 13 > n$ or $n < 16$, and $13 + n > 3$ or $n > -10$.
To satisfy all three inequalities, n must be greater than 10 and less than 16.
Thus, the possible values of n for which a triangle has side lengths $3, 13, n$ are $n = 11, 12, 14, 15$ ($n \neq 13$ since 13 is in the list).

In a triangle with side lengths $4, 10, n$, we get $4 + n > 10$ or $n > 6$, and $4 + 10 > n$ or $n < 14$, and $10 + n > 4$ or $n > -6$.
To satisfy all three inequalities, n must be greater than 6 and less than 14.
Thus, the possible values of n for which a triangle has side lengths $4, 10, n$ are $n = 7, 8, 9, 11, 12$.

In a triangle with side lengths $4, 13, n$, we get $4 + n > 13$ or $n > 9$, and $4 + 13 > n$ or $n < 17$, and $13 + n > 4$ or $n > -9$.
To satisfy all three inequalities, n must be greater than 9 and less than 17.

Thus, the possible values of n for which a triangle has side lengths $4, 13, n$ are $n = 11, 12, 14, 15, 16$.

In a triangle with side lengths $10, 13, n$, we get $10+n > 13$ or $n > 3$, and $10+13 > n$ or $n < 23$, and $13 + n > 10$ or $n > -3$.
To satisfy all three inequalities, n must be greater than 3 and less than 23.
Thus, the possible values of n for which a triangle has side lengths $10, 13, n$ are $n = 5, 6, 7, 8, 9, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22$.

Recall that for each value of n, we need exactly three of the six triangles to satisfy the triangle inequality (the triangle with side lengths $4, 10, 13$ is the fourth).
Clearly for values of n less than 7, there are too few triangles, and similarly for values of n greater than 16, there are also too few triangles. (There are at most two triangles in each of these two cases.)
In the table below, we summarize our work by placing a checkmark if the triangle satisfies the triangle inequality and then counting the number of such triangles.

n	$(3, 4, n)$	$(3, 10, n)$	$(3, 13, n)$	$(4, 10, n)$	$(4, 13, n)$	$(10, 13, n)$	$(4, 10, 13)$	number of triangles
7				✓		✓	✓	3
8		✓		✓		✓	✓	4
9		✓		✓		✓	✓	4
11		✓	✓	✓	✓	✓	✓	6
12		✓	✓	✓	✓	✓	✓	6
14			✓		✓	✓	✓	4
15			✓		✓	✓	✓	4
16					✓	✓	✓	3

Therefore, there are exactly four different values of n that satisfy the given conditions, and the sum of these values of n is $8 + 9 + 14 + 15 = 46$.

Answer: (A)