



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Wednesday, February 22, 2023
(in North America and South America)

Thursday, February 23, 2023
(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 60 minutes

©2022 University of Waterloo

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as “7”) must be coded with a leading zero (“07”).
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

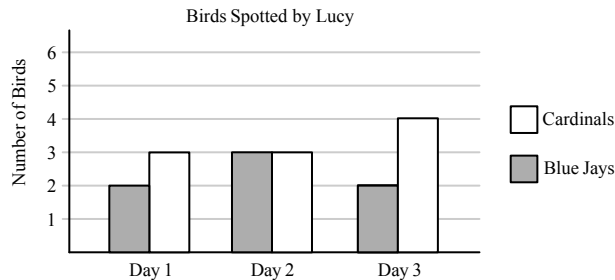
Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

- What is the value of $\frac{1}{1} + \frac{2}{2} + \frac{3}{3}$?
 (A) 1 (B) 0 (C) 3 (D) 9 (E) 10
- If $3n = 9 + 9 + 9$, then n equals
 (A) 3 (B) 6 (C) 18 (D) 9 (E) 27
- One movie is 1 hour and 48 minutes long. A second movie is 25 minutes longer than the first. How long is the second movie?
 (A) 2 hours and 13 minutes
 (B) 1 hour and 48 minutes
 (C) 2 hours and 25 minutes
 (D) 2 hours and 3 minutes
 (E) 2 hours and 48 minutes
- Lucy tracked the number of cardinals and the number of blue jays that she saw each day for three days. The graph below summarizes the birds that she saw.



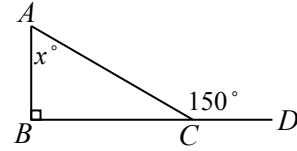
How many more cardinals than blue jays did she see over the three days?

- (A) 7 (B) 3 (C) 6 (D) 8 (E) 1
- The number 2023 is written on one side of a classroom window using the style of numbers shown. If you were to stand on the other side of the window, how would the number appear?



- (A) 3202
 (B) 5056
 (C) 6505
 (D) 5053
 (E) 3032

6. In the diagram, points B , C and D lie on a line. Also, $\angle ABC = 90^\circ$ and $\angle ACD = 150^\circ$. The value of x is
- (A) 30 (B) 45 (C) 90
(D) 150 (E) 60



7. The surface area of a cube is 24. The volume of the cube is
- (A) 4 (B) $3\sqrt{3}$ (C) 9 (D) 16 (E) 8
8. Charlie is making a necklace with yellow beads and green beads. She has already used 4 green beads and 0 yellow beads. How many yellow beads will she have to add so that $\frac{4}{5}$ of the total number of beads are yellow?
- (A) 16 (B) 4 (C) 20 (D) 24 (E) 12
9. A positive number is increased by 60%. By what percentage should the result be decreased to return to the original value?
- (A) 57.5% (B) 40% (C) 62.5% (D) 50% (E) 37.5%
10. Each of five doors is randomly either open or closed. What is the probability that exactly two of the five doors are open?
- (A) $\frac{5}{16}$ (B) $\frac{13}{32}$ (C) $\frac{9}{25}$ (D) $\frac{1}{2}$ (E) $\frac{7}{16}$

Part B: Each correct answer is worth 6.

11. Karim has 23 candies. He eats n candies and divides the remaining candies equally among his three children so that each child gets an integer number of candies. Which of the following is not a possible value of n ?
- (A) 2 (B) 5 (C) 9 (D) 11 (E) 14
12. A 6 m by 8 m rectangular field has a fence around it. There is a post at each of the four corners of the field. Starting at each corner, there is a post every 2 m along each side of the fence. How many posts are there?
- (A) 12 (B) 14 (C) 16 (D) 18 (E) 20
13. The integer 2023 is equal to 7×17^2 . Which of the following is the smallest positive perfect square that is a multiple of 2023?
- (A) 2023^2 (B) 4×2023 (C) 7×2023 (D) 17×2023 (E) $7 \times 17 \times 2023$
14. Points A , B , C , and D are on a line in that order.
The distance from A to D is 24.
The distance from B to D is 3 times the distance from A to B .
Point C is halfway between B and D .
What is the distance from A to C ?
- (A) 12 (B) 8 (C) 16 (D) 9 (E) 15

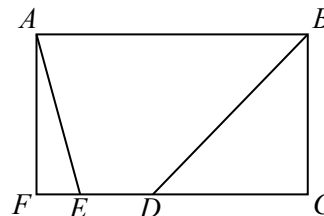
15. Suppose that $a = \frac{1}{n}$, where n is a positive integer with $n > 1$.

Which of the following statements is true?

