The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Canadian Intermediate Mathematics Contest

Wednesday, November 15, 2023
(in North America and South America)

Thursday, November 16, 2023
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 2 hours

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Do not open this booklet until instructed to do so.
There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Write your name, school name, and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

Canadian Intermediate Mathematics Contest

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

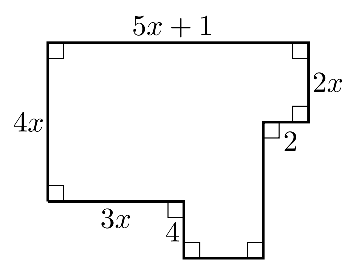
PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

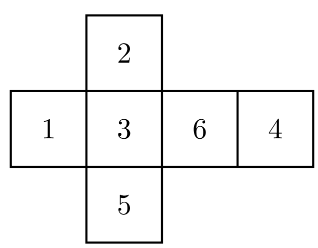
1. A bus leaves the station at exactly 7:43 a.m. and arrives at its destination at exactly 8:22 a.m. on the same day. How long, in minutes, was the bus trip?

2. A new mathematical operation, $\blacklozenge$, is defined by $a \blacklozenge b = \dfrac{2a}{b}$.
   For example, $6 \blacklozenge 3 = \dfrac{2 \times 6}{3} = 4$. If $a \blacklozenge 4 = 18$, what is the value of a?

3. The figure shown is formed using eight line segments. Whenever two of these line segments meet, they form a right angle. The perimeter of the figure is 82. What is the value of x?



4. A net of a cube is shown with one integer on each face. A larger cube is constructed using 27 copies of this cube. What is the minimum possible sum of all of the integers showing on the six faces of the larger cube?

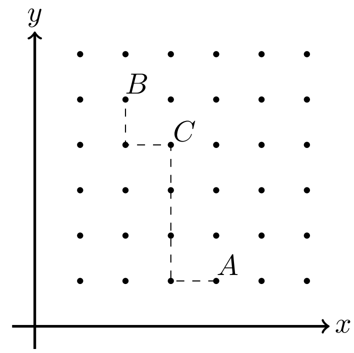


5. A hiking trail leaves from point P. The trail consists of a flat section from point P to point Q, followed by an uphill section from Q to a viewing platform at point R.

   A hiker walked from P to Q to R and back from R to Q to P. The hiker's speed was 4 km/h on the flat section in both directions, 3 km/h while walking uphill, and 6 km/h while walking downhill. If the hiker left P at 1:00 p.m., spent exactly 1 hour at R, and returned to P at 8:00 p.m. on the same day, what was the total distance walked by the hiker?

6. Consider the two points $A(4,1)$ and $B(2,5)$. For each point C with positive integer coordinates, we define d_C to be the shortest distance needed to travel from A to C to B moving only horizontally and/or vertically. For example, for the point $C(3,4)$, we compute d_C as follows:

To get from A to C moving only horizontally and/or vertically, we can move 1 unit to the left then 3 units up for a total distance of $1+3=4$. (There are other such ways to get from A to C, but no shorter ways.) The shortest path from C to B moving only horizontally and/or vertically is to move 1 unit left and then 1 unit up (or 1 unit up then 1 unit left) for a total distance of $1+1=2$. Thus, for $C(3,4)$, we have that $d_C = 4+2=6$. This is demonstrated in the diagram.

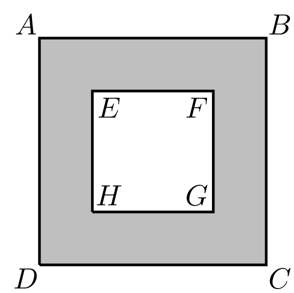


The positive integer N has the property that there are exactly 2023 points $C(x,y)$ with $x>0$ and $y>0$ and $d_C=N$. What is the value of N?

PART B

For each question in Part B, your solution must be well-organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. In the diagram, square $EFGH$ is centred inside square $ABCD$ and the region between the two squares is shaded.



   (a) Determine the area of the shaded region if the length of AB is 10 cm and the length of EF is 8 cm.

   (b) Determine the length of EF if the length of AB is 13 cm and the area of the shaded region is 120 cm^2.

   (c) Determine the length of EF if the length of AB is 18 cm and the area of the shaded region is $\frac{3}{4}$ of the area of $ABCD$.

   (d) Suppose that the area of the shaded region is 64% the area of $ABCD$. If the length of AB is a cm and the length of EF is b cm, determine the value of $\frac{a}{b}$.

2. For a four-digit positive integer $ABCD$, where A, B, C, and D are digits with $A \neq 0$ and $D \neq 0$, we define the *Reverse Digit sum* (RD sum) of $ABCD$ to be the sum of $ABCD$ and $DCBA$. For example, the RD sum of 1205 is $1205 + 5021 = 6226$, while the integer 2300 does not have an RD sum.

   Note that the four-digit positive integer $ABCD$ is equal to $1000A + 100B + 10C + D$.

   (a) Determine the RD sum of 4281.

   (b) There are positive integers m and n with the property that the RD sum of the integer $ABCD$ is always equal to $m \times (A + D) + n \times (B + C)$. State the value of m and the value of n.

   (c) Determine the number of four-digit integers whose RD sum is 3883.

   (d) Determine the number of four-digit integers that are equal to the RD sum of a four-digit integer.

3. The positive integers are written into rows so that Row n includes every integer m with the following properties:

   (i) m is a multiple of n,

   (ii) $m \leq n^2$, and

   (iii) m is not in an earlier row.

   The table below shows the first six rows.

Row 1	1
Row 2	2, 4
Row 3	3, 6, 9
Row 4	8, 12, 16
Row 5	5, 10, 15, 20, 25
Row 6	18, 24, 30, 36

   (a) Determine the smallest integer in Row 10.

   (b) Show that, for all positive integers $n \geq 3$, Row n includes each of $n^2 - n$ and $n^2 - 2n$.

   (c) Determine the largest positive integer n with the property that Row n does not include $n^2 - 10n$.