Problem of the Month
Problem 3: December 2022

This month's problem is an extension of Problem 6 from the November 2022 Canadian Senior Mathematics Contest. Here is the original problem.

A bag contains exactly 15 marbles of which 3 are red, 5 are blue, and 7 are green. The marbles are chosen at random and removed one at a time from the bag until all of the marbles are removed. One colour of marble is the first to have 0 remaining in the bag. What is the probability that this colour is red?

Note: It might be useful to familiarize yourself with the notation of *binomial coefficients* before attempting this problem.

(a) Suppose there are r red marbles and b blue marbles. As in the original problem, the marbles are chosen at random and removed from the bag one at a time until all marbles are removed. One colour of marble is the first to have 0 marbles remaining in the bag. What is the probability that this colour is red?

(b) Suppose there are r red marbles, b blue marbles, and g green marbles. The marbles are chosen at random and removed one at a time until all marbles are removed. What is the probability that red is the colour of marble that is first to be completely removed from the bag?

(c) Suppose there are r red marbles, b blue marbles, and g green marbles with $r < b < g$. Let $p(r)$ be the probability that the red marbles are the first to be completely removed from the bag and define $p(b)$ and $p(g)$ similarly. Determine which of $p(r)$, $p(b)$, and $p(g)$ is the smallest and which is the largest. Does the result agree with your intuition?

(d) Show that the values of $p(r)$, $p(b)$, and $p(g)$ depend only on the proportions of r, b, and g to the total number of marbles. For example, if one bag has r red, b blue, and g green marbles and another has $7r$ red, $7b$ blue, and $7g$ green marbles, then the probability that the red are removed first is the same for both bags.