- (A) $a < \frac{1}{a} < a^2$ (B) $a^2 < a < \frac{1}{a}$ (C) $a < a^2 < \frac{1}{a}$
 (D) $\frac{1}{a} < a < a^2$ (E) $\frac{1}{a} < a^2 < a$

16. In the diagram, $ABCF$ is a rectangle with $AB = 30$ cm and $AF = 14$ cm. Points E and D are on FC so that $FE = 5$ cm and the area of quadrilateral $ABDE$ is 266 cm². The length of DC is

- (A) 17 cm (B) 16 cm (C) 19 cm
 (D) 20 cm (E) 18 cm



17. Megan and Hana raced their remote control cars for 100 m.

The two cars started at the same time.

The average speed of Megan's car was $\frac{5}{4}$ m/s.

Hana's car finished 5 seconds before Megan's car.

What was the average speed of Hana's car?

- (A) $\frac{4}{5}$ m/s (B) $\frac{5}{2}$ m/s (C) $\frac{4}{3}$ m/s (D) $\frac{6}{5}$ m/s (E) $\frac{3}{2}$ m/s

18. A factory makes chocolate bars. Five boxes, labelled V, W, X, Y, Z , are each packed with 20 bars. Each of the bars in three of the boxes has a mass of 100 g. Each of the bars in the other two boxes has a mass of 90 g. One bar is taken from box V , two bars are taken from box W , four bars are taken from box X , eight bars are taken from box Y , and sixteen bars are taken from box Z . The total mass of these bars taken from the boxes is 2920 g. The boxes containing the 90 g bars are labelled

- (A) V and W (B) W and Z (C) X and Y (D) V and Z (E) W and Y

19. The average of a, b and c is 16. The average of c, d and e is 26. The average of $a, b, c, d,$ and e is 20. The value of c is

- (A) 10 (B) 20 (C) 21 (D) 26 (E) 30

20. A robotic grasshopper jumps 1 cm to the east, then 2 cm to the north, then 3 cm to the west, then 4 cm to the south. After every fourth jump, the grasshopper restarts the sequence of jumps: 1 cm to the east, then 2 cm to the north, then 3 cm to the west, then 4 cm to the south. After a total of n jumps, the position of the grasshopper is 162 cm to the west and 158 cm to the south of its original position. The sum of the squares of the digits of n is

- (A) 22 (B) 29 (C) 17 (D) 14 (E) 13

Part C: Each correct answer is worth 8.

Each correct answer is an integer from 0 to 99, inclusive.

A one-digit answer (such as “7”) must be coded with a leading zero (“07”).

Note: The integer formed by the rightmost two digits of 12 345 is 45.

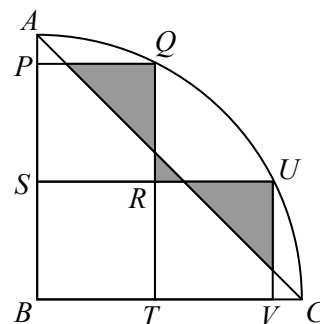
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. A line has equation $y = mx - 50$ for some positive integer m . The line passes through the point $(a, 0)$ for some positive integer a . What is the sum of all possible values of m ?
22. The integers 1, 2, 4, 5, 6, 9, 10, 11, 13 are to be placed in the circles and squares below with one number in each shape.



Each integer must be used exactly once and the integer in each circle must be equal to the sum of the integers in the two neighbouring squares. If the integer x is placed in the leftmost square and the integer y is placed in the rightmost square, what is the largest possible value of $x + y$?

23. In the diagram, ABC is a quarter-circle centred at B . Each of square $PQRS$, square $SRTB$ and square $RUVT$ has side length 10. Points P and S are on AB , points T and V are on BC , and points Q and U are on the quarter-circle. Line segment AC is drawn. Three triangular regions are shaded, as shown. What is the integer closest to the total area of the shaded regions?



24. Carina is in a tournament in which no game can end in a tie. She continues to play games until she loses 2 games, at which point she is eliminated and plays no more games. The probability of Carina winning the first game is $\frac{1}{2}$. After she wins a game, the probability of Carina winning the next game is $\frac{3}{4}$. After she loses a game, the probability of Carina winning the next game is $\frac{1}{3}$. The probability that Carina wins 3 games before being eliminated from the tournament equals $\frac{a}{b}$, where the fraction $\frac{a}{b}$ is in lowest terms. What is the value of $a + b$?
25. A set consists of five different odd positive integers, each greater than 2. When these five integers are multiplied together, their product is a five-digit integer of the form $AB0AB$, where A and B are digits with $A \neq 0$ and $A \neq B$. (The hundreds digit of the product is zero.) For example, the integers in the set $\{3, 5, 7, 13, 33\}$ have a product of 45045. In total, how many different sets of five different odd positive integers have these properties?



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

For students...

Thank you for writing the 2023 Cayley Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website cemc.uwaterloo.ca to find

- More information about the Galois Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